

# Chapter 2

## Maximizing Profits and Maximizing Resource Providers' Wealth

In microeconomics, the firm is set to maximize excess profits. In corporate finance, the firm is supposed to maximize shareholders' wealth. Some even argue that the firm should maximize stakeholders' wealth. In this chapter, I show that under certainty with no transaction costs, maximizing excess profits, maximizing shareholders' wealth and maximizing stakeholders' wealth are equivalent. Modigliani and Miller's (1958) capital structure irrelevancy proposition is a restatement of the Coase theorem. With fixed investment, higher debt brings higher rate of return on equity but brings no risk to the equityholder.

### 2.1 The Coase Theorem and the Modigliani-Miller Propositions

Assume a one-period model: In the beginning of the period, the firm employs labor ( $L$ ) and capital ( $K$ ) to produce output ( $q = q(L, K)$ ). At the end of the period, the capital has no scrap value, and the firm sells outputs and liquidates. The firm's profit-maximizing problem is:

$$\underset{L, K}{Max} \pi = p(q(L, K)) \cdot q(L, K) - wL - (1 + r)K, \quad (2.1)$$

where  $w$  is the wage rate,  $1 + r$  is the rental price of capital. In Eq. (2.1), a Schumpeterian entrepreneur owns and tries to maximize excess profits,  $\pi$ , by choosing optimal labor and capital inputs. The resource providers: the labor and the capitalist can only earn opportunity costs:  $w$  and  $1 + r$ , respectively.

The first-order conditions for Eq. (2.1) are:

$$\frac{\partial \pi}{\partial K} = \frac{\partial}{\partial K} [p(q(L, K)) \cdot q(L, K)] - (1 + r) \equiv 0 \quad (2.2)$$

$$\frac{\partial \pi}{\partial L} = \frac{\partial}{\partial L} [p(q(L, K)) \cdot q(L, K)] - w \equiv 0. \quad (2.3)$$

From Eq. (2.2) we get  $K = K(L, r)$  and from Eq. (2.3),  $L = L(K, w)$ . The optimal inputs:  $L^* = L(r, w)$  and  $K^* = K(r, w)$  can be obtained by jointly solving Eqs. (2.2) and (2.3).

We can also use a two-step approach to solve the profit-maximization problem. First, solve Eq. (2.3) for  $L = L(K)$ , and substitute it into Eq. (2.1):

$$\text{Max}_K \pi(K) = p(q(L(K), K)) \cdot q(L(K), K) - w \cdot L(K) - (1 + r) \cdot K, \quad (2.4)$$

where “ $p(q(L(K), K)) \cdot q(L(K), K) - w \cdot L(K)$ ” is the capitalist’s quasi-rent. Note that deriving  $L = L(K)$  from Eq. (2.3) implies that for any given capital  $K$ , the capital provider can choose an optimal  $L$  to maximize excess profits  $\pi$ . Thus, Eq. (2.4) is the capital provider’s wealth-maximizing problem, i.e., it assumes that the capital provider owns the property rights of the production (i.e., in addition to the opportunity costs of the capital input, the capital provider also owns the excess profits). The capitalist chooses optimal capital input  $K^*$  to maximize profits by equalizing the marginal revenue (i.e., marginal quasi-rent) with the marginal cost (i.e., the cost of capital):

$$\left. \frac{\partial \pi(K)}{\partial K} \right|_{K=K^*} = \left. \frac{\partial [p(q(L(K), K)) \cdot q(L(K), K) - w \cdot L(K)]}{\partial K} \right|_{K=K^*} - (1 + r) \equiv 0. \quad (2.5)$$

After deriving the optimal capital input  $K^* = K(r, w)$ , the optimal labor input  $L^* = L(r, w)$  can be calculated from  $L = L(K)$ . Comparing Eq. (2.1) with Eq. (2.4), we can conclude that in the absence of transaction costs, no matter who (either the capitalist or the entrepreneur) owns the property rights of production (i.e., excess profits) the value of production remains the same.

