Chapter 2
The Basis of Microwave Imaging Radar

Synthetic aperture radar (SAR) obtains the two-dimensional high-resolution image by means of pulse compression technology in range dimension and synthetic aperture processing in azimuth dimension. The high range resolution is achieved by transmitting linear frequency modulation (LFM) signal and using pulse compression technology. Due to the movement of the platform, radar echo in azimuth has got the characteristic of LFM signal, and the azimuth resolution is realized by azimuth focusing. LFM signal and pulse compression technology are the basis of SAR imaging.

2.1 Principle of LFM Signal Pulse Compressing

There has been a lot of research on the theory of radar signal for better detection ability and resolution. It is found that ranging accuracy and range resolution are mainly determined by the structure of the signal frequency. As a result, a large bandwidth signal is needed to improve the ranging accuracy and range resolution, and a high power signal is needed to improve the ability of target detection. For the limitation of the radar peak transmit power, the high average transmission power can only be obtained by increasing the time width of the signals. LFM signal is a typical example of a large time–bandwidth product pulse compression signal, which is the earliest and most widely used pulse compression signal also.

The LFM signal with the normalized amplitude can be expressed by

\[ x(t) = \text{rect}\left(\frac{t}{T_p}\right) \cos(\omega_0 t + \pi kr^2) \]  

(2.1)
where \( T_p \) is the pulse length, \( \text{rect}\left(\frac{t}{T_p}\right) \) is rectangular function, which is defined as

\[
\text{rect}\left(\frac{t}{T_p}\right) = \begin{cases} 
1 & \frac{T_p}{2} \leq t \leq \frac{T_p}{2} \\
0 & \text{其他}
\end{cases}
\]

The instantaneous angular frequency

\[
\omega(t) = \frac{d}{dt}(\omega_0 t + \pi k t^2) = \omega_0 + 2\pi k t
\]

where \( k \) is the chirp rate. During the pulse duration, the scope of the signal frequency is the bandwidth.

\[
B = k T_p
\]

(2.2)

The LFM signal is represented as a complex form

\[
s(t) = \text{rect}\left(\frac{t}{T_p}\right) e^{j(\omega_0 t + \pi k t^2)}
\]

(2.3)

The LFM pulse compression can be achieved by matched filter. According to the matched filter theory, the matched filter corrects the phase shift of each component in the signal spectrum and adds coherently at a certain time together. Due to the signal energy unchanged during this period, the matched filter makes the time width of LFM signals compressed greatly, and forms a high signal spike. The matched filter is

\[
h(t) = s^*(t_0 - t)
\]

(2.4)

After the matched filter, the output signal is

\[
s_0(t) = T_p \cdot \sin\left[\frac{\pi k T_p (t - t_0)}{\pi k T_p (t - t_0)}\right], e^{j\omega_0 (t - t_0)}
\]

(2.5)

The peak of output signal appears at \( t = t_0 \), and the waveform is in the form of sinc function shown in Fig. 2.1.

Time resolution can be defined as the width of output pulse at peak fall to 4 dB

\[
\rho_T = \frac{1}{B}
\]

(2.6)
Time resolution is usually defined as the width of output pulse at peak fall to 3 dB

\[ \rho_\tau = \frac{0.886}{B} \]  

\[ (2.7) \]

### 2.2 Principle of Synthetic Aperture Radar Imaging

The radar is mounted on the flight vehicle to make a straight line motion along with the platform. The direction of flight is defined as the azimuth dimension, and perpendicular to the flight direction is the range dimension. Schematic diagram of the side-looking SAR is shown in Fig. 2.2.

In Fig. 2.2, the range extent \( W_s \) of radar beam is called swath width, and the azimuth extent \( L_s \) of radar beam at the same slant range \( R \) is called synthetic aperture length, the time \( T_s \) of radar flying over synthetic aperture length is called synthetic aperture time.

The principle of synthetic aperture radar imaging can be explained from several points of view.

- **Doppler Sharpening:** Using Doppler processing to sharpen the narrow azimuth beam of antenna.
- **Pulse Doppler:** Using Doppler filter to distinguish the targets from different positions.
- **Synthetic Aperture:** Using a small moving antenna synthesizes a large antenna aperture to form a narrow beam.
- **Matched Filter:** Using Matched filter to optimize detection of signals.
Holography: SAR is regarded as a kind of microwave holographic imaging technology from the point of optical holography.

