ERRATA for An Introduction to Homological Algebra 2nd Ed.

June 3, 2011

Here are all the errata that I know (aside from misspellings). If you have found any errors not listed below, please send them to me at rotman@math.uiuc.edu

Page 3 line 17. change $da$ to $dx$ (two times)

Page 4 line 2. change ... $\cup [a,c]$ to ... $\cup [a,b]$

Page 26 lines $-4,-3$. should read: For all $C \in \text{obj}(C)$.

\[(\sigma \tau)_C = \sigma_C \tau_C.\]

Page 30 line 3. change $t_n$ to $t_{n-1}$

Page 38 line 10. ... and $\alpha \beta(y) = \sum_{x \in G} \alpha(x) \beta(x^{-1} y)$.

line 17. Elements of $G$, when viewed as elements of $kG$, multiply as

Page 39 Prop 2.4(i) should read:

\[(f + g)_* = f_* + g_* \text{ for all } f, g \in \text{Hom}_R(A,B).\]

Page 40 line 17. Definition of additive functor should be placed on page 39.

Page 47 line 12. should read: $0 \rightarrow S/T \rightarrow M/T \rightarrow M/S \rightarrow 0$

Page 48 line $-10$. interchange "injection" and "projection"

Page 49 line 14. Change $\varphi$ to $\psi$.

Page 49 line $-10$. Should be: (iv) $\Rightarrow$ (v)

Page 51 line 7. should read: $\ker p = (T + S')/S'$

Page 52 In Prop 2.26, should be $S_i \cap (S_1 + \cdots + \hat{S}_i + \cdots + S_n) = \{0\}$

Page 53 line $-18$ should read:

is the map $\lambda_j : A_j \rightarrow B$, defined by $a_i \mapsto$

Page 65 Exercise 2.8(i) Note that $i q$ is a retraction

Page 78 line 9 should be: $\text{Hom}_R(\square, B)$

Page 83 line 11. change $k$-trilinear to $k$-triadditive

Page 85 line $-16$. should read: $f : (a,b') \mapsto a \otimes b + E$

Page 87 line $-17$. replace line; should read: $A \otimes_R (\bigoplus B_i)$. There is a ho-
Page 93 line 8. change $\text{Hom}_S(\square \otimes_S B, C)$ to $\text{Hom}_S(B \otimes_R \square, C)$

Page 119 line 7 change $E_i$ to $E_k$

Page 121 line 10 change $rf(1)$ to $rf(a)/a$

Page 133 line 12 should be: $= \sum \kappa(m_j) \otimes a_j$

Page 142 line 7 change $A$ to $B$

Page 147 line 3 should read: Let $0 \to A \to$


Page 148 line 3. change $\{a_i + K : i = 1, \ldots, m\}$ to $\{a_j + K : j = 1, \ldots, n\}$

Page 149 line 5 change $a_i$ to $a_1$

Page 172 line 4 change basis of $V$ to basis of $B$

Page 173 line 9 change Theorem 4.35 to Theorem 4.34

Page 181 line 1 change $g^k$ to $g^{k-1}$

Page 185 line 8 change essential to superfluous

Page 192 line 5 because $s''$ is not a zero-divisor,

Page 195 line 6 change $\mathfrak{p}R \mathfrak{p}$ to $\mathfrak{p}R \mathfrak{p}$

Page 196 line 5 change $S^{-1} \otimes M$ to $S^{-1}R \otimes M$

Page 198 line 8. the composite should have another functor on the right, namely, the change of rings functor $S^{-1}R \text{Mod} \to R \text{Mod}$ induced by the localization map $h: R \to S^{-1}R$

Page 207 line 3 change $f(x)$ to $f(y)$

Page 217 line 1 change $C$ to $B$ (3 times)

