Chapter 2
Elastic Material Behavior

Abstract This chapter reviews the basics of elastic material behavior. Starting from simple load cases, i.e. uniaxial tension, pure shear and hydrostatic compression, basic material parameters are derived from experimental results. The next part presents the three-dimensional constitutive law for isotropic and linear-elastic material behavior. The chapter closes with two important cases of two-dimensional formulations, i.e. the plane stress and the plane strain case.

2.1 Simple Load Cases

2.1.1 Uniaxial Tensile Test

Let us consider in the following a tensile sample of length L and diameter d which is loaded by a linearly increasing external force F, see Fig. 2.1. In addition to the applied force F, the elongation ΔL and change in diameter Δd is recorded during the test.

Using the definitions of engineering stress and strain as given in Eqs. (1.1) and (1.2), i.e.

$$\sigma = \frac{F}{A_0}, \tag{2.1}$$

and the longitudinal strain

$$\varepsilon_x = \frac{\Delta L}{L}, \tag{2.2}$$

the engineering stress-strain diagram in the linear-elastic region results as shown in Fig. 2.2a. For most of the engineering materials, a straight line is observed and its slope is equal to YOUNG's modulus:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon_x}. \tag{2.3}$$

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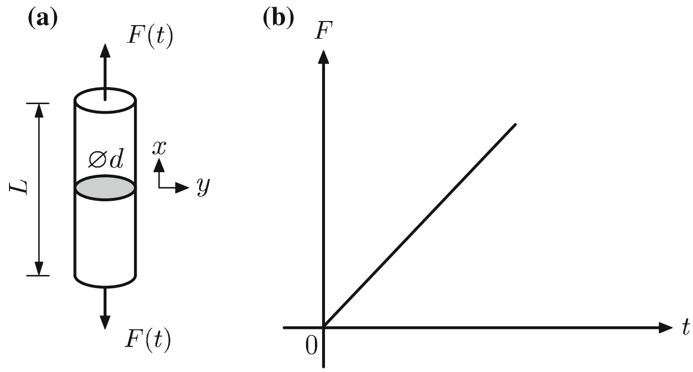

Fig. 2.1 Uniaxial tensile test: **a** idealized specimen and **b** force-time distribution

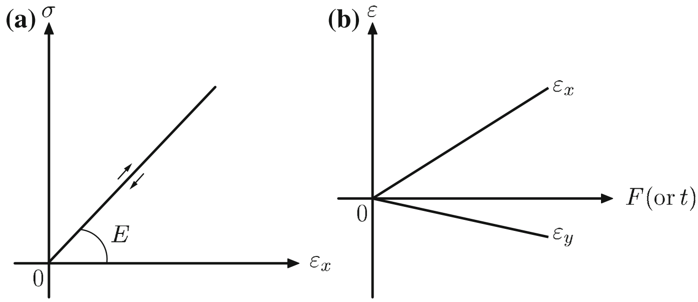

Fig. 2.2 Linear-elastic material behaviour: **a** engineering stress-strain diagram; **b** course of longitudinal and transversal strain

If the specimen is elongated, the cross-sectional area of all classical engineering materials is reduced and the transversal strain is defined as:

$$\varepsilon_y = \frac{\Delta d}{d}. \tag{2.4}$$

The course of the longitudinal and transversal strain is shown in Fig. 2.2b.

Relating the transversal to the longitudinal strain defines POISSON's ratio:

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}. \tag{2.5}$$

Mechanical and physical reference values[1] of some typical engineering materials are summarised in Table 2.1.

[1]Conversion from GPa to MPa: value times 1000; conversion from $g \times cm^{-3}$ to $kg \times m^{-3}$: value times 1000; 1 Pa = 1 N/m^2.

