Logic and structure, by Dirk van Dalen. Pp. 263. f23. 2004. ISBN 3540208798 (Springer-Verlag).

This book teaches logic to mathematicians in just the way I would have wished. But it is worth quoting how the introduction motivates the mathematician to trouble himself over logic by citing the Tarski-Los theorem:

> 'It' is .well known that a subset of a group which is c10sed under multiplication and inverse is a group; however, a subset of an algebraically c10sed field which is c10sed under sum, product, minus and inverse, is in general not an algebraically c10sed field. This phenomenon is an instance of something quite general: an axiomatizable class of structures is axiomatised by a set of universal sentences ... if and only if it is closed under substructures. If we check the axioms of group theory we see that indeed all axioms are universal, while not all the axioms of algebraically c10sed fields are universal.

Beginning with the propositional calculus by means of truth-tables, i.e. the semantics, it proceeds to the syntactics in the form of Gentzen's natural deduction. The predicate calculus is suitably motivated bythe needs of mathematicians in setting up structures - groups, sets and so on - there are many examples given. Then natural deduction again, initially for the universal quantifier only, then a full discussion of the existential quantifier and identity. Chapter 3 establishes .completeness and the Skolem-Löwenheim theorems so as to lead quickly to non-standard models of arithmetic and the real numbers. In a fairly brief discussion of second order logic, suitable health warnings are given. Unsurprisingly in a book based on lectures in Utrecht, there is an excellent chapter on intuitionistic logic. This begins with Heyting's proof-interpretation of Brouwer, takes over the appropriate parts of the natural deduction and proves completeness by Kripke models.

So much for the basic argument. Then cut-elimination is expounded as a means of efficiency in both classical and intuitionistic cases; and this fourth edition has a longfinal chapter added, on Gödel's incompleteness theorem. lts length is because the necessary recursive function theory and some details of Peano arithmetic have not been set up earlier. The chapter follows traditional lines but preserves the excellent clarity of the earlier chapters. This is a delightful textbook, with plenty of examples for the reader.

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