Handbook of geomatics. 2 Volumes.

Recently, the search for efficient strategies of protections against threats of a changing Earth, and the exceptional situation of getting terrestrial, airborne as well as spaceborne data of ever better quality, explain the strong need for new mathematical descriptions, tools and methods, i.e., geomatics describing the complex Earth system, a system comprising besides mathematics also the knowledge of informatics, physics, chemistry, biology, and also economics and geography.

The present exhaustive work consists of six parts written by 78 internationally distinguished scientists. Volume 1 contains general issues, key technologies, data acquisition, and modeling the system Earth. Volume 2 is also dedicated to modeling, it considers analytical and statistical methods, computational and numerical algorithms. With this work the authors contribute significantly to the understanding of the planet Earth, its climate and environment, and the expected shortage of natural resources. The book offers the key parameters for the study of the Earth’s dynamics such that interactions of its solid part with ice, oceans, and the atmosphere become more and more accessible.

The book starts with an introductory part (chapters 1 and 2) on the role, the aims and the potential of geomatics. Here geomatics is exemplified by two classical problems, the determination of the gravity field from terrestrial deflections from the vertical, and ocean flow modeling from satellite (altimeter measured) ocean topography. It is underlined that geomatics is no new science. It began as mankind realized that it walks across a sphere-like Earth and that this has to be taken into account in measurements and computations. So Erastothenes may be seen as an early geomatician as he tried to determine the circumference of the Earth by measurements of the Sun’s position. A review of the use of magnets for navigational purposes gives a deep insight into the nature of scientific inventions. Gilbert’s dip theory and the unhappy idea to link the geomagnetic latitude to the dip is a paradigm of what can go wrong in mathematical modeling. The computation of the dip table is, however, a brilliant piece of trigonometry by Henry Briggs.

Part 2 of the book deals with observational and measurement key technologies. Here first an overview on Earth observation satellites is given, describing the end-to-end elements of the missions. Some significant results obtained using data from European missions (ERS, Envisat) are provided. It is shown that the European Space Agency (ESA) Living Planet Programme is composed of two components, a science and research element in form of Earth Explorer missions, and an operational element known as Earth Watch designed to facilitate the delivery of Earth observational data for use in operational service. In chapter 4, the gravitational gravimetry in the GOCE satellite is explained. The aim of GOCE is the detailed determination of the spatial variations of the Earth’s gravitational field, with applications in oceanography, geophysics, geodesy, glaciology, and climatology. Several strategies exist for the determination of the gravity field at the Earth surface using measured tensor components at altitude. The geomagnetic field measured at the Earth’s surface or on board of satellites is the sum of contributions from many different sources below and above the Earth’s surface, which have different spatio-temporal properties. At the end of part 2 a general overview of these sources and observations is given.

The modeling of the system Earth, of the geosphere, cryosphere, hydrosphere, atmosphere, biosphere, and anthroposphere, is explained in part 3. The part starts with a chapter on classical physical geodesy, the science of the Earth and its gravitational field. At present, the accuracy of the determination of the Earth’s surface is $10^{-8} - 10^{-9}$ (that means a few cm globally). This requires that effects of special and general relativity, as well as the effects of a non-rigid surface, such as tides and irregularities in the Earth’s rotation, have to be taken into account. Since spherical harmonic series are the standard mathematical tool for representing the Earth’s gravitational field, their convergence is thoroughly studied. Further, the extensively used method of data combination to determine the geopotential in 3D-space, the least-squares collocation (developed as extension of least-squares gravity interpolation together with least squares adjustment by Krarup and others), is briefly presented. However, using ellipsoidal harmonics and ellipsoidal wavelets, approximations for planetary bodies and their gravity field indeed converge three times faster than descriptions by spherical harmonics and wavelets. Thus next, various effects, e.g. boundary-value problems, the gravity field, and vertical deflections in gravity and geometry spaces are treated considering the Earth to be ellipsoidal.

