A Study on the Power Functions of the Shewhart $\bar{X}$ Chart via Monte Carlo Simulation

M.B.C. Khoo

Abstract The Shewhart $\bar{X}$ control chart is used to monitor shifts in the process mean. However, it is less sensitive to small shifts. The Shewhart $\bar{X}$ chart's sensitivity can be enhanced by reducing the width of the control limits, increasing the subgroup size and using detection rules to enhance the chart's sensitivity. However, these actions will influence the power functions of the Shewhart $\bar{X}$ chart. A probability table providing the probabilities of detecting shifts in the mean, calculated using the formulae is recommended. However, the main setback is that the calculations of the probabilities using the formulae are complicated, laborious and time consuming. In this paper, a Monte Carlo simulation using the Statistical Analysis System (SAS) is conducted to compute these probabilities. The probabilities computed via Monte Carlo simulation are closed to that obtained using formulae. Therefore, the Monte Carlo simulation method is recommended as it provides savings, in terms of time and cost. In addition, the Monte Carlo simulation method is also more flexible in calculating the probabilities, for different combinations of the detection rules. The results obtained will enable practitioners to design and implement the Shewhart $\bar{X}$ chart more effectively.

1 Introduction

The control chart for monitoring process quality was introduced by Dr. Walter A. Shewhart in the 1920s. Shewhart defined the product attributes, types of product variations and proposed methods to collect, plot and analyze data [1].

The Shewhart $\bar{X}$ control chart is an important tool in Statistical Process Control (SPC). It detects assignable causes in process control so that process investigation and corrective actions can be made before many defective products are produced [2].

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The main objective of applying the Shewhart $\bar{X}$ chart is to prevent failure so that production of low quality products will not occur. This enables cost savings and the production of high quality products.

The Shewhart $\bar{X}$ chart is powerful in detecting large process mean shifts but it is slow in detecting small shifts. Many researches have been made by quality experts to improve the Shewhart $\bar{X}$ chart's sensitivity. Some of these improvements include the synthetic $\bar{X}$ charts by [3–5], the sequential probability ratio test (SPRT) chart by [6], and the time weighted control charts (see [7–9], to name a few). However, more common approaches to increase the sensitivity of the $\bar{X}$ chart are by increasing the subgroup size, employing tighter control limits and applying sensitizing rules, such as those by [10–13]. All these approaches influence the power functions of the Shewhart $\bar{X}$ chart. Tables showing the probabilities of detecting shifts in the mean, computed using the formulae were presented by [14] to guide quality practitioners to construct the Shewhart $\bar{X}$ chart. In this work, a Monte Carlo simulation method is given, where similar results are obtained. This research is motivated by the fact that the Shewhart $\bar{X}$ chart is the most widely used control chart among practitioners, hence a good understanding of the power function of this chart enables the chart to be used more efficiently in process monitoring.

This chapter is organized as follows: The Shewhart $\bar{X}$ chart is discussed in Sect. 2. In Sect. 3, the power functions of the Shewhart $\bar{X}$ chart are presented. The probability table of the power functions of the Shewhart $\bar{X}$ chart is explained in Sect. 4. Section 5 compares the performance between the probabilities obtained via formulae and that computed using Monte Carlo simulation. Lastly, conclusions are drawn in Sect. 6.