Although the interpretations of the above points of view are different, the essence is the same.

1) **Doppler Frequency analysis**

Geometry of SAR moving along track from S to S1 and passing over the target P is presented in Fig. 2.3. The closest slant range is $R_0$ when radar is at position S ($t = 0$). If the influence of earth autorotation can be ignored, at time $t$, radar moves to the S1, and the distance between the radar and the target is $R(t)$. 

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Fig. 2.2 Schematic diagram of the side-looking SAR

Fig. 2.3 Geometry of SAR imaging in the slant plane
Radar echo is represented as a complex form

\[ s_r(t) = \text{rect} \left[ \frac{t - \frac{2R(t)}{c}}{T_p} \right] e^{j \left\{ \omega_0 \left[ t - \frac{2R(t)}{c} \right] + \pi k_r \left[ t - \frac{2R(t)}{c} \right]^2 \right\} } \] (2.8)

where \( c \) is the velocity of light, since \( x \ll R_0 \), the distance from \( S \) to \( P \) is approximated as

\[ R(t) = \sqrt{R_0^2 + x^2} = \sqrt{R_0^2 + (v_st)^2} \approx R_0 + \frac{(v_st)^2}{2R_0} \] (2.9)

where \( v_s \) is the velocity of flight vehicle, the phase \( \phi(t) \) is

\[ \phi(t) \approx \omega_0 \left( t - \frac{2R_0}{c} \right) + \pi k_r \left( t - \frac{2R_0}{c} \right)^2 - \frac{2\pi}{\lambda R_0} (v_st)^2 - 2\pi k_r \left( t - \frac{2R_0}{c} \right) \frac{(v_st)^2}{cR_0} \] (2.10)

where the first and second terms compose a LFM signal in range, the third term is azimuth Doppler signal; the fourth term is coupling between range and azimuth. In the airborne SAR processing, the coupling phase term is small enough to satisfy the condition of two dimensions independent and separated between the range and azimuth, and can be ignored. However in the spaceborne SAR processing, this coupling phase term cannot be ignored. The main purpose of difference imaging algorithm is to compensate the effect of this coupling phase term. After compensated, the phase of echo signal is

\[ \phi(t) \approx \omega_0 \left( t - \frac{2R_0}{c} \right) + \pi k_r \left( t - \frac{2R_0}{c} \right)^2 - \frac{2\pi}{\lambda R_0} (v_st)^2 \] (2.11)

Azimuth instantaneous Doppler frequency in the third term is

\[ f_d(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ -\frac{2\pi}{\lambda R_0} (v_st)^2 \right] = -\frac{2v_s^2}{\lambda R_0} t \] (2.12)

The echo in azimuth is a linear frequency modulated signal, and the chirp rate is

\[ k_a = -\frac{2v_s^2}{\lambda R_0} \] (2.13)
The difference of the instantaneous Doppler frequency between two targets apart \( \Delta x \) in azimuth is

\[
\Delta f_d = \frac{2v_s \cdot \Delta x}{\lambda R_0} \quad (2.14)
\]

The Doppler bandwidth and time duration of echo for each point target are

\[
B_a = \frac{2v_s L_s}{\lambda R_0} \quad (2.15)
\]

\[
T_S = \frac{L_s}{v_s} \quad (2.16)
\]

where \( L_s = \frac{\lambda R_0}{D_a} \) is synthetic aperture length, \( D_a \) is antenna aperture length in azimuth.

In order to distinguish the two point targets, the echoes can be multiplied by the LFM signal with the same chirp rate of Eq. (2.13), and the output signals are two pulses modulated by different frequencies with pulse duration \( T_S \). According to the different modulation frequency, the different bandpass filters can be used to separate the output signals, as shown in Fig. 2.4.

The frequency spectrum of the single frequency pulse signal with a time length of \( T_S \) is present in the form of sinc function, the spectrum width is

\[
B = \frac{1}{T_S} = \frac{v_s}{L_s} \quad (2.17)
\]

The bandwidth of bandpass filter is B. If the difference between the instantaneous Doppler frequencies for two point targets is less than B, the two targets cannot be distinguished. The azimuth resolution is defined as

\[
p_a = \Delta x = \frac{\lambda R_0}{2L_s} = \frac{D_a}{2} \quad (2.18)
\]

![Fig. 2.4 Doppler frequency analysis processes](image-url)
The azimuth resolution is half of the length of the antenna aperture, which is independent of the distance from the radar to the target and the working wavelength. This is one of the unique advantages of SAR. The reasonable antenna size design can provide a higher azimuth resolution.