Table 2.1 Reference values of typical engineering materials (all values are given near room temperature)

Material	E in GPa	ν in –	Density in g $\times$ cm^{-3}
Aluminium	70	0.33	2.700
Steel	210	0.30	7.874
Magnesium	45	0.35	1.738
Titanium	110	0.34	4.506

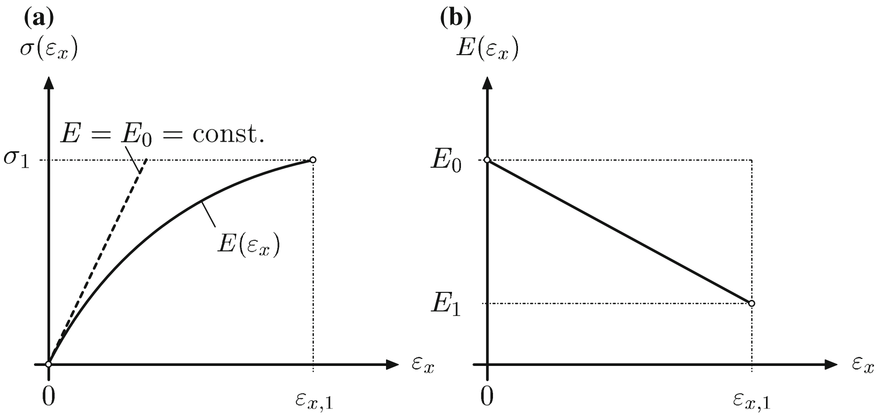

Fig. 2.3 **a** Nonlinear-elastic stress-strain diagram; **b** strain-dependent modulus of elasticity

Some materials show an elastic material behavior, but the course of the stress-strain diagram is nonlinear, see Fig. 2.3a.

In such a case, the YOUNG's modulus is not constant, i.e. a function of the acting strain. Thus, Eq. (2.3) must be modified to a differential definition as:

$$E(\varepsilon_x) = \frac{d\sigma}{d\varepsilon_x}. \tag{2.6}$$

2.1.2 Pure Shear Test

Let us consider in the following a cylindrical specimen of length L and diameter d which is loaded by a linearly increasing external torsional moment M_T, see Fig. 2.4. In addition to the applied moment M_T, the total twist angle ϑ_L at the end of the specimen ($x = L$) is recorded during the test. This test can be realized in a specific torsion testing machine or in a classical universal testing machine under usage of a special jig which creates the load by a lever arm.

Plotting the torsional moment over the twist angle results in a straight line as shown in Fig. 2.5a. However, these quantities are still dependent on the size of the specimen, similar to the force and elongation in the case of the tensile test.

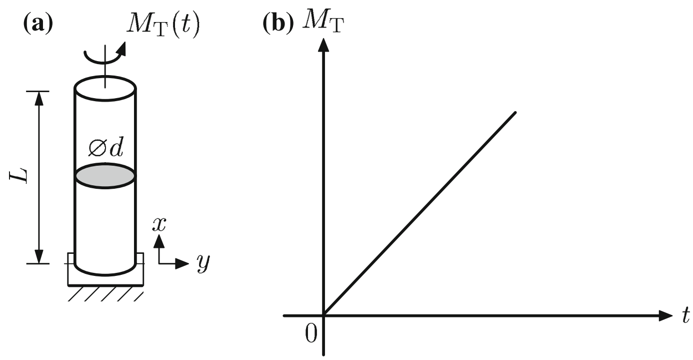

Fig. 2.4 Pure shear test: **a** idealized specimen and **b** moment-time distribution

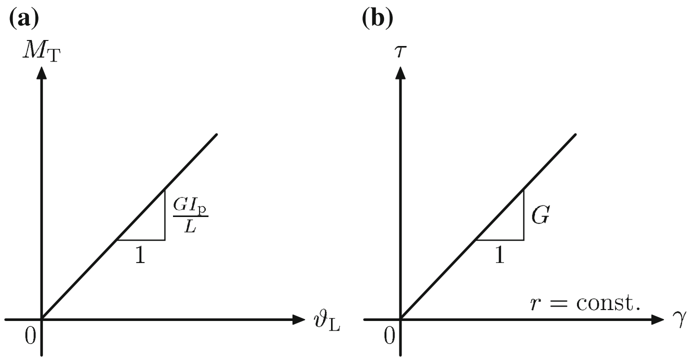

Fig. 2.5 Pure shear test: **a** moment—twisting angle distribution **b** shear stress—shear strain diagram

