Figure 4.1: For $\rho = 1$ the $x, y$ in Tab. 4.1 represent the four arms of equilateral hyperbolas $|x^2 - y^2| = 1$. Here we indicate how each arm is traversed as the parameter $\theta$ goes from $-\infty$ to $+\infty$. In particular there is a symmetry with respect to axis bisectors.

4.1.2 Hyperbolic Rotations as Lorentz Transformations of Special Relativity

Let us write a space-time vector as a hyperbolic variable, $w = t + h x$ and a hyperbolic constant $a = a_r + h a_h$ with $a_r > a_h$ in the exponential form

$$a_r + h a_h \equiv \rho_a \exp[h \theta_a] \equiv \rho_a \left( \cosh \theta_a + h \sinh \theta_a \right)$$

where $\rho_a = \sqrt{(a_r^2 - a_h^2)}$; $\theta_a = \tanh^{-1}(a_h/a_r)$.

Then the multiplicative group, $w' \equiv t' + h x' = a w$ becomes

$$t' + h x' = \sqrt{(a_r^2 - a_h^2)} \left[ t \cosh \theta_a + x \sinh \theta_a + h(t \sinh \theta_a + x \cosh \theta_a) \right]. \quad (4.1.7)$$

In this equation, by letting $(a_r^2 - a_h^2) = 1$ and considering as equal the coefficients of versors “1” and “h”, as we do in complex analysis, we get the Lorentz transformation of two-dimensional special relativity [55] and [62]. It is interesting to

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1In all the problems which refer to Special Relativity we change the symbols by indicating the variables with letters reflecting their physical meaning $x, y \Rightarrow t, x$, i.e., $t$ is a normalized time variable (light velocity $c = 1$) and $x$ a space variable.
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