(2) Synthetic Aperture

The concept of linear array antenna is used for reference in understanding synthetic aperture radar.

Figure 2.5 shows the schematic diagram of linear array antenna. The array element spacing is \( d \), and off-axis angle is \( \theta \). Assume that each element is same, echo from each element in-phase stacked in the \( \theta = 0 \) direction reaches the maximum. The array pattern is

\[
F(\theta) = \frac{\sin\left(\frac{\pi Nd}{\lambda} \sin \theta\right)}{N \sin\left(\frac{\pi d}{\lambda} \sin \theta\right)}
\]

(2.19)

when \( \theta \) is very small, the array pattern is approximate to

\[
F(\theta) \approx \frac{\sin\left(\frac{\pi Nd}{\lambda} \sin \theta\right)}{\frac{\pi Nd}{\lambda} \sin \theta} = \text{sinc} \left(\frac{\pi Nd}{\lambda} \sin \theta\right)
\]

(2.20)

Synthetic aperture radar transmits signals periodically along with the movement of the platform along the track. Figure 2.6 shows the position of the radar transmitting signal that is arranged into a linear array in azimuth and each element is the real antenna. Array element spacing is \( \Delta x = \frac{v \text{ PRF}}{N} \), and array length is \( L_s = \frac{\lambda R_0}{D_s} \).
In terms of Eq. (2.8), the azimuth echo in each space position can be expressed as

\[ s_a(n \Delta x) = e^{-j\frac{2\pi}{\lambda R_0} (n \Delta x)^2} \]  

(2.21)

Coherent processing in azimuth needs to correct the phase of the echo signal for different azimuth positions. The phase compensation function is as

\[ \phi(n \Delta x) = \frac{2\pi}{\lambda R_0} (n \Delta x)^2 \]  

(2.22)

After phase correction, the echo in different azimuth positions can be stacked in same phase. Considering the echo round-trip phase shift between the target and radar, the beamwidth of synthetic antenna array is

\[ \beta_{az} = \frac{\lambda}{2L_a} \]  

(2.23)

So the azimuth resolution is defined as

\[ \rho_a = \beta_{az} R_0 = \frac{\lambda R_0}{2L_s} = \frac{D_a}{2} \]  

(2.24)

The azimuth resolution in Eq. (2.24) from the viewpoint of synthetic aperture is the same as Eq. (2.18).

(3) **Matched filter**

Equation (2.11) represents the phase of point target echo for the synthetic aperture radar. The third term for azimuth instantaneous Doppler frequency shows that the echo in azimuth can be viewed as LFM signal, and the duration is the synthetic aperture time \( T_s \). The Doppler bandwidth is defined as the scope of the Doppler frequency changing over the synthetic aperture length, which can be expressed as

\[ B_a = |k_a| \cdot T_s = |k_a| \cdot \frac{L_s}{v_s} = \frac{2v_s}{D_a}, \]  

(2.25)

where \( L_a \) is the azimuth length of antenna. The echo in azimuth can be compressed by the matched filter with Doppler bandwidth \( B_a \), the impulse response of matched filter is

\[ h_a(t) = e^{j\frac{2\pi}{\lambda R_0}(v_s t)^2} \]  

(2.26)

After compressing processing of matched filter, the azimuth resolution is
The azimuth resolution in Eq. (2.27) from the viewpoint of the matched filter remains the same with Eq. (2.18).

Synthetic aperture radar can also be understood from the point of view of microwave holography. The microwave holography draws lessons from the concept of optical holography, and has played a prominent role in proposing and improving the early synthetic aperture radar optical signal processing.

2.3 Theory Model of Synthetic Aperture Radar [1, 2]

Synthetic aperture radar is a kind of microwave imaging radar, and radar image can reflect microwave backscattering characteristics of the terrain. If the microwave backscattering coefficient is expressed as \( \sigma_0(x, r) \), the synthetic aperture radar can be equivalent to a two-dimensional filtering network. Based on the network theory, if the impulse response function (IRF) of a linear network is \( h(x, r) \), the output of the system is the radar image \( \tilde{\sigma}_0(x, r) \).

\[
\tilde{\sigma}_0(x, r) = \sigma_0(x, r) \otimes h(x, r)
\] (2.28)

Only if the IRF is \( \delta \) function, radar image can be the correct reproduction of backscattering coefficient of the terrain. In order to measure the radar back scatter of targets, it is necessary that the IRF of SAR will close to \( \delta \) function as far as possible.