Page 218 line 2 change $X$ to $C$

Page 230 lines 1, 2. should read: inverse system in $C$ over $I$

Example 5.16(i): interchange $A$ and $C$ in the diagram

Page 233 line 11 change $\psi_N^M$ to $\psi_M^N$

Page 235 line 5 change $b_n + JM$ to $b_n + J^nM$
Page 237  top diagram  vertical arrows should point down
  line 11  should read: $\varphi^i_j: M_i \to M_j$
Page 258  line 11  change $\text{Hom}(A, B \otimes C)$ to $\text{Hom}(A \otimes B, C)$
Page 283  line 23  should read
  $$\lim_{\substack{\to \exists x}} P_1(U) \to \lim_{\substack{\to \exists x}} P_2(U):$$
Page 286  line −9  should read
  because, if $x$ is a closed point, then all the stalks of $x_*A$ are $\{0\}$
  except $(x_*A)_x$, which is $A$.
Page 302  Exercise 5.42  should read
  Give an example of a presheaf of abelian groups on a discrete
  space $X$ which is not a sheaf.
Page 316  line 15  should read $F: A \to \underline{R\text{Mod}}$ for some ring $R$.
Page 321  Exercise 5.57  in every abelian category
Page 327  lines 6, 7  change $\pi$ to $p$ (2 times)
Page 333  line 12, 13  should read  $= dic' = id'c,$
Page 335  line 2  change $\text{Comp}(A)$ to $A$.
Page 336  top diagram  remove subscript * from vertical arrows $f, g, h$
  line 3 below top diagram
  $$f_*\partial\text{cls}(z'') = \text{cls}(fz').$$
Page 337  Change 1c to $(1c)_*.$
Page 343  line 17  change $\partial^*_n$ and $\partial^*_{n-1}$ to $\partial^*_{n+1}$ and $\partial^*_n$
Page 350  line −3 of first paragraph  change $K_0$ to $V$ (twice)
  bottom: Change $\delta_{n+1}$ to $d_{n+1}$ (twice)
Page 359  first diagram: interchange $\partial$ and $\partial'$
Page 360  line −8.  change $H_q$ to $H_n$
  Use Proposition 2.70 to prove the case $n = 1$
Page 361  third diagram  change $\tau_{n-1,K}$ on far right to $T_{n-1}g$
Page 365  line −9.  $B \in \text{obj}(A)$
Page 366  line 3.  change $\text{im} \eta$ to $\text{im} d_0$ and $\text{im} d_{n-1}$ to $\text{im} d^n$
Page 373  line 9  change Ext to ext
Prof. A. Azizi found a gap in the proof of Theorem 7.5; on page 408, line 6, why is $H_1(F_A, B) \cong \text{im } \gamma$? He also sent me a corrected version which I have rewritten.

Page 406 Replace the paragraph beginning “We are going to use” by

The following two results will be used in the next proof. The first is a generalization of the First Isomorphism Theorem.

*If* $f : A \to B$ *is a homomorphism and* $N \subseteq \ker f$, then $\tilde{f} : A/N \to B$, given by $\tilde{f} : a + N \mapsto fa$, *is a well-defined homomorphism with* $\ker \tilde{f} = \ker f/N$ *and* $\text{im } \tilde{f} = \text{im } f$.

The second result is a variation of the Snake Lemma, due to Cartier–Weil.

*If* $f : A \to B$ *and* $g : B \to C$ *are homomorphisms, then there is an exact sequence*

$$0 \to \ker f \to \ker(gf) \to \ker g \to \coker f \to \coker(gf) \to \coker g \to 0.$$ 

We sketch a proof. There is a commutative diagram with exact rows

$$
\begin{array}{ccccccccc}
0 & \to & A & \xrightarrow{f} & A \oplus B & \xrightarrow{h} & B & \xrightarrow{g} & 0 \\
& & \downarrow{f} & \downarrow{h} & \downarrow{g} & & \\
0 & \to & B & \xrightarrow{0} & B \oplus C & \xrightarrow{0} & C & \to & 0
\end{array}
$$

the maps in the rows are the usual injections and projections of direct sums, while $h(a, b) = (fa - b, gb)$. The Snake Lemma gives exactness of

$$0 \to \ker f \to \ker h \to \ker g \to \text{coker } f \to \text{coker } (gf) \to \text{coker } g \to 0,$$

and it is easy to see that $\ker h \cong \ker(gf)$ and $\text{coker } h \cong \text{coker } (gf)$.

Page 407, line 11 through Page 408, line 7 Replace with the following.

For $n = 1$, let $h = d_1 \otimes 1 : F_1 \otimes_R B \to F_0 \otimes_R B$. Since $\text{im}(d_2 \otimes 1) \subseteq \ker(d_1 \otimes 1)$, the generalized First Isomorphism Theorem says that $\tilde{h} : F_1 \otimes_R B / \text{im}(d_2 \otimes 1) \to F_0 \otimes_R B$ has kernel

$$\ker \tilde{h} = \frac{\ker(d_1 \otimes 1)}{\text{im}(d_2 \otimes 1)} = H_1(F_A, B).$$ (1)

Let $Y = \text{im } d_1$, let $i : Y \to F_0$ be the inclusion, and let $d'_1 : F_1 \to Y$ be given by $d'_1 : x \mapsto d_1(x)$ [so $d'_1$ differs from $d_1$ only in its target]. Of course, $id'_1 = d_1$.

Define $f = d'_1 \otimes 1 : F_1 \otimes_R B \to Y \otimes_R B$. Note that $d'_1$ is surjective, so that right exactness of $\square \otimes_R B$ gives $f = d'_1 \otimes 1$ surjective; that is, $Y \otimes_R B = (\text{im } d_1) \otimes_R B = \text{im}(d'_1 \otimes 1)$. Since $\text{im}(d_2 \otimes 1) \subseteq \ker(d'_1 \otimes 1)$, the generalized First Isomorphism Theorem gives $\tilde{f} : F_1 \otimes B / \text{im}(d_2 \otimes 1) \to Y \otimes B$ surjective and

