As each transformation is a 4×4 matrix, we can now multiply them together (see [Craig 2003]) to find the complete manipulator transition from base to tool – something we could not have done without homogeneous coordinates ($c_{123}$ is an abbreviation for $\cos(\theta_1+\theta_2+\theta_3)$; equivalent for $s_{123}, c_{12}, s_{12}$):

$$
\begin{align*}
_{0}^{1}T &= \begin{bmatrix}
c\theta_1 & -s\theta_1 & 0 & 0 \\
s\theta_1 & c\theta_1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \\
_{1}^{2}T &= \begin{bmatrix}
c\theta_2 & -s\theta_2 & 0 & L_1 \\
s\theta_2 & c\theta_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \\
_{3}^{2}T &= \begin{bmatrix}
c\theta_3 & -s\theta_3 & 0 & L_2 \\
s\theta_3 & c\theta_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
$$

This transformation can be applied to a point $p$ on the tool tip of the manipulator (relative to joint 3) and will return its 3D pose (position and orientation). If the manipulator moves, we do not have to recalculate the 4×4 matrix, we only have to update the matrix’s parameters (here: $\theta_1, \theta_2, \theta_3$).

If we assume the sample configuration $\theta_1 = 0^\circ, \theta_2 = 90^\circ, \theta_3 = -90^\circ$, then we can calculate the end-effector position $L_3$ as shown in Figure 14.7 (the top index of point $P$ indicates the relative coordinate system used, so $^0P$ is using the base coordinate system).

$$
\begin{align*}
_{0}^{3}T &= \begin{bmatrix}
c_{123} & -s_{123} & 0 & L_1 \cdot c_1 + L_2 \cdot c_{12} \\
s_{123} & c_{123} & 0 & L_1 \cdot s_1 + L_2 \cdot s_{12} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \\
_{0}^{3}P &= _{0}^{3}T \cdot ^{3}P = \begin{bmatrix}
1 & 0 & 0 & L_1 \\
0 & 1 & 0 & L_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
L_3 \\
0 \\
0 \\
1
\end{bmatrix} = \begin{bmatrix}
L_1 + L_3 \\
L_2 \\
0 \\
1
\end{bmatrix}
\end{align*}
$$

*Figure 14.7: Sample configuration*

### 14.2.2 Inverse Kinematics

Performing the inverse kinematics calculation proves to be much more complex than the forward kinematics and there has been at least anecdotal evidence of manipulator manufacturers changing their mechanical design in order to simplify the inverse kinematics solution. On the other hand, inverse kinematics is the more important task, as we usually need to know how to get to a
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