The IRF of SAR is illustrated by two-dimensional pulse compression in Fig. 2.7. The LFM signal transmitted by radar is

\[
S(t) = \text{Re}[\mu(t) \exp(j2\pi f_0 t)] = \text{Re} \left[ A \exp \left( j \cdot \frac{1}{2} k_r t^2 \right) \exp(j2\pi f_0 t) \right],
\] (2.29)

where \( u(t) \) is complex amplitude, which is a function of time. \( A \) is the amplitude of LFM signal, and \( k_r \) is the chirp rate. The echo backscattered from a point target is

\[
S_r(t) = \text{Re}[k \mu(t - t_d) \exp(j2\pi f_0 (t - t_d))]
\] (2.30)

Using Eq. (2.9) to express the delay time \( t_d \), the following can be obtained:

\[
t_d = \frac{2R}{c} = \frac{2R_0}{c} + \frac{(x - X_p)^2}{cR_0} = \frac{2R_0}{c} \left[ 1 + \frac{(x - X_p)^2}{2R_0^2} \right],
\] (2.31)

where \( X_t = v_s t \).
From Eq. (2.30) and Eq. (2.31), two kinds of time concepts are included in the signal complex amplitude and the delay. The former Eq. (2.30) reflects the change within the signal duration, and the latter Eq. (2.31) reflects the position change caused by the movement of the vehicle. Usually, in the signal duration the radar position change caused by the vehicle going forward is not worth mentioning, and can be ignored. So it can be defined that the time concept in the complex amplitude as “fast” time, and the time in the signal delay as “slow” time. Since the vehicle movement can be ignored in signal duration, by considering the signal duration as “fast” time, the “slow” time change can be viewed as a zero. It is to say the “fast” time and the “slow” time can be separated by processing under certain condition.

As we know, the signal included in complex amplitude changing with the “fast” time determines the range resolution, and the signal included in echo delay changing with “slow” time determines the azimuth resolution.

The echo signal is expressed as the two-variable function of the “fast” changing time $t$ and the position of the vehicle $x$. 

---

**Fig. 2.7** Two-dimensional pulse compression of SAR
Similar to the derivation of Sect. 2.2, the echo signal is matched filtered and self-correlation integrated after synchronization demodulation and amplitude normalization, the output signal is the IRF $h(x, r)$ of SAR.

\[
S_r(x, t) = \text{Re} \left[ k \mu \left( t - \frac{2R_0}{c} \left[ 1 + \frac{(x - X_p)^2}{2R_0^2} \right] \right) \exp \left( j2\pi f_0 \left( t - \frac{2R_0}{c} \left[ 1 + \frac{(x - X_p)^2}{2R_0^2} \right] \right) \right) \right] \quad (2.32)
\]

The envelope of the IRF is

\[
E[h(x, t)] = \left\{ T \cdot e^{-j\frac{\lambda}{2} t^2} \cdot \sin \left( \frac{\pi t}{\tau_r} \right) \right\} \cdot \left\{ L_a e^{j\frac{\lambda}{2} x^2} \cdot \sin \left( \frac{\pi x}{\rho_a} \right) \right\} \quad (2.34)
\]

Formula (2.34) shows that the envelope of IRF has the shape of sinc function both in azimuth and in range for the unweighted SAR. Due to the independence of the two variables, the IRF of SAR can be regarded as the product of two impulse responses in range dimension and azimuth dimension. That is

\[
h(x, t) = h_r(x, t) \cdot h_a(x, t) \quad (2.35)
\]

The theoretical model of SAR is represented in Fig. 2.8. SAR signal processing can be divided into two steps: pulse compression in range and beam sharpening in azimuth.

Based on the analysis above, the echo of SAR will be divided into the independent two-dimensional signal with the “fast” time in range and the “slow” time in azimuth while the position of vehicle can be ignored in the duration of transmitting pulse. The range processing and the azimuth processing can be independent deal with; However, there is the coupling between signals of range and azimuth and the coupling cannot be ignored in most cases, especially for spaceborne SAR, which will result in serious range migration.
2.4 Pulse Compression Technology and Synthetic Aperture Processing

High resolution of SAR is achieved by pulse compression in range and synthetic aperture processing in azimuth. In general, the radar transmitting signal (range) and the synthetic aperture signal (azimuth) have the character of linear frequency modulation. In fact, the signal processing of SAR is pulse compression of LFM signal by matched filtering in range and azimuth dimensions, which provides fine delay time resolution in range and fine Doppler frequency resolution in azimuth.