Normalizing these quantities results in a shear stress versus shear strain diagram (see Fig. 2.5b) and the so-called shear modulus can be identified as the slope:

$$G = \left.\frac{\tau}{\gamma}\right|_{r = \text{const.}} \tag{2.7}$$

In the case of a cylindrical specimen which is clamped at one end and the other end is loaded by a torsional moment, the following relation can be derived [24, 64][2]:

$$G = \frac{M_T L}{I_p \vartheta_L}, \tag{2.8}$$

[2]See supplementary Problem 2.4.

where I_p is the polar second moment of area (in the case of a cylinder: $I_\mathrm{p} = \frac{\pi d^4}{32}$) and ϑ_L is the total twist angle at the end of the cylindrical specimen. Equation (2.8) can be written as

$$G = \frac{\frac{M_\mathrm{T}}{I_\mathrm{p}}}{\frac{\vartheta_\mathrm{L}}{L}}, \tag{2.9}$$

which corresponds to the formulation of the YOUNG's modulus: $E = \frac{F/A}{\Delta L/L}$.

2.1.3 Hydrostatic Compression Test

Let us consider in the following a cubic sample which is loaded from all three sides by linear increasing external compressive forces $F(\sim \sigma)$, see Fig. 2.6. In addition to the applied forces F, the shortening is recorded during the test. These quantities can be easily converted into a compressive stress and a compressive (axial) strain which is, due to the symmetry of the problem in all three directions (x, y, z), the same. Such a test can be realized in a triaxial testing machine, i.e. a machine which allows simultaneous loading of specimens along three axes perpendicular to each other, or by enclosing specimens in a hydraulic cylinder where a fluid pressure is applied by a piston.

Plotting the mean stress over the volumetric strain,[3] a linear dependency is obtained which allows the definition of the bulk modulus.

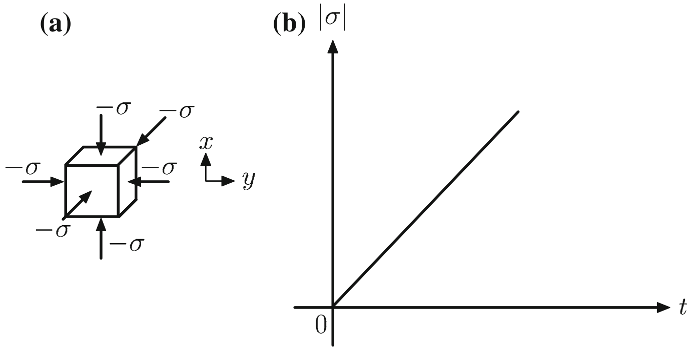

Fig. 2.6 Hydrostatic compression test: **a** idealized specimen and **b** stress-time distribution

[3] See supplementary Problems 2.2 and 2.6.

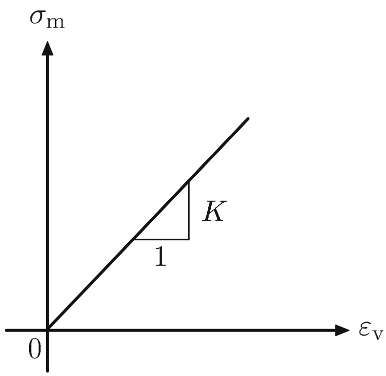

Fig. 2.7 Mean normal stress versus volumetric strain

The bulk modulus K is then defined as the ratio between the mean normal stress (hydrostatic stress) and the corresponding volume change [15] (Fig. 2.7):

$$K = \frac{\sigma_{\mathrm{m}}}{\varepsilon_{\mathrm{v}}} = \frac{\frac{1}{3}\,\sigma_{kk}}{\varepsilon_{kk}}. \tag{2.10}$$

2.2 Three-Dimensional Hooke's Law

Let us first look on the general concept of continuum mechanical modelling of material behavior, see Fig. 2.8. The equilibrium equation relates the external forces to the internal reactions and is a measure for the loading of the material. The kinematics