The final topic of chapter 7 starts with a review of the curvilinear datum problem referring to ellipsoidal harmonics (a seven-parameter transformation). A characteristic example in terms of ellipsoidal harmonics for an ellipsoid of revolution transformed to another one is given. A direct measurement of the amount of
mass that is redistributed at or near the surface of the Earth by oceanic and atmospheric circulation and through the hydrological cycle is possible observing the time variability of the gravity field. Thus, after reconsidering the relation between gravity and mass change, chapter 8 discusses the hypothetical surface mass change and its relation to the 3D deformation field of the elastic Earth. Then the inversion of the gravity field to density is mentioned.

Chapter 9 explains satellite gravity gradiometry as ultra-sensitive detection technique of the space gravitational gradient (i.e., the Hesse tensor of the Earth’s gravitational potential) from potential theoretic point of view, and as ill-posed pseudo-differential equation the solution of which is dealt with classical regularization methods. A very promising method is worked out for developing an immediate interrelation between the Earth’s gravitational potential at the Earth’s surface and the known gravitational tensor. Chapter 10 considers a compositionally and entropically stratified, compressible, rotating fluid Earth and studies gravitational-viscoelastic perturbations of its hydrostatic initial state.

A multiresolution analysis of temporal and spatial variations of the Earth’s gravitational potential by the use of tensor product wavelets which are built up by Legendre and spherical wavelets for the time and space domain, respectively, is presented in chapter 11. The multiresolution is performed for satellite and hydrological data, and based on the results correlation coefficients between both data sets are computed. This helps to develop a filter for the extraction of an improved hydrology model from the satellite data. Time varying costal and global sea levels are analysed in chapter 12. The variability of the trend of costal sea level appears to be related to the interannual time scale of variability of the Northern Atlantic oscillation and of the El-Niño-Southern Atlantic oscillation climatic indices.

The current status of large-scale ocean modeling on unstructured meshes is discussed in chapter 13 in the context of climate applications. The focus is on consistency and performance issues which are much easier to achieve with finite volume methods. Numerical methods in support of advanced tsunami early warning are presented in chapter 14. The behaviour of tsunami waves is usually modeled by simplifications of the Navier-Stokes equations.

In chapter 15, a brief overview is given on existing upscaling, multiscale, and multiphysics methods to describe transport processes in porous media including multiple fluid phases. Chapter 16 presents a short introduction into the basics of modern dynamo simulations. Then it discusses the fundamental force balances and addressed the question how well modern dynamo models reproduce the geomagnetic field. It is shown that only low Ekman number models seem to retain the huge dipole dominance of the geomagnetic field once the Rayleigh number has been increased to values where field reversals happen. These models also better reproduce the low-latitude fields found at Earth’s core-mantle boundary. Geomagnetic field models, derived from observational data, form the basis for the separation of magnetic fields produced by different sources and the extrapolation of the fields to places where they cannot be directly measured. Chapter 17 reviews the mathematical foundation of global magnetic field modeling. The spatial modeling of the field in spherical coordinates is especially considered. It is pointed out that the joint use of contemporary ground-based and satellite-born observations enables the modeling of magnetic fields originating above and below the Earth’s surface.

A report on recent applications of multiscale techniques to the modeling of geomagnetic problems is given in chapter 18. Two approaches are presented: a spherical harmonics-oriented one, using frequency packages, and a spatially-oriented one, using regularizations of the single layer kernel and Green’s function with respect to the Beltrami operator. The separation of the magnetic field with respect to interior and exterior sources and the reconstruction of radial ionospheric currents are especially considered. Detailed mathematical derivations of forward and adjoint sensitivity methods are presented in chapter 19 for the computing of the electromagnetic induction response of a 2D heterogeneous conducting sphere to a transient external electric current excitation. As application conductivity structures of the Earth are obtained based on CHAMP magnetic data. Atmospheric flows, e.g. the propagation of internal gravity or Rossby waves, baroclinic instabilities, cloud formation and moist convection, (anti-)cyclonic weather patterns, and hurricanes, possess length scales of $10^{-5} - 10^5$ m and time scales from microseconds to weeks and more. Chapter 20 demonstrates how many well-known reduced sets of model equations for specific scale-dependent atmospheric flow phenomena may be derived in a unified and transparent fashion from the full compressible atmospheric flow equations using standard techniques of formal asymptotics. It also discusses an example for the limitations of this approach.