Since the echo-Doppler signal can be approximated as a LFM signal, the process of focused SAR is similar to the pulse compression. The similarity comparison is shown in Table 2.1.

Because the echo-Doppler signal is generated by uniform speed and linear forward motion of the vehicle, so its parameters are different from that of the normal LFM signal. There are three main differences:

(1) Echo-Doppler signal duration (synthetic aperture time) is generally in the second order, and the normal LFM signal pulse duration is usually in the microsecond order.

(2) The echo-Doppler signal bandwidth is very small, generally a few hundred Hertz, while the normal LFM signal bandwidth is greater than MHz. Although the time–bandwidth product of the echo-Doppler signal in azimuth and the normal LFM signal in range is equivalent to the order of magnitude, the time duration and bandwidth are very different.

(3) The frequency modulation slope of the echo-Doppler signal varies inversely with the slant range, while the chirp rate the normal LFM signal is constant.

For convenient next analysis, some expression forms of azimuth resolution are as follows:

Using Doppler bandwidth \( B_d \) representation

\[
B_d = \frac{2 v_s}{D_a}
\]

\[
T_s = \frac{\lambda R}{D_a v_s}
\]

\[
f_{dr} = -2 \frac{v_s^2}{(\lambda R)}
\]

\[
B T_p = B_d T_s
\]

\[
K = \frac{B}{T_p}
\]

\[
f_{dr} = -2 \frac{v_s^2}{(\lambda R)}
\]

\[
B T_p = B_d T_s
\]

\[
D = \frac{c}{2 B}
\]

\[
\frac{v_s}{B_d}
\]

\[
1/B_d
\]

Table 2.1 The similarity comparison between synthetic aperture processing and pulse compression

<table>
<thead>
<tr>
<th></th>
<th>Pulse compression</th>
<th>Synthetic aperture processing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>( B )</td>
<td>Doppler bandwidth ( B_d = 2 v_s /D_a )</td>
</tr>
<tr>
<td>Time duration</td>
<td>( T_p )</td>
<td>Synthetic aperture time ( T_s = \lambda R/D_a v_s )</td>
</tr>
<tr>
<td>Chirp rate</td>
<td>( K = B/T_p )</td>
<td>( f_{dr} = -2 \frac{v_s^2}{(\lambda R)} )</td>
</tr>
<tr>
<td>Time–bandwidth product</td>
<td>( B T_p )</td>
<td>( B_d T_s )</td>
</tr>
<tr>
<td>Velocity</td>
<td>Light velocity ( c )</td>
<td>Carrier velocity ( v_s )</td>
</tr>
<tr>
<td>Time resolution</td>
<td>( 1/B )</td>
<td>( 1/B_d )</td>
</tr>
<tr>
<td>Space resolution</td>
<td>( c/2B )</td>
<td>( v_s /B_d )</td>
</tr>
</tbody>
</table>
\[ \rho_a = \frac{v_s}{B_d} \]  

(2.36)

Using synthetic aperture \( L_a \) representation

\[ \rho_a = \frac{\lambda R}{2L_s} \]  

(2.37)

Using synthetic time \( T_s \) representation

\[ \rho_a = \frac{\lambda R}{2v_f T_s} \]  

(2.38)

Using real antenna length \( D_a \) representation,

\[ \rho_a = \frac{D_a}{2} \]  

(2.39)

### 2.5 Focus Depth and Range Migration [3, 4]

The matched filtering can be also called as focusing process, which compensates fully the quadratic phase of the echo signal, and got the best azimuth resolution \( D/2 \). There is the focus depth for focus SAR.

The frequency modulation slope of SAR azimuth echo is inversely proportional to the slant range, so the matched filter has to change with the range for focus processing. In the actual processing, the matched filter parameter is not necessary to change with each range gate due to the complexity of hardware; it can use the same matched filter parameter to focus processing within a certain range.

The depth of focus is an important concept in SAR imaging processing. The depth of focus is the variation of the detection range that is adapted to the same matched filter in azimuth to satisfy the requirement of given resolution. The depth of focus determines the number of different reference signals that the processor

![Fig. 2.9 The depth of focus](image)
needs. If the depth of focus is large enough, the full range of action in the system requires only a reference signal; otherwise many different reference functions are needed.