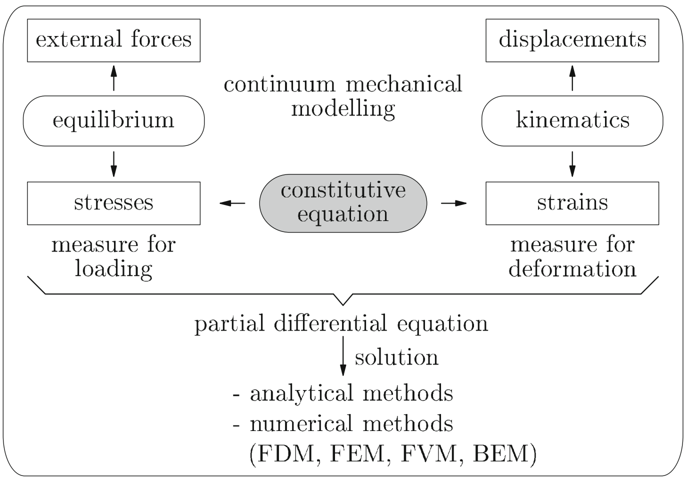

Fig. 2.8 Continuum mechanical modelling

equation relates the displacements to the strains and is a measure for the deformation of the body. The constitutive equation relates stress and strain. The following section treats the simplest three-dimensional case, i.e. HOOKE's law for isotropic linear-elastic material behavior.

Combining these three equations, i.e. equilibrium, kinematics and constitution, results in a partial differential equation which describes the entire problem at any location of the material domain. For simple geometries (e.g. beams or rods), analytical solutions can be derived which offer an exact description of the problem under the given assumptions and simplifications. For more complicated cases, approximate solutions can be obtained based on numerical techniques such as the finite element method (FEM), see [43, 44].

Let us look in the following at a three-dimensional body which is sufficiently supported and loaded as shown in Fig. 2.9a. Isolating a differential volume element, the internal stresses—which are in equilibrium with the external loads—occur, see Fig. 2.9b.

The nine stresses[4] σ_{ij} are the components of the second-order stress tensor and are arranged in the following way:

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad \rightarrow \quad \boldsymbol{\sigma} = \{\sigma_x \ \sigma_y \ \sigma_z \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{xz}\}^{\mathrm{T}}, \tag{2.11}$$

where $\boldsymbol{\sigma}$ is the column matrix of the stress components which contains only the independent variables (engineering notation). This reduces the required disk space to store the stress values. In a similar way, the corresponding strains are arranged:

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \quad \rightarrow \quad \boldsymbol{\varepsilon} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ 2\varepsilon_{xy} \ 2\varepsilon_{yz} \ 2\varepsilon_{xz}\}^{\mathrm{T}}. \tag{2.12}$$

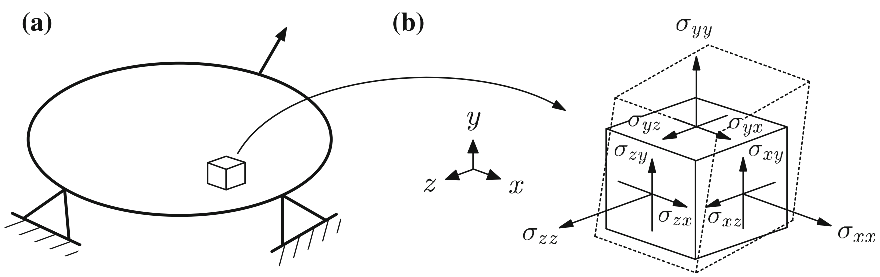

Fig. 2.9 **a** Three-dimensional body under arbitrary load; **b** differential volume element with acting stresses

[4]The first index i indicates that the stress acts on a plane normal to the i-axis and the second index j denotes the direction in which the stress acts.

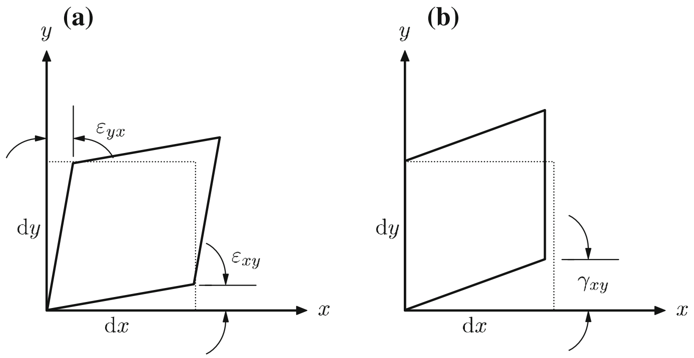

Fig. 2.10 Definition of shear strain: **a** tensor definition as $\varepsilon_{xy} \approx \partial u_y/\partial x$ and $\varepsilon_{yx} \approx \partial u_x/\partial y$; **b** engineering definition as the total $\gamma_{xy} \approx \partial u_y/\partial x + \partial u_x/\partial y$