$$\ker \tilde{f} = \frac{\ker(d'_1 \otimes 1)}{\text{im}(d_2 \otimes 1)}.$$
Let \( g = i \otimes 1: Y \otimes_R B \to F_0 \otimes_R B \). The Cartier–Weil variation gives exactness of
\[
\ker \tilde{f} \to \ker(g \tilde{f}) \to \ker g \to \coker \tilde{f}.
\]
We have seen that \( \tilde{f} \) is surjective, so that \( \coker \tilde{f} = \{0\} \). Moreover, exactness of \( F_2 \to F_1 \to Y \to 0 \) gives exactness of
\[
F_2 \otimes_R B \xrightarrow{d_2 \otimes 1} F_1 \otimes_R B \xrightarrow{d_1 \otimes 1} Y \otimes_R B \to 0,
\]
because \( \square \otimes_R B \) is right exact. Hence, \( \text{im}(d_2 \otimes 1) = \ker(d'_1 \otimes 1) \), and so \( \ker \tilde{f} = \ker(d'_1 \otimes 1)/\text{im}(d_2 \otimes 1) = \{0\} \). Thus, \( \ker(g \tilde{f}) \cong \ker g \). But \( g \tilde{f} = \tilde{h} \), and Eq. (1) gives \( \ker h \cong H_1(F_A, B) \). Therefore,
\[
H_1(F_A, B) \cong \ker g = \ker(i \otimes 1).
\]
Page 411 line −8 change essential to superfluous.

Page 427 line just below diagram. change \( GR \cong C \) to \( GR \cong A \).

Page 429, 430 Formula II: change \( e(C, A') \) to \( \text{Ext}^1(C, A') \).

Formula III: change \( e(C', A) \) to \( \text{Ext}^1(C', A) \).

Page 453 line −6. should read
\[
\text{Ext}_R^n(A, B) = \frac{\ker d_{n+1}^*}{\text{im} d_n^*}.
\]

Page 465 line 9. change \( fd(R) \) to \( fd(A) \).

Page 466 Exercise 8.5 0 \( \to M' \to M \to M'' \to 0 \).

Page 474 line 1. Change \( pd_R(K^*) \leq 2 \) to \( pd_R(M^*) \leq 2 \).

Page 475 line 5. Change \( R \text{Mod} \) to \( \text{Mod}_R \).

Page 477 line −12. change \( (d_2, p_2, 0) \) to \( (d_2 p_2, 0) \).

Page 481 line 12. delete “left” two times

Page 482 lines 16–18. Should read:
\[
f(x) \in p \subseteq R[x] \text{ of least degree, and consider the exact sequence } 0 \to (f) \to p \to p/(f) \to 0. \text{ Now } (f) \cong R[x], \text{ since } R[x] \text{ is a domain, and } \text{ann}(p/(f)) \neq \{0\}; \text{ if } Q = \text{Frac}(R), \text{ then } pQ[x] \text{ is generated by } f, \text{ so that if } g \in p, \text{ there is } c \in R \text{ with } cg \in Q[x].
\]

Page 486 line −5. \( pd(M) \leq n \).

Page 487 line −2. Change \( y_i + m \) to \( y_i + (x) \).
Page 487 line −1. Change mod \( m \) to mod \( m^2 \)

Page 488 line −6. change \{ f/g \in k[V] \} to \{ f/g \in k(V) \}

Page 489 Lemma 8.59 add hypothesis: \( R \) is noetherian

Page 490 Proposition 8.61 add hypothesis: \( R \) is noetherian

Page 519 line 8. change \( \text{Hom}_G(A, B) \) to \( \text{Hom}_\mathbb{Z}(A, B) \)

Page 560 bottom 3 lines should read

\[
\text{Hom}_S(\mathbb{Z}, A) \text{ is a (left) } G\text{-module as follows: if } y \in G \text{ and } g: \mathbb{Z}G \to A, \text{ define }
\]
\[
yg: x \mapsto g(y^{-1}x).
\]

Page 614 line −10. a subscript \( Q \) should be \( q \):

\[
\Delta'_{p} \otimes 1_{B_n}
\]

Page 629 line −8. change subscript on \((F^{p-1})_n\) to \( n - 1 \)

Page 632, 633 interchange the rows in the \( 2 \times 2 \) matrices

Page 667 lines −7, −6

\[
\text{Hom}_S(B \otimes_R A, C) \cong \text{Hom}_R(A, \text{Hom}_S(B, C)).
\]

If \( G = B \otimes_R \square \) and \( F = \text{Hom}_S(\square, C) \),

Page 678 line 10. change "isomorphic" to "chain equivalent"

Page 679 line 3. also assume that the complex \( Z \) of cycles is flat

Page 692 add references


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**Additional errata, verified and added by Springer**

*Mar 30, 2017*

Page 626 line 13 Replace “Since \( F^s = 0 \) and \( F^t = C, \ldots \)” with “Since \( F^s = 0 \) and \( F^t = C \) in each degree, \ldots \)”

Page 627 In (i), replace all instances of \( F^p \) with \( F^p_n \), and \( F^{p-1} \) with \( F^{p-1}_n \)
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