Figure 2.9 shows the depth of focus. The target P is at the distance R far from the radar. When the aircraft flies from \(-L_s/2\) to \(L_s/2\) along the coordinates x, all echoes from the target P can be received. The wave front of the echo for P point target is a sphere (round is for plane). As shown in Fig. 2.9, the maximum phase error occurs at both ends of the point. Er can be expressed as

\[
Er = \sqrt{R^2 + \left(\frac{L_s}{2}\right)^2} - R \approx \frac{L_s^2}{8R}
\]  

(2.40)

Maximum phase error is

\[
\frac{4\pi}{\lambda} \cdot Er = \frac{\pi L_s^2}{2\lambda R}
\]  

(2.41)

If the azimuth resolution \(\rho_a\) is known, the synthetic aperture length \(L_s\) is

\[
L_s = \frac{\lambda R}{2\rho_a}
\]  

(2.42)

Equation (2.41) can be rewritten as

\[
\frac{4\pi}{\lambda} \cdot Er = \frac{\pi \lambda R}{8\rho_a^2}
\]  

(2.43)

The Doppler phase \(\frac{4\pi}{\lambda} \cdot Er\) will be corrected to stack in the same phase during synthetic aperture focusing. Equation (2.43) shows that \(\frac{4\pi}{\lambda} \cdot Er\) is proportional to slant range. If the processor is focused on the target at the distance \(R\), the focusing performance will get worse for target at the distance \(R + \Delta R\).

The variation of the Doppler phase due to \(\Delta R\) is

\[
\frac{4\pi}{\lambda} \Delta Er = \frac{\pi \lambda}{8\rho_a^2} \Delta R
\]  

(2.44)

In order to focus processing approximately for the target at the distance \(R + \Delta R\), and the azimuth resolution cannot be decreased remarkable, the maximum allowable quadratic phase error will be \(\pi/4\). That is

\[
\frac{\pi \lambda \cdot \Delta R}{8\rho_a^2} \leq \frac{\pi}{4}
\]  

(2.45)
The variation of slant range $\Delta R$ is

$$\Delta R \leq \frac{2\rho_a^2}{\lambda} \quad (2.46)$$

The depth of focus is

$$\Delta R_m = \frac{2\rho_a^2}{\lambda} \quad (2.47)$$

Equation (4.27) shows that the depth of focus depends on azimuth resolution and wavelength. The lower the resolution, the shorter the wavelength, the greater the depth of focus, and the simpler the focusing processor. Otherwise, the higher resolution will make the higher requirement on the maximum phase error, the depth of focus becomes smaller and the filter parameters will be updated faster.

It is assumed that the echo from the target at the distance $R$ is received and stored within the same range gate. But that is not always the case. When the LFM signals are transmitted from one pulse to the next one along the track, the echoes of one static point target are not sampled always in the same range gates. Due to the coupling between the range and azimuth dimensions, the peak of range compression responses will appear at different range gates (Fig. 2.10), that the meaning of range migration. In order to synthetic aperture processing, it necessary to sample the echo at the right range gate and compensate the range migration. The range migration is divided into two parts: the range walk and the range curvature (Fig. 2.10).

The range migration is

$$R(s) \approx R_0 + R'_0(s - s_0) + R''_0(s - s_0)^2 / 2 \quad (2.48)$$

**Fig. 2.10** Point target position after range compression in the $(s, t)$ plane
where \( s \) is the “slow” time in azimuth, \( R(s) \) is second-order Taylor expansion of the slant range from the radar to target in synthetic aperture time, and \( s_0 \) is the midpoint of the synthetic aperture time. The Doppler phase is

\[
\phi(s) = -\frac{4\pi}{\lambda} R(s) 
\]

The Doppler centroid frequency is

\[
f_{dc} = \frac{\phi'(s)}{2\pi} = -\frac{2}{\lambda} R'(s_0) 
\]

The Doppler frequency rate is

\[
f_{dr} = \frac{\phi''(s)}{2\pi} = -\frac{2}{\lambda} R''(s_0) 
\]

So Eq. (2.48) can be rewritten as

\[
R(s) = R_0 - \frac{\dot{\phi}_{dc}}{2} (s - s_0) - \frac{\dot{\phi}_{dr}}{4} (s - s_0)^2 
\]

It is defined that the linear item is range walk and the quadratic term is range curve in Eq. (2.52). When the synthetic aperture length is comparable to the slant range, the range curve is particularly evident. The total range change is called as the
range migration $\Delta R = R(s) - R_0$, which is inevitable in SAR processing, and it changes with system parameters.