2 Shewhart $\bar{X}$ Chart

The Shewhart $\bar{X}$ chart is a times series plot which provides assistance in identifying whether a process is in a state of statistical control. It is a variables control chart with no memory since it uses only the recent data in its control statistics. Having this property, the Shewhart $\bar{X}$ chart performs well when we are interested in the detection of large shifts. The Shewhart $\bar{X}$ contains three crucial decision lines, i.e., the center line (CL), upper control limit (UCL) and lower control limit (LCL). UCL and LCL are occasionally known as the "natural process limits" as they shows threshold at which the quality characteristic of a process being monitored is regarded statistically "unlikely". The Shewhart $\bar{X}$ chart is constructed based on some statistical principles. As a common practice, the limits of the Shewhart $\bar{X}$ chart are taken as $\pm 3\sigma$ from the CL. That is, the UCL is drawn $+3\sigma$ above the CL whereas the LCL is drawn -3σ below the CL. The $\pm 3\sigma$ limits are chosen to strike a balance between the risk of the Type-I and Type-II errors. Here, the value of the mean of the statistic is represented by the CL. All the plotted sample points on the Shewhart $\bar{X}$ chart are connected so that the quality practitioner can have a better view on how a process evolves over time. An out-of-control signal or action signal will be given when a plotted point falls outside the UCL or LCL limit. When a process has shifted, we wish to detect the assignable cause as soon as possible and on the other hand, we wish to have a minimum rate of false alarms when the process is in-control [8]. This is because a slow response to an out-of-control process can cause quality deterioration and increase quality costs while too high false alarm rates can give rise to unnecessary process adjustments and loss of confidence in the control charting methods.

Assume that a quality characteristic is normally distributed with mean μ and standard deviation σ. Here, both μ and σ are assumed known. If $X_1, X_2, \ldots, X_n$ is a sample of size n, then the mean of this sample can be computed as [9]

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} \tag{1}$$

and we know that $\bar{X}$ is normally distributed with mean μ and standard deviation $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$. The probability that any sample mean will fall between

$$\mu + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \tag{2a}$$

and

$$\mu - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \tag{2b}$$

is $1 - \alpha$. Equations (2a) and (2b) can be taken as the UCL and LCL of the Shewhart $\bar{X}$ chart if μ and σ are known. To use the $\pm 3\sigma$ limits, we can just simply replace the $Z_{\alpha/2}$ by 3.

When the sample mean falls outside these limits, the process mean is considered as out-of-control. Corrective actions to search and eliminate the assignable causes are taken so that the process returns to its in-control state again. Note that the above results are approximately correct when the assumption of normality is violated due to the central limit theorem. The values of μ and σ are usually unknown in real situations and therefore, these parameters are required to be estimated from a set of in-control historical Phase-I data. When estimates are used in place of known parameters, at least 20–25 samples are required for generating better and reliable estimations. Note that a revision of the control limits periodically is necessary to ensure an effective use of the control chart.

3 Shewhart $\bar{X}$ Chart's Power Functions

In the literature of statistical quality control, hypothesis testing is always used to summarize an inference on the mean of a population which is given as:

$$\begin{aligned} H_0 &: \mu = \mu_0, \\ H_1 &: \mu \neq \mu_0. \end{aligned} \tag{3}$$

[9] defined the power as

$$\begin{aligned} \text{Power} &= 1 - \beta \\ &= P\{\text{reject } H_0 | H_0 \text{ is false}\}, \end{aligned} \tag{4}$$

where β is the Type-II error probability. In industries, quality users employ a control chart to prevent the production of defective outputs so that quality costs can be minimized [10]. Hence, an in-depth understanding of the method that helps to increase the power of a control chart in detecting process shifts is deemed indispensable.

The calculation of the control limits and sample standard deviation involves the subgroup size, n. So, n is one of the factors that can affect the power of a control chart. The n is proportional to the power of a control chart. As n increases, the power of a control chart increases and vice versa. However, using a large n is impractical in the industrial setting as it will inflate the quality cost. The control chart's sensitivity in detecting process mean shifts can be enhanced via the application of detection rules. Many authors have contributed to new methods on detection rules (also known as runs rules). Recent works on detection rules, were made by [11–15].

When more detection rules are used, the overall Type-I error probability is given by [9, 16]. The overall Type-I error probability is expressed as

$$\alpha = 1 - \prod_{i=1}^{r} (1 - \alpha_i), \tag{5}$$

where

α = probability of Type-I error,

r = number of detection rules used,

α_i = probability of Type-I error of the ith rule, for $i = 1, 2, \ldots, r$.