An overview on modern numerical weather prediction is given in chapter 21. The chapter sketches the mathematical formulation of the underlying physical problem and its numerical treatment and gives an outlook on statistical weather forecasting. Special emphasis is given to the Kyrill event in order to demonstrate the application of the different methods. The aim of chapter 22 is the presentation of the main methods for mathematical modeling of reservoir detection and flow simulation related to geothermal projects. From
spring to summer period, a large number of lakes are laced with thick layers of algae representing a serious problem for the biological diversity of species.

Chapter 23 gives a survey on recent progress in modeling and numerical simulation of plankton spring bloom situations caused by eutrophication via phosphorus accumulation. Due to the underlying processes, shallow water equations are employed and coupled with additional equations describing biogeochemical processes of interest within both the water layer and the sediment. The dynamical simulations are based on a second-order finite volume scheme extended by a specific formulation of the modified Patankar approach to satisfy the natural requirements to be unconditionally positivity preserving as well as conservative due to stiff transition terms.

Volume 2 of the work starts with part 4 dealing with analytic, algebraic, and operator theoretical methods. The standard view of noise in ill-posed problems is that it is either deterministic and small (strongly bounded noise) or random and large (not necessarily small). Following Eggermont, LaRiccia and Nashed (2009), in chapter 24 a new noise model is investigated, wherein the noise is weakly bounded, that means local averages of the noise are small. Many geophysical imaging problems are ill-posed in the sense that the solution does not depend continuously on the measured data. Therefore their solutions cannot be computed directly, but instead require the application of regularization.

Chapter 25 reviews recent mathematical results for sparsity regularization, which often represents real objects much better than solutions with small $L^2$ norm. Quantitative remote sensing is an appropriate way to estimate structural parameters and spectral component signatures of Earth surface cover type. The problem for quantitative remote sensing is the inversion, a typically ill-posed problem without solution or with a solution of infinite dimension, or with a incontinuous solution. Chapter 26 addresses the theory and methods from the viewpoint that the quantitative remote sensing inverse problems can be represented by kernel-based operator equations and solved by coupling regularization and optimization methods.

The downward continuation of spaceborne gravity data is considered in chapter 27. The ill-posed nature of this problem is analysed and some approaches to its treatment are described. Thereat the chapter focuses on the multiparameter regularization approach. The next chapter develops the mathematical theory of correlation functions of the Earth's gravity field in classical geodesy, as well as covariance functions under a statistical interpretation of the field, for functions and processes on the sphere and plane, with formulation of the corresponding power spectral densities in the respective frequency domains, and with extensions into the third dimension for harmonic functions. Modeling the subsurface of the Earth can be widely reduced by means of the purpose or target response.

In chapter 28, a distance between two subsurface models is defined, which are created using different input parameters. Many of the Cartesian-based Earth modeling problems and methodologies are redefined, such as inverse modeling, stochastic simulation and estimation, model selection and screening, model updating and response uncertainty evaluation in metric space. The analyses lead to the conclusion that modeling does not require a Cartesian framework; a distance defining a metric is sufficient. In geosciences, one may be interested in spectrally modeling a time series defined only on a certain interval, or one wants to characterize specific geographical area observed using an effectively bandlimited measurement device. It is clear that analysing and representing data of this kind will be facilitated if a basis of functions can be found that are spacio-spectrally concentrated. Chapter 30 gives a theoretical overview of one particular approach to this concentration problem, as originally proposed for time series by Slepian and coworkers in 1960.

Chapter 31 reports on the current activities and recent progress in the field of special functions of mathematical geosciences. It is focused on trial systems of polynomial (i.e., spherical harmonics) and polynomially based (i.e., zonal kernel) types. The essential outcome is a better understanding of the constructive approximation in terms of zonal kernel functions such as splines and wavelets. A brief survey of three different approaches for the approximation of functions in the 3D-ball is presented in chapter 32: the expansion in an orthonormal (polynomial) basis, a reproducing kernel based spline interpolation/approximation, and a wavelet-based multiscale analysis. In addition, some geomathematical tomography problems are discussed as applications. Chapter 33 attempts to present a self-contained and comprehensive survey of the mathematics and physics of material behavior of rocks in terms of texture and anisotropy. The chapter is in particular an account of modern mathematical texture analysis, explicitly clarifying terms, providing definitions and justifying their application, and emphasizing its major insights. Mathematical texture is brought back to the realm of spherical Radon and Fourier transform, spherical approximation, spherical probability, i.e., to the mathematics of spherical topography.