When $s - s_0 = \pm T_s/2$, the range migration reaches the maximum for orthogonal side-looking SAR that is expressed as

$$\Delta R = \frac{\lambda f_{dr}}{4} \left( \frac{T_s}{2} \right)^2 = \frac{\lambda^2 R_0}{32 \rho_a^2}$$

(2.53)

Generally, if the maximum range migration is less than $\rho_r/4$,

$$\left( \frac{\rho_a}{\lambda} \right)^2 \geq \frac{R_0}{8 \rho_t}$$

(2.54)

The range migration does not have to be compensated. The migration is proportional to illumination time and slant range. If the migration is less than one range gate for short or medium range, this range migration effects can be ignored.

### 2.6 The Main Technical Parameters of SAR

Due to high altitude and wide coverage, the spaceborne SAR has been paid great attention by lot of countries in the world. The final output of SAR is a grayscale image of the terrain, and the image quality depends on the system specifications, such as resolution, side lobe ratio, swath, slant range, system sensitivity, ambiguity, radiometric resolution, radiation precision, dynamic range, etc. The main technical parameters of SAR system are introduced briefly as follows.

#### 2.6.1 Resolution [5]

Resolution is defined as the ability of the imaging radar to distinguish the two closely spaced point targets. Figure 2.11 shows the magnitude of two point signals which present the same radar section at the receive output. It is the different possible definitions of the radar resolution.

The first definition: Nominal resolution

The nominal resolution is defined as the signal peak width of a single point target echo measured at 3 dB below its maximum value. This is the simplest definition of the radar resolution, which is only related to the echo of a single point target.

The second definition: Two-point target echo

Resolution is defined as the minimum interval required between two point targets with the same RCS in order to observe a trough between the two pulse
compression peaks. It corresponds to the peak width of a single point target echo measured at 6 dB below its maximum value.

Suppose the amplitude of the two identical point targets echo is \( a \), with a position shift equal to the 6 dB peak width, and random phase shift \( \varphi \), the trough amplitude between the two signals is

\[
|u(0)_{a,\varphi}| = \left| \frac{a}{2} + \frac{a}{2} e^{i\varphi} \right| = a \sqrt{\cos^2(\varphi) + \cos^2(\varphi) \frac{1}{2}}. \tag{2.55}
\]

Equation (2.55) has the maximum value \( a \) only for \( \varphi = 0 \), this means that for any value of phase shift between both target echo, \( |u(0)_{a,\varphi}| \leq a \). There will be a trough between both peaks if the two target position shift is greater than 6 dB peak width.

In most cases, the definition has a limited operational interest, because the trough is not deep enough to be observed. Therefore, the third definition is given as follows.

The third definition:

Resolution is defined as the minimum interval required between two point targets with the same RCS, in order to give a trough of more than 3 dB between the two pulse compression peaks. The definition gives a value about two times as great as the resolution at 3 dB of the first definition.

In order to establish a link between the first and the third definition, suppose that the phase shift of the matched filter outputs of is \( \varphi \), when \( \varphi = 0 \), the two echoes are added. If the intersection of the two peaks is 9 dB below their maximum value, then the trough between the two peaks is 3 dB. Under any other relation phase between signals from the two targets, the trough is greater than 3 dB. Therefore, the third definition of resolution will be corresponding to 9 dB width of the peak at the output of the matched filter for the first definition.

2.6.1.1 Range Resolution [6, 7]

The range resolution of SAR is mainly determined by the bandwidth \( B \) of LFM signal, that is

\[
\rho_s = \frac{c}{2B}. \tag{2.56}
\]

The wider bandwidth, the higher the range resolution. In Eq. (2.56), \( \rho_s \) is the slant range resolution, the ground range resolution \( \rho_g \) can be obtained.

\[
\rho_g = \frac{\rho_s}{\sin \eta}, \tag{2.57}
\]

where \( \eta \) is incident angle.
2.6.1.2 Azimuth Resolution [8]

The azimuth resolution of SAR, independent to the slant range and wavelength, is half of the antenna length.

\[ \rho_a = \frac{D_a}{2} \]  

(2.58)

The azimuth Doppler bandwidth is \( B_a = \frac{2v_s}{D_a} \). In order to satisfy the sampling theorem, the system pulse repetition frequency (\( f_{\text{PRF}} \)) must be satisfied

\[ f_{\text{PRF}} > B_a = \frac{2v_s}{D_a} \Rightarrow \frac{1}{f_{\text{PRF}}} < \frac{D_a}{2v_s} \]  

(2.59)

Azimuth sampling interval is

\[ \Delta \chi = \frac{v_s}{f_{\text{PRF}}} < \frac{D_a}{2} \]  

(2.60)

In order to satisfy the requirement of sampling theorem, the azimuth sampling interval must be less than azimuth resolution.