The r detection rules in (5) are assumed to be independent of one another.

Based on the four detection rules given in Table 1, [14] had estimated the occurrence of a Type-I error probability in the first k subgroups. The following explains the four detection rules considered by [14]:

Rule 1 When one or more points fall beyond the +3σ control limit, a process is declared as out-of-control.

Table 1 Type-I error probabilities for numerous combinations of detection Rules 1–4 [14]

Detection rules	Number of subgroups									
	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$	$k = 10$
1	0.003	0.005	0.008	0.011	0.013	0.016	0.019	0.021	0.024	0.027
1 and 2	0.003	0.006	0.011	0.015	0.020	0.024	0.028	0.034	0.039	0.043
1, 2 and 3	0.003	0.006	0.011	0.016	0.025	0.032	0.040	0.050	0.060	0.060
1, 2, 3 and 4	0.003	0.006	0.011	0.016	0.025	0.032	0.040	0.060	0.070	0.080

Rule 2 When 2 of 3 successive points fall in the same zone and these points are between the $+2\sigma$ and $+3\sigma$ control limits, a process is declared as out-of-control.

Rule 3 When 4 of 5 consecutive points fall in the same zone and these points are beyond the $+1\sigma$ control limit, a process is declared as out-of-control.

Rule 4 When 8 consecutive points fall in the same zone, either above or below the CL, a process is declared as out-of-control.

The implementation of more detection rules will enhance the detection power of a control chart but at the expense of inflating the probability of the overall Type-I error.

4 Shewhart $\bar{X}$ Chart's Probability Tables

A detailed discussion regarding the formulae to compute the probabilities of several combinations of detection rules applied on the Shewhart $\bar{X}$ chart is provided in this section. We will also describe the probability table proposed by [14].

4.1 Formulae and Computation of Probabilities for the Power Functions

The following defines the symbols that will be used in this study:

k = number of subgroups.

$PDS(k)$ = probability of detecting an off-target signal within k subgroups.

p_k = probability of detecting an off-target signal at the kth subgroup.

Note that the subsequent discussions still consider the four detection rules employed by [14] as described in Sect. 3.

4.1.1 Using Detection Rule 1

Let "e" represents the probability that an $\bar{X}$ sample falls beyond the $+3\sigma$ limit (Fig. 1).

Wheeler [14] gave the following fundamental formula for the detection of an off-target signal within k subgroups:

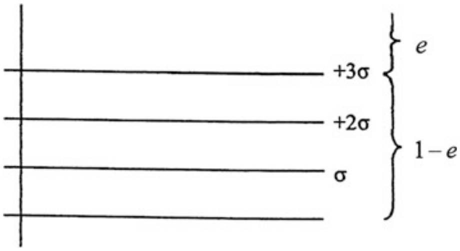

Fig. 1 Probabilities of an $\bar{X}$ sample on the Shewhart $\bar{X}$ chart, using detection Rule 1

$$\begin{aligned}PDS(k) &= \sum_{i=1}^{k} p_i \\ &= e + e(1-e) + e(1-e)^2 + e(1-e)^3 + \cdots + e(1-e)^{k-1} \\ &= \sum_{i=1}^{k} e(1-e)^{i-1} \\ &= 1 - (1-e)^k.\end{aligned} \quad (6)$$

The case discussed here is for the upper sided control chart.

4.1.2 Using Detection Rules 1 and 2

Let

e = the probability that an $\bar{X}$ sample plots beyond the +3σ limit,
f = the probability that an $\bar{X}$ sample plots between the +2σ and the +3σ limits,
g = the probability that an $\bar{X}$ sample plots between the CL and the +2σ limit (Fig. 2).

At least 2 subgroups are needed in performing the analysis if detection Rule 2 is considered. We can use (6) to compute the probability, except for cases where $k = 2$ or more.