If on the other hand, we solve Eq. (2.2) first for  $K = K(L)$ , and then substitute it into Eq. (2.1):

$$\text{Max}_L \pi(L) = p(q(L, K(L))) \cdot q(L, K(L)) - (1 + r) \cdot K(L) - w \cdot L, \quad (2.6)$$

where “ $p(q(L, K(L))) \cdot q(L, K(L)) - (1 + r) \cdot K(L)$ ” is the labor’s quasi-rent. Deriving  $K = K(L)$  from Eq. (2.2) implies that for any given  $L$  the labor can choose an optimal  $K$  to maximize excess profits  $\pi$ . Equation (2.6) is the labor’s wealth-maximizing problem, i.e., it assumes that the labor owns the property rights of the production (i.e., in addition to the opportunity costs of the labor input, the labor also owns the excess profits). The labor chooses optimal labor input  $L^*$  to maximize profits by equalizing the marginal revenue (marginal quasi-rent) with the marginal cost (the wage rate):

$$\left. \frac{\partial \pi(L)}{\partial L} \right|_{L=L^*} = \left. \frac{\partial [p(q(L, K(L))) \cdot q(L, K(L)) - (1+r) \cdot K(L)]}{\partial L} \right|_{L=L^*} - w \equiv 0. \quad (2.7)$$

After deriving the optimal labor input  $L^* = L(r, w)$ , the optimal capital input  $K^* = K(r, w)$  can be calculated from  $K = K(L)$ . Comparing Eq. (2.1) with Eq. (2.6), we conclude that in the absence of transaction costs, no matter who (either the labor or the entrepreneur) owns the property rights of production (i.e., excess profits) the value of production is the same.

In sum, in the absence of transaction costs, maximizing Schumpeterian entrepreneur's excess profits and maximizing any resource provider's wealth are equivalent. For example, assuming in Eq. (2.1),  $q(L, K) = L^{1/2}K^{1/2}$ ,  $w = 1$ ,  $r = 0.5625$ , and  $p(q) = 10 - q$ . Solve any of Eqs. (2.1), (2.4) or (2.6), we get the same  $\pi^* = 14.0625$ ,  $L^* = 4.6875$ , and  $K^* = 3$ . These are the results of the Coase theorem: in the absence of transaction costs, who (or even no one) owns the property rights of production (excess profits) is irrelevant to the value of production, and "Professor Steven N. S. Cheung has even argued that, if transaction costs are zero, 'the assumption of property rights can be dropped without in the least negating the Coase Theorem' and he is no doubt right" (Coase 1988, pp. 14–15).

In the above example, suppose that the capital provider owns the property rights of the production (i.e., Eq. (2.4)) and invests three units of  $K$  of his own money in the production, i.e., no debt. Then the rate of return on equity is:  $(14.0625 + 3 \times 1.5625)/3 = 625\%$ , and the market value of the firm, i.e., the share which belongs to fund providers, is:  $14.0625 + 3 \times 1.5625 = 18.75$ . If the capital provider invests one unit of  $K$  of his own money and borrows two units of  $K$  from the capital market, then the rate of return on equity increases to:  $(14.0625 + 1 \times 1.5625)/1 = 1,562.5\%$ , but the market value of the firm is still the same:  $14.0625 + 3 \times 1.5625 = 18.75$ .<sup>1</sup> These results show that in the absence of transaction costs, Modigliani-Miller's first proposition (i.e., the market value of the firm is independent of the firm's capital structure) is just the Coase theorem. Also, Modigliani-Miller's second proposition (i.e., the rate of return on equity increases with the firm's debt-equity ratio) holds, but it has nothing to do with risk.

Suppose that the labor provider owns the property rights of the production (i.e., Eq. (2.6)) and invests 4.6875 units of  $L$  in the production. Then the rate of return on the labor-owner's input is:  $(14.0625 + 4.6875 \times 1)/4.6875 = 400\%$ , and the market value of the firm, i.e., the share which belongs to labor providers, is:  $14.0625 + 4.6875 \times 1 = 18.75$ . If the labor-owner invests only one unit of  $L$  and hires 3.6875 units of  $L$  from the labor market, then the rate of return on the labor-owner's input increases to:  $(14.0625 + 1 \times 1)/1 = 1,506.25\%$ , but the market value

<sup>1</sup> Suppose the capital provider has only one unit of  $K$  and decides not to borrow. Then the capitalist's wealth-maximization becomes:  $Max_L \pi = (10 - L^{1/2} \cdot 1^{1/2})(L^{1/2} \cdot 1^{1/2}) - (1 + 0.5625)(1) - 1 \cdot L$ ; and  $L^* = 6.25$ ,  $K = 1$  and  $\pi^* = 10.9375$ . The capitalist's wealth is:  $\$12.5 (= 10.9375 + 1.5625)$  which is less than  $\$15.625 (= 14.0625 + 1.5625)$  if she borrows two more units of  $K$  from the capital market.