The azimuth resolution given by Eq. (2.60) is the best resolution of SAR. In fact, due to the weight of the antenna pattern, the azimuth Doppler spectrum does not have the shape of the rectangular spectrum, which makes the main lobe broadening and the resolution decreased. On the other hand, the processing bandwidth in the actual system does not necessarily select the whole Doppler bandwidth. Taking into account the different operating modes of SAR system, the signal processing bandwidth may only select a part of the Doppler bandwidth, which will also cause a decrease in the azimuth resolution.

The synthetic aperture length illuminated by the 3 dB main lobe of the uniform aperture antenna is

\[ L_S = K_A \cdot \frac{\lambda R_0}{D_a} \]  

(2.61)

where \( K_A \) is antenna pattern factor in azimuth, and \( K_A = 0.886 \). The Doppler bandwidth is

\[ B_a = K_A \cdot \frac{2v_s}{D_a} \]  

(2.62)
If $K_P$ is the ratio between signal processing bandwidth $B_P$ and Doppler bandwidth $B_a$, $K_P \leq 1$, $B_P$ is

$$B_P = K_P \cdot B_a = K_P K_A \frac{2v_s}{D_a}$$  \hspace{1cm} (2.63)

The broadening factor of azimuth compression output due to antenna pattern weighting is expressed as $K_a$, $K_a > 1$,

$$\rho_a = K_a K_A \frac{v_s}{B_P} = K_a \frac{D_a}{K_P} \cdot \frac{1}{2}$$  \hspace{1cm} (2.64)

### 2.6.2 Peak Side Lobe Ratio and Integrated Side Lobe Ratio [5]

Peak-to-side lobe ratio (PSLR) is a power ratio between the main peak and the side lobes located in an interval of 10 times the peak width, which is usually expressed in dB. Figure 2.12 shows the definition of PSLR.

Secondary side lobe ratio (SSLR) is a power ratio between the main peak and the side lobes located in an interval between 10 times and 20 times the peak width, which is usually expressed in dB. Figure 2.13 shows the definition of SSLR.

Integrated side lobe ratio (ISLR) is defined as a power ratio between the main peak lobe and the total power of all the side lobes, which is usually expressed in dB. Figure 2.14 shows the definition of ISLR.

![PSLR Definition](image)
The mathematical formal definition of $f_{\text{ISLR}}$ is

$$f_{\text{ISLR}} = \frac{\int_{-T}^{-1/B} |u(\tau)|^2 d\tau}{\int_{-1/B}^{1/B} |u(\tau)|^2 d\tau + \int_{1/B}^{T} |u(\tau)|^2 d\tau},$$

(2.65)

where $|u(\tau)|$ is the impulse response function (IRF) at the output of matched filter, $T$ is the pulse length. The length of the impulse response function (IRF) is $2T$. In practice, the far side lobe is too low to be measured, so it is necessary to introduce the second definition of ISLR. The side-lobe integration is limited to an interval of 20 times the main lobe width; Measurement of ISLR is illustrated in Fig. 2.15.
2.6.3 Swath Width

The swath of SAR is the ground coverage area in range direction illuminated by antenna beam, which depends on the range beamwidth, the incident angle, the height of platform and PRF, etc.

The swath is defined as the coverage area illuminated by antenna in 3 dB beamwidth.

\[
W_g = \frac{\beta_e R_0}{\cos \eta} = \frac{\lambda R_0}{H_a \cdot \cos \eta},
\]

where \( \beta_e \) is the range beamwidth of the antenna, \( \eta \) is the incident angle, \( H_a \) is the width of antenna. For the radar to operate correctly, the echo time in swath \( W_g \) should be less than pulse repetition time (PRT), the relationship is given by

\[
W_g < \frac{c}{2 \cdot f_{PRF} \cdot \sin \