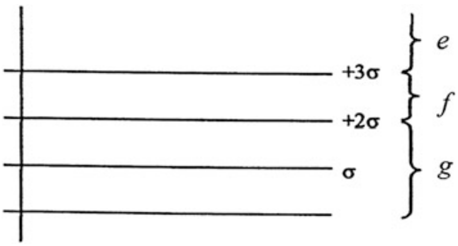

Fig. 2 Probabilities of an $\bar{X}$ sample on the Shewhart $\bar{X}$ chart, using detection Rules 1 and 2

For case k = 2, p_2 and $PDS(2)$ are [14]

$$p_2 = ge + fe + f^2 \tag{7}$$

and

$$\begin{aligned} PDS(2) &= p_1 + p_2 \\ &= e + ge + fe + f^2 \\ &= e + f^2 + (f + g)e. \end{aligned} \tag{8}$$

For case k = 3, p_3 and $PDS(3)$ can be expressed as [14]

$$p_3 = 2efg + 2f^2g + eg^2 \tag{9}$$

and

$$\begin{aligned} PDS(3) &= p_1 + p_2 + p_3 \\ &= e + f^2 + (f + g)e + 2efg + 2f^2g + eg^2 \\ &= e + f^2 + eg + ef(1 + 2g) + 2f^2g + eg^2. \end{aligned} \tag{10}$$

Using the same method, p_k, for k = 4, 5, …, 10 can be obtained using the following equations [14]:

$$p_4 = 3efg^2 + 2f^2g^2 + eg^3. \tag{11}$$

$$p_5 = ef^2g^2 + f^3g^2 + 4efg^3 + 2f^2g^3 + eg^4. \tag{12}$$

$$p_6 = 3ef^2g^3 + 3f^3g^3 + 5efg^4 + 2f^2g^4 + eg^5. \tag{13}$$

$$p_7 = 6ef^2g^4 + 5f^3g^4 + 6efg^5 + 2f^2g^5 + eg^6. \tag{14}$$

$$p_8 = ef^3g^4 + f^4g^4 + 10ef^2g^5 + 5f^3g^5 + 7efg^6 + 4f^2g^6 + eg^7. \tag{15}$$

$$p_9 = 4ef^3g^5 + 4f^4g^5 + 15ef^2g^6 + 9f^3g^6 + 8efg^7 + 2f^2g^7 + eg^8. \tag{16}$$

$$p_{10} = 11ef^3g^6 + 10f^4g^6 + 21ef^2g^7 + 10f^3g^7 + 8efg^8 + 2f^2g^8 + eg^9. \tag{17}$$

4.1.3 Using Detection Rules 1, 2 and 3

Let

e = the probability that an $\bar{X}$ sample plots beyond the +3σ limit,

f = the probability that an $\bar{X}$ sample plots between the +2σ and the +3σ limits,

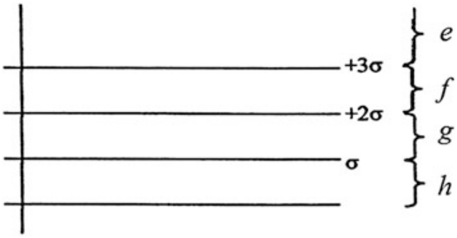

Fig. 3 Probabilities of an $\bar{X}$ sample on the Shewhart $\bar{X}$ chart, using detection Rules 1, 2 and 3

g = the probability that an $\bar{X}$ sample plots between the +σ and the +2σ limits,
h = the probability that an $\bar{X}$ sample plots between the CL and the +σ limit (Fig. 3)