It should be noted here that there are two definitions for the shear strain in use: The tensor definition ε_{ij} (for $i \neq j$) and the engineering shear strain $\gamma_{ij} = 2\varepsilon_{ij}$ (for $i \neq j$), see Fig. 2.10.

In the following description, the tensor notation is abandoned and the engineering notation (VOIGT notation) is now consistently introduced and observed, i.e. the components of the second-order stress tensor σ_{ij} and the strain tensor ε_{ij} are arranged into column matrices $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$. Since the stress and strain tensor are symmetric (e.g. $\sigma_{ij} = \sigma_{ji}$), it is more convenient and economic to store only the six independent components in a single column matrix. The constitutive equation which relates the stresses and strains (see Fig. 2.8) can be generally expressed as

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\varepsilon}\,, \tag{2.13}$$

or

$$\boldsymbol{\varepsilon} = \boldsymbol{D}\boldsymbol{\sigma}\,, \tag{2.14}$$

where $\boldsymbol{C}$ is the so-called elasticity matrix and $\boldsymbol{D}$ the so-called elastic compliance matrix. Let us assume the following simplifications for the material under consideration:

- linear elastic,
- isotropic,
- homogeneous,
- isothermal conditions.

In extension to HOOKE's law ($\sigma = E\varepsilon$) of the year 1678, the following generalized formulation based on the engineering constants YOUNG's modulus E and POISSON's ratio ν can be given:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\,\varepsilon_{xy} \\ 2\,\varepsilon_{yz} \\ 2\,\varepsilon_{xz} \end{bmatrix}. \tag{2.15}$$

Rearranging the elastic stiffness form given in Eq. (2.15) for the strains gives the elastic compliance form:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\,\varepsilon_{xy} \\ 2\,\varepsilon_{yz} \\ 2\,\varepsilon_{xz} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}. \tag{2.16}$$

In a simple tension test, the only nonzero stress component σ_x causes axial strain ε_x and transverse strains $\varepsilon_y = \varepsilon_z$. Thus, Eqs. (2.15) and (2.16) yield

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad \text{and} \qquad \varepsilon_y = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E}. \tag{2.17}$$

By using Eq. (2.17), one can experimentally determine the elastic constants, i.e. YOUNG's modulus E and POISSON's ratio ν, from a uniaxial tension or compression test, see Chap. 1 and Sect. 2.1.1.

Replacing the YOUNG's modulus E and POISSON's ν ratio by the shear modulus G and bulk modulus K gives:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix} = \begin{bmatrix} K+\frac{4}{3}G & K-\frac{2}{3}G & K-\frac{2}{3}G & 0 & 0 & 0 \\ K-\frac{2}{3}G & K+\frac{4}{3}G & K-\frac{2}{3}G & 0 & 0 & 0 \\ K-\frac{2}{3}G & K-\frac{2}{3}G & K+\frac{4}{3}G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\,\varepsilon_{xy} \\ 2\,\varepsilon_{yz} \\ 2\,\varepsilon_{xz} \end{bmatrix}, \tag{2.18}$$

or as the elastic compliance form:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\,\varepsilon_{xy} \\ 2\,\varepsilon_{yz} \\ 2\,\varepsilon_{xz} \end{bmatrix} = \frac{1}{18KG} \begin{bmatrix} 6K+2G & -3K+2G & -3K+2G & 0 & 0 & 0 \\ -3K+2G & 6K+2G & -3K+2G & 0 & 0 & 0 \\ -3K+2G & -3K+2G & 6K+2G & 0 & 0 & 0 \\ 0 & 0 & 0 & 18K & 0 & 0 \\ 0 & 0 & 0 & 0 & 18K & 0 \\ 0 & 0 & 0 & 0 & 0 & 18K \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix}. \tag{2.19}$$