The objective of chapter 34 is to highlight the current research activities and recent progress in the area of dimensionality reduction of hyperspectral geological/geographical imagery data, which are widely used in image sedimentation and feature classification. The focus is on four topics: hyperspectral image (HSI) data processing, similarity/dissimilarity definition of HSI data, construction of dimensionality reduction (DR)
Part 5 of the work deals with statistical and stochastic methods. First it is reported on the current state of weak solutions to oblique boundary problems for the Poisson equation. Deterministic as well as stochastic inhomogeneities are treated, and existence and uniqueness results for corresponding weak solutions are presented. Next, there are presented current activities and recent progress in the field of geodetic deformation analysis if a refined uncertainty budget is considered. This is meaningful in the context of a thorough system-theoretical assessment of geodetic monitoring and it leads to a more complete formulation of the modeling and analysis chain. It is focused on the mathematical modeling of an extended uncertainty budget, the adequate adaption of estimation and analysis methods, and the consequences for one outstanding step of geodetic deformation analysis, the test of a linear hypothesis. The promise of a future broader multi-frequency, multi-signal Global Navigation Satellite System (GNSS) has the potential of enabling a much wider range of demanding applications compared to the current GPS situation. A successful exploitation of GNSS properties (like multi-satellite system tracking, mm-level measurement precision, frequency diversity, and the integer ambiguities of the carrier phases) results in an accuracy improvement of the GNSS parameters of two orders of magnitude. The theory that underpins this ultraprecise GNSS parameter estimation, the theory of integer inference, is the topic of chapter 37.

Chapter 38 provides a space geodesy tutorial on mixed integer linear models. First real-valued and mixed integer linear models are classified, and then, accordingly the corresponding conventional and mixed integer least squares problems are defined. Integer unknown parameters are solved under a general framework of integer programming and represented from the statistical point of view. As a fundamental element of integer least squares estimator, the Voronoi cell is considered. The aim of chapter 39 is the statistical analysis of climate time series. The data sets analysed consist of monthly temperature means and monthly precipitation amounts gained at three German weather stations. Emphasis lies on the method of time series analysis, comprising plotting, modelling, and predicting climate values in the near future.

Computational and numerical methods are explained in part 6. There, chapter 40 gives a survey of the general problem of numerical integration on the sphere in $\mathbb{R}^3$. After the discussion of some basic facts about numerical integration rules with positive weights, special important rules are explained in detail: rules with a specified polynomial degree of precision, including longitude-latitude rules; rules using scattered data points; rules based on equal-area partitions; and rules for numerical integration over subsets of the sphere. The concept of multiscale approximations of functions by means of wavelet expansions is briefly recalled in chapter 41. A short review on the basic construction principles is given and the most important properties of wavelets such as characterizations of function spaces are discussed. It is explained how wavelets can be used in signal/image analysis, in particular for compression and denoising. The main goal of chapter 42 is the discussion of perspectives of applications based on solving underdetermined systems of linear equations (SLE). This includes the interconnection of underdetermined SLE with global problems of information theory and with data measuring and representation. The preference is given to the description of the hypothetic destination point of the undertaken efforts, the current status of the problem, and possible methods to overcome difficulties on the way to that destination point. Algorithms that compress 2D and 3D seismic data arrays are presented in chapter 43.

These arrays are piece-wise smooth in the horizontal direction and have oscillating events in the vertical direction. The transform part of the compression process is an algorithm that combines wavelet and local cosine transforms. The quantization and the entropy coding parts of the compression are taken from wavelet-based coding such as set partitioning in hierarchical trees and embedded zerotree wavelet encoding/decoding schemes that efficiently use the multiscale structure of the wavelet transform coefficients. Chapter 44 gives an overview on theoretical and technological developments that have shaped and reshaped maps and cartography. It analyzes the nature of cartography and introduces three fundamental transformations involved in the cartographic process from the real world to a mental reality in user’s brain. In chapter 45, the scientific background of geoinformatics is reflected and research issues are described, together with examples and an extensive list of references.

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