At least 4 subgroups are required in the analysis when using detection Rule 3. The probabilities obtained are the same as the equations given in Sect. 4.1.2, except for cases where k = 4 or more. When $k \geq 4$, p_k is obtained using the following equations [14]:

$$\begin{aligned} p_4 = {} & 3efg^2 + 6efgh + 3efh^2 + 3eg^2h + 3egh^2 \\ & + eg^3 + eh^3 + 3f^2h^2 + 4f^2gh + 2f^2h^2 + g^4 + 4fh^3. \end{aligned} \tag{18}$$

$$\begin{aligned} p_5 = {} & 2ef^2gh + 2f^3gh + 12efg^2h + 14f^2g^2h + 16fg^3h \\ & + 4eg^3h + 4g^4h + ef^2h^2 + f^3h^2 + 12efgh^2 + 6f^2gh^2 \\ & + 6ef^2h^2 + 4efh^3 + 2f^2h^3 + 4egh^3 + eh^4. \end{aligned} \tag{19}$$

$$\begin{aligned} p_6 = {} & 9ef^2gh^2 + 9f^3gh^2 + 30efg^2h^2 + 20f^2g^2h^2 + 10eg^3h^2 \\ & + 16fg^3h^2 + 4g^4h^2 + 3ef^2h^3 + 3f^3h^3 + 20efgh^3 + 8f^2gh^3 \\ & + 10eg^2h^3 + 5efh^4 + 2f^2h^4 + 5egh^4 + eh^5. \end{aligned} \tag{20}$$

$$\begin{aligned} p_7 = {} & 19ef^2g^2h^2 + 18f^3g^2h^2 + 24efg^3h^2 + 25f^2g^3h^2 \\ & + 15fg^4h^2 + 6eg^4h^2 + 3g^5h^2 + 24ef^2gh^3 + 20f^3gh^3 \\ & + 60efg^2h^3 + 28f^2g^2h^3 + 20ef^3g^3 + 16fg^3h^3 + 4g^4h^3 \\ & + 6ef^2h^4 + 5f^3h^4 + 30efgh^4 + 10f^2gh^4 + 15eg^2h^4 \\ & + 6efh^5 + 2f^2h^5 + 6egh^5 + eh^6. \end{aligned} \tag{21}$$

Due to the complexity, the Monte Carlo simulation is employed in the computation of the probabilities p_8, p_9 and p_{10}.

4.1.4 Using Detection Rules 1, 2, 3 and 4

When detection Rule 4 is considered, at least 8 subgroups are called for in the analysis. Thus, the probabilities obtained are similar to that in the equations given in Sect. 4.1.3, except for cases where $k = 8$ or more. Owing to the complexity of the equations [14], reported that a total of 12,000 combinations are needed in the computation for detecting a shift. For this reason, the Monte Carlo simulation method is used in place of the formulae to calculate the probabilities for $k = 8$, 9 and 10.

4.2 Probability Tables

Wheeler [14] introduced a probability table obtained using the formulae. The probability table provides the probability of detecting sustained process mean shifts. The probability table includes four major characteristics defined as follows:

- Shift size:
  The probability table provides a total of 11 sizes of shifts. They are 0.42σ, 0.67σ, 0.95σ, 1.25σ, 1.52σ, 1.72σ, 1.96σ, 2.16σ, 2.48σ, 2.75σ and 3.00σ.
- Number of subgroups, k:
  The number of subgroups, $k = 1$ to $k = 10$ are shown in the probability table.
- Subgroup size, n:
  Different subgroup sizes are indicated in the probability table of [14]. They are $n = 1$ to $n = 10$, $n = 12$, $n = 15$ and $n = 20$.
- Detection rules:
  The necessary probabilities are computed based on several combinations of detection rules.

5 A Study on the Performance

The Statistical Analysis System (SAS) software is used in this study. The SAS computer programs are developed according to the framework of the probability table and the formulae (see also Sects. 4.1.1, 4.1.2, 4.1.3 and 4.1.4) given in [14], to compute the probabilities of detecting a sustained mean shift within particular number of subgroups following the shift. The results obtained are tabulated in Tables 2 and 3.

As can be observed from Tables 2 and 3,

- the probability of detecting a shift in the process mean becomes larger as the size of the shift increases, regardless of the values of n and k, and the combination of the detection rules being used.