Table 2.2 Conversion of elastic constants λ, μ LAMÉ's constants; K bulk modulus; G shear modulus; E YOUNG's modulus; ν POISSON's ratio, [14]

	λ, μ	E, ν	μ, ν	E, μ	K, ν	G, ν	K, G
λ	λ	$\frac{\nu E}{(1+\nu)(1-2\nu)}$	$\frac{2\mu\nu}{1-2\nu}$	$\frac{\mu(E-2\mu)}{3\mu-E}$	$\frac{3K\nu}{1+\nu}$	$\frac{2G\nu}{1-2\nu}$	$K - \frac{2G}{3}$
μ	μ	$\frac{E}{2(1+\nu)}$	μ	μ	$\frac{3K(1-2\nu)}{2(1+\nu)}$	G	G
K	$\lambda + \frac{2}{3}\mu$	$\frac{E}{3(1-2\nu)}$	$\frac{2\mu(1+\nu)}{3(1-2\nu)}$	$\frac{\mu E}{3(3\mu-E)}$	K	$\frac{2G(1+\nu)}{3(1-2\nu)}$	K
E	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	E	$2\mu(1+\nu)$	E	$3K(1-2\nu)$	$2G(1+\nu)$	$\frac{9KG}{3K+G}$
ν	$\frac{\lambda}{2(\lambda+\mu)}$	ν	ν	$\frac{E}{2\mu} - 1$	ν	ν	$\frac{3K-2G}{2(3K+G)}$
G	μ	$\frac{E}{2(1+\nu)}$	μ	μ	$\frac{3K(1-2\nu)}{2(1+\nu)}$	G	G

Let us note at the end of this section that the general characteristic of HOOKE's law in the form of, for example, Eqs. (2.15) and (2.16) is that two independent material parameters are used. In addition to the YOUNG's modulus E and POISSON's ratio ν, other elastic parameters can be used to form the set of two independent material parameters, and the following Table 2.2 summarizes the conversion between the common material parameters.

2.3 Plane Stress and Plane Strain Case

2.3.1 Plane Stress Case

The two-dimensional plane stress case ($\sigma_z = \sigma_{yz} = \sigma_{xz} = 0, \varepsilon_z \neq 0$) shown in Fig. 2.11 is commonly used for the analysis of thin, flat plates loaded in the plane of the plate (x-y plane).

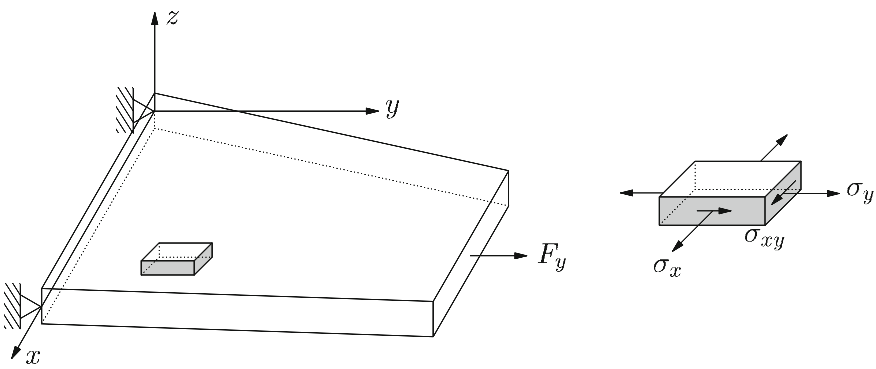

Fig. 2.11 Two-dimensional problem: plane stress

By imposing the condition $\sigma_z = 0$, we get from Eq. (2.15)

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)} \cdot \Big(\nu(\varepsilon_x + \varepsilon_y) + (1-\nu)\varepsilon_z\Big) = 0 \tag{2.20}$$

and

$$\varepsilon_z = -\frac{\nu}{1-\nu} \cdot \Big(\varepsilon_x + \varepsilon_y\Big). \tag{2.21}$$

Substituting now ε_z into Eq. (2.15) and respecting $\sigma_z = \sigma_{yz} = \sigma_{xz} = 0$, we obtain the following form of HOOKE's law:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\,\varepsilon_{xy} \end{bmatrix}. \tag{2.22}$$

Rearranging the elastic stiffness form given in Eq. (2.22) for the strains gives the elastic compliance form

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\,\varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(\nu+1) \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}. \tag{2.23}$$

The general characteristic of plane HOOKE's law in the form of Eqs. (2.22) and (2.23) is that two independent material parameters are used.