of the firm remains the same:  $14.0625 + 4.6875 \times 1 = 18.75$ .<sup>2</sup> Thus, if the labor provider owns the property rights of production, we can rewrite the Modigliani-Miller first proposition as: in the absence of transaction costs, the market value of the firm is independent of the ratio of hired labor's input to labor-owner's input, and rewrite the Modigliani-Miller second proposition as: the rate of return on the labor-owner's input increases with the ratio of hired labor's input to labor-owner's input, but it has nothing to do with risk.

## 2.2 A Simple Example of the Modigliani-Miller Second Proposition

Suppose you have a patent (a particular technology) so that if you invest \$8,000 now, you will have \$1,200 annually (i.e., rate of return on equity is:  $15\% = 1,200/8,000$ ). Banks provide  $10\%$  risk-free interest rate. You will find that if you borrow \$4,000 from a bank to invest, the rate of return on equity is:  $20\% = 800/4,000$ . If you borrow \$8,000, the rate of return on equity increases to  $\infty = 400/0$ . This example shows that with fixed investment, higher debt brings higher rate of return on equity but brings no risk to the equityholder.<sup>3</sup>

The above example can be generalized as follows. The cash flow of the levered firm  $X$  (i.e., \$1,200) belongs to and is distributed to the debtholders and equityholders:

$$X \equiv X_B + X_S \quad (2.8)$$

where  $X_B$  is the cash flow for debtholders, and  $X_S$ , the cash flow for equityholders. Equation (2.8) is equality by definition. Define  $V_L \equiv S_L + B$ ,  $X \equiv (r_{WACC})(S_L + B)$ ,  $X_B \equiv r_B B$ , and  $X_S \equiv r_S S_L$ , where  $V_L$  (i.e., \$8,000) is the market value of the firm;  $S_L$  is the market value of equity;  $B$  is the market value of debt;  $r_{WACC}$  is the weighted average cost of capital on the levered firm's assets;  $r_B$  is the rate of return on debt; and  $r_S$  is the rate of return on equity. Equation (2.8) can be rewritten as:

$$(r_{WACC})(S_L + B) \equiv r_B B + r_S S_L, \quad (2.9)$$

<sup>2</sup> Suppose the labor provider has only one unit of  $L$  and decides not to hire more. Then the labor's wealth-maximization becomes:  $Max_K \pi = (10 - 1^{1/2} \cdot K^{1/2})(1^{1/2} \cdot K^{1/2}) - (1 + 0.5625)(K) - 1 \cdot (1)$ ; and  $K^* = 3.8072576$ ,  $L = 1$  and  $\pi^* = 11.656157$ . The labor's wealth is:  $\$12.656157 (= 11.656157 + 1)$  which is less than  $\$15.0625 (= 14.0625 + 1)$  if she hires 3.6875 more units of  $L$  from the labor market.

<sup>3</sup> In this case, higher or lower debt will not affect the equityholder's wealth (or welfare). For example, suppose the firm asks the equityholder to withdraw \$4,000 from the firm and borrows \$4,000 from the money market. Then the equityholder can deposit this \$4,000 in a bank, and annually get \$400 from the bank and \$800 from the firm.

or

$$r_S = r_{WACC} + (B/S_L)(r_{WACC} - r_B). \quad (2.10)$$

The cash flow of the firm,  $X$ , is independent of the debt-equity ratio ( $B/S_L$ ). The firm's value  $V_L \equiv S_L + B$  is also independent of the debt-equity ratio. Hence,  $r_{WACC}$  must be independent of the debt-equity ratio. The Modigliani-Miller second proposition can thus be derived from Eq. (2.10): As long as  $r_{WACC}$  is greater than  $r_B$ , increasing debt-equity ratio increases the rate of return on equity. However, increasing debt does not increase any risk to equityholder.

## References

- Coase R (1988) The firm, the market and the law. The University of Chicago Press, Chicago  
Modigliani F, Miller M (1958) The cost of capital, corporation finance and the theory of investment. Am Econ Rev 48:261–297



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