Table 2 Probabilities of detecting a sustained shift in the process mean within k subgroups, following a shift, for $n = 2$, using different combinations of detection rules

Detection rule	Shift size (σ)	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$	$k = 8$	$k = 9$	$k = 10$
1 only	0.42	0.010	0.017	0.024	0.033	0.041	0.048	0.056	0.064	0.070	0.078
	0.67	0.022	0.041	0.058	0.078	0.096	0.116	0.132	0.150	0.165	0.186
	1.72	0.289	0.488	0.629	0.735	0.812	0.867	0.902	0.928	0.948	0.963
	3.00	0.893	0.988	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1 and 2	0.42	0.010	0.021	0.037	0.054	0.071	0.087	0.104	0.118	0.133	0.147
	0.67	0.022	0.054	0.097	0.142	0.184	0.219	0.251	0.289	0.320	0.354
	1.72	0.289	0.626	0.831	0.910	0.954	0.978	0.989	0.994	0.998	0.999
	3.00	0.893	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1, 2 and 3	0.42	0.010	0.021	0.037	0.064	0.112	0.143	0.171	0.196	0.227	0.260
	0.67	0.022	0.054	0.097	0.172	0.285	0.348	0.403	0.462	0.507	0.558
	1.72	0.289	0.626	0.831	0.951	0.991	0.997	0.999	1.000	1.000	1.000
	3.00	0.893	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1, 2, 3 and 4	0.42	0.010	0.021	0.037	0.064	0.112	0.143	0.171	0.233	0.275	0.320
	0.67	0.022	0.054	0.097	0.172	0.285	0.348	0.403	0.541	0.593	0.643
	1.72	0.289	0.626	0.831	0.951	0.991	0.997	0.999	1.000	1.000	1.000
	3.00	0.893	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 3 Probabilities of detecting a sustained shift in the process mean within k subgroups, following a shift, for n = 10, using different combinations of detection rules

Detection rule	Shift size (σ)	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6	k = 7	k = 8	k = 9	k = 10
1 only	0.42	0.047	0.090	0.132	0.173	0.210	0.253	0.287	0.318	0.350	0.383
	0.67	0.189	0.340	0.466	0.568	0.652	0.715	0.769	0.810	0.843	0.872
	1.72	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1 and 2	0.42	0.047	0.132	0.234	0.317	0.388	0.452	0.519	0.572	0.619	0.656
	0.67	0.189	0.466	0.682	0.797	0.876	0.921	0.950	0.966	0.979	0.988
	1.72	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1, 2 and 3	0.42	0.047	0.132	0.234	0.389	0.566	0.645	0.715	0.771	0.817	0.855
	0.67	0.189	0.466	0.682	0.868	0.957	0.979	0.990	0.995	0.998	0.999
	1.72	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1, 2, 3 and 4	0.42	0.047	0.132	0.234	0.389	0.566	0.645	0.715	0.846	0.883	0.914
	0.67	0.189	0.466	0.682	0.868	0.957	0.979	0.990	0.999	0.999	0.999
	1.72	0.993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	3.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4 Differences in probabilities, in terms of percentage, for subgroup size, $n = 2$ using different combinations of detection rules

Detection rule	Shift size (σ)	$k = 1$ (%)	$k = 2$ (%)	$k = 3$ (%)	$k = 4$ (%)	$k = 5$ (%)	$k = 6$ (%)	$k = 7$ (%)	$k = 8$ (%)	$k = 9$ (%)	$k = 10$ (%)
1 only	0.42	**20**	**5**	**2**	**3**	**1**	**3**	**3**	**1**	**0**	0
	0.67	**9**	**1**	**2**	**1**	0	**1**	0	0	**1**	**2**
	1.72	**1**	0	**1**	**1**	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1 and 2	0.42	**20**	**2**	**5**	**1**	**1**	0	**2**	**4**	**4**	**3**
	0.67	**9**	**3**	**5**	**1**	**1**	0	**1**	**3**	**3**	**2**
	1.72	**1**	**1**	**1**	**1**	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1, 2 and 3	0.42	**20**	**2**	**5**	**2**	**3**	**1**	**1**	**2**	**1**	0
	0.67	**9**	**3**	**5**	**2**	0	**1**	**2**	**2**	**3**	**2**
	1.72	**1**	**1**	**1**	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1, 2, 3 and 4	0.42	**20**	**2**	**5**	**2**	**3**	**1**	**1**	**7**	**5**	0
	0.67	**9**	**3**	**5**	2	0	**1**	**2**	**2**	**1**	**3**
	1.72	**1**	**1**	**1**	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0