It should be finally noted that the thickness strain ε_z can be obtained based on the two in-plane normal strains ε_x and ε_y as:

$$\varepsilon_z = -\frac{\nu}{1-\nu} \cdot \Big(\varepsilon_x + \varepsilon_y\Big). \tag{2.24}$$

The last equation can be derived from the tree-dimensional formulation, see Sect. 2.2.

2.3.2 Plane Strain Case

The two-dimensional plane strain case ($\varepsilon_z = \varepsilon_{yz} = \varepsilon_{xz} = 0$) shown in Fig. 2.12 is commonly used for the analysis of elongated prismatic bodies of uniform cross section subjected to uniform loading along their radial axis but without any component in direction of the z-axis (e. g. pressure p_1 and p_2), such as in the case of tunnels, soil slopes, and retaining walls.

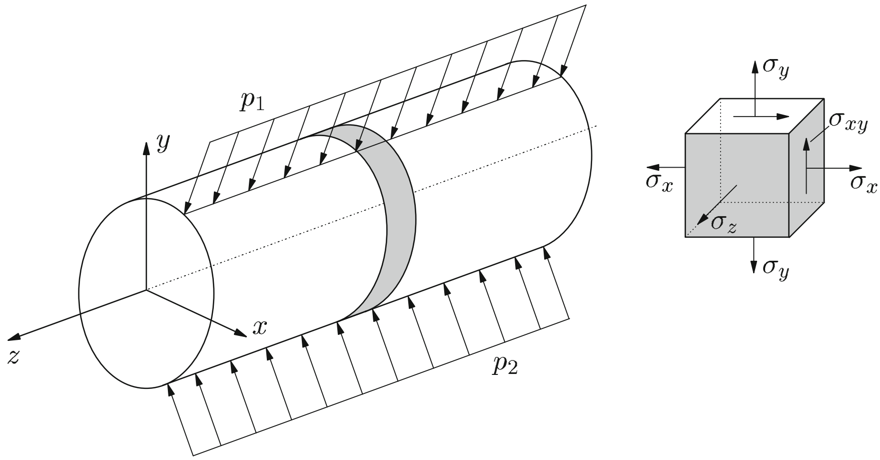

Fig. 2.12 Two-dimensional problem: plane strain

Considering that the stress components σ_{yz} and σ_{xz} are zero, Eq. (2.15) can be directly reduced to the plane strain form:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\,\varepsilon_{xy} \end{bmatrix}. \tag{2.25}$$

Rearranging the elastic stiffness form given in Eq. (2.25) for the strains gives the elastic compliance form

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\,\varepsilon_{xy} \end{bmatrix} = \frac{1-\nu^2}{E} \begin{bmatrix} 1 & -\frac{\nu}{1-\nu} & 0 \\ -\frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{2}{1-\nu} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}. \tag{2.26}$$

By imposing the condition $\varepsilon_z = 0$, we get from Eq. (2.16)

$$\varepsilon_z = \frac{1}{E} \cdot \Big(-\nu(\sigma_x + \sigma_y) + \sigma_z \Big) = 0\,, \tag{2.27}$$

and the stress component σ_z can be obtained based on the two in-plane normal stresses σ_x and σ_y as:

$$\sigma_z = \nu(\sigma_x + \sigma_y)\,. \tag{2.28}$$

2.4 Supplementary Problems

2.1 Knowledge questions

- How many material parameters are required for one-dimensional HOOKE's law?
- State the one-dimensional HOOKE's law for a pure *normal* stress and *normal* strain state.
- State the one-dimensional HOOKE's law for a pure *shear* stress and shear strain state.
- How many material parameters are required for the three-dimensional HOOKE's law?
- Explain the assumptions for an (a) 'isotropic' and (b) 'homogeneous' material.
- State the major characteristic of an *elastic* material.
- State approximate values for the YOUNG's modulus of (a) steel and (b) aluminium alloys.
- Which quantities does the constitutive law relate to?
- How many independent stress components occur in a general three-dimensional stress state?
- How many independent stress and strain components are acting in a plane stress state?
- How many independent stress and strain components are acting in a plane strain state?

2.2 Approximation of the volume change in a hydrostatic compression test

Consider a cuboid with dimensions $a \times b \times c$ which is compressed in a hydrostatic compression test. Derive an approximate equation for the volume change $\frac{\Delta V}{V}$ which depends only on the three normal strains.

2.3 Mohr's circle for simple load cases

Draw MOHR's circle for a uniaxial tensile and compression test (see Fig. 2.1), a pure shear test (see Fig. 2.4), and a hydrostatic compression test (see Fig. 2.6).

2.4 Derivation of the evaluation equation for the torsion test

Derive the equation to evaluate the shear modulus G from the pure shear test as shown in Fig. 2.4a, i.e. $G = \frac{M_{\mathrm{T}} L}{I_{\mathrm{p}} \vartheta_{\mathrm{L}}}$.

2.5 Alternative realization of pure shear test

Sketch MOHR's circle for pure shear test (torsional loading of a cylindrical specimen) and derive a strategy for a pure shear test which is only based on normal stresses.

2.6 Evaluation of isostatic compression test

Simplify the generalized HOOKE's law in terms of shear and bulk modulus to evaluate an isostatic compression test, i.e. to derive the bulk modulus based on $K = \frac{\sigma_{\mathrm{m}}}{\varepsilon_{\mathrm{v}}} = \frac{1/3\sigma_{kk}}{\varepsilon_{kk}}$.

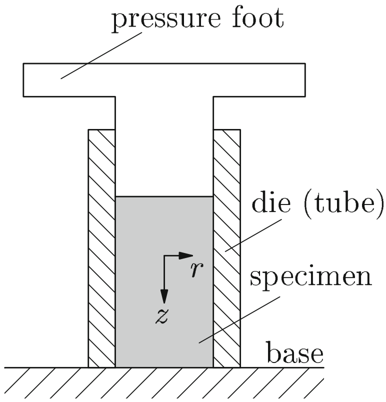

Fig. 2.13 Schematic representation of the axial compaction experiment

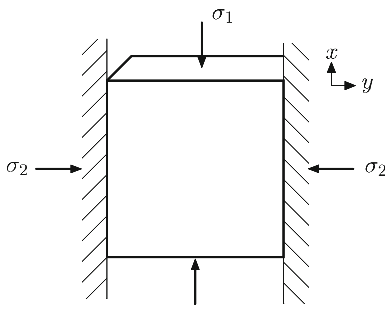

Fig. 2.14 Schematic representation of the plane strain experiment

2.7 Axial compaction

Consider the axial compaction as schematically shown in Fig. 2.13. Derive under the assumption that the wall friction can be neglected an equation for the gradient $\mathrm{d}\sigma_z/\mathrm{d}\varepsilon_z$.

2.8 Plane strain experiment

Consider the plane strain experiment as schematically shown in Fig. 2.14. A specimen is compressed in the x-direction while the deformation in the y-direction is constrained to zero ($\varepsilon_y = 0$). The z-direction is free, i.e. $\varepsilon_z \neq 0$. Derive from the generalized HOOKE's law a simple equation which allows to calculate POISSON's ratio based on the stress fraction σ_1 and σ_2 where σ_2 is the stress required to prevent any deformation in the y-direction.

2.9 Hooke's law in terms of Lamé's constants

Use a computer algebra system (e.g. Maple®, Mathematica® or Matlab®) to derive the generalized HOOKE's law for a linear isotropic material in terms of LAMÉ'S constants μ and λ in the elastic stiffness form ($\boldsymbol{\sigma} = \boldsymbol{\sigma}(\varepsilon)$) and the elastic compliance form ($\varepsilon = \varepsilon(\boldsymbol{\sigma})$).

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