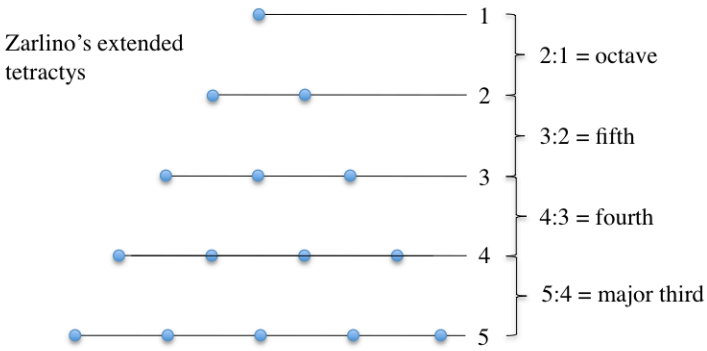

Fig. 2.6. The extension of the Pythagorean tetractys by amplification with a fifth row, yielding the new ratio 5 : 4 of the just-tuned major third interval.

there wasn't a standard tuning system between different instrument types such as strings, winds, and keyboards. The formula he discovered allowed pitch-pipes in an equal-tempered scale of twelve equal-ratio semitones per octave, which led to a revolution of music and physics.

The problem of how to modulate between different keys bothered many theorists, until Zhu solved it. This solution shows that music should not be separated from mathematics. Independently, only 150 years after Zhu's theory, in the 18th century, the equal-temperament became the basis of Western composition. Equal-tempered tuning is widely used today because it is the best tuning system for modulation in performance. Zhu commented on his theory of equal-temperament tuning that scholars would have to *be well acquainted both with acoustics and with mathematical calculation.* Fifty-two years after Zhu published this equal-tempered tuning system, theorist Père Mersenne discovered exactly the same principles, using knowledge of mathematics to solve music problems.

Fig. 2.7. Zaiyu Zhu (1536-1611).

Zhu inherited his peerage "Prince Zheng" in 1553 as the first son of Zheng, when Emperor Zheng died. Although he could have had power, he decided to live in self-imposed retirement to focus on his research instead. Zhu's famous books include (乐律全书) *On the Equal Temperament*, 1584, [119], (律吕精义) *A Clear Explanation of That Which Concerns the Equal Temperament*, 1595/96 [120], and (算学新说) *Reflection on Mathematics*, 1603, [121]. Theorist Fritz A. Kuttner describes him as "one of the most important historians of his nation's music" [56].

This system is a great invention, specifically for tonal modulation. However, in our contemporary practice it is still used in combinations of different tuning systems depending on the instrumental setup.

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