Table 5 Differences in probabilities, in terms of percentage, for subgroup size, $n = 10$ using different combinations of detection rules

Detection rule	Shift size (σ)	$k = 1$ (%)	$k = 2$ (%)	$k = 3$ (%)	$k = 4$ (%)	$k = 5$ (%)	$k = 6$ (%)	$k = 7$ (%)	$k = 8$ (%)	$k = 9$ (%)	$k = 10$ (%)
1 only	0.42	0	**3**	**2**	**2**	**2**	0	0	**1**	**1**	0
	0.67	0	**1**	0	0	0	0	0	0	**1**	**1**
	1.72	0	0	0	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1 and 2	0.42	0	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	0.67	0	**1**	**1**	0	**1**	0	0	0	0	0
	1.72	0	0	0	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1, 2 and 3	0.42	0	**1**	**1**	0	0	**1**	**1**	0	0	**1**
	0.67	0	**1**	**1**	0	0	0	0	**1**	0	0
	1.72	0	0	0	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0
1, 2, 3 and 4	0.42	0	**1**	**1**	0	0	**1**	**1**	**1**	0	0
	0.67	0	**1**	**1**	0	0	0	0	0	0	0
	1.72	0	0	0	0	0	0	0	0	0	0
	3.00	0	0	0	0	0	0	0	0	0	0

- for all n, sizes of shift and combinations of the detection rules, the detection probability within k subgroups rises as k increases.
- The implementation of more detection rules can increase the sensitivity of a control chart, for any k, n and sizes of shifts.

A detailed comparison between Tables 2 and 3 shows that the probabilities in Table 2 are generally less than the corresponding probabilities in Table 3. This indicates that an increase in the subgroup size will result in an enhancement in the power of a control chart, irrespective of the value of k, shift size and detection rules being used.

The difference in terms of percentage is computed to enable us to make a more meaningful comparison between Wheeler's formulae method [14] and the Monte Carlo simulation method. Each probability table is compared in terms of percentages and the percentage difference can be computed as follows:

$$\text{Difference in terms of percentage} = \left|\frac{u - v}{v}\right| \times 100\,\%, \tag{22}$$

where

u = probability obtained via Monte Carlo simulation,

v = probability calculated via formulae.

Tables 4 and 5 show the results for the percentage difference computed using (22). From Tables 4 and 5, we can observe that the probabilities computed by Monte Carlo simulation are quite closed to that obtained from the formulae approach. The existence of differences in the probabilities between the two methods are indicated by the boldfaced entries in Tables 4 and 5. It is very interesting to point out that the differences in the probabilities between the two methods are small. Indeed, for practical applications, the magnitude of these differences can be omitted.

6 Concluding Remarks

In SPC, the Shewhart $\bar{X}$ chart is a very popular process monitoring tool. The implementation of this chart is relatively simple. To enable a fast and effective design and implementation of the Shewhart $\bar{X}$ chart so that a more efficient system of process monitoring can be achieved, Tables 2, 3, 4 and 5 are provided. In fact, the approach by means of formulae is rather cumbersome. When detection Rule 4 is considered (see Sect. 4.1.4), the computation becomes very complicated which increases the chance of computation error. The findings of this study reveal that the Monte Carlo simulation method can favorably replace the formulae approach given by [14]. This is because the Monte Carlo simulation method allows the quality users to calculate the probabilities of detecting a sustained process mean shift within k subgroups, for the various detection rules more easily and quickly with the same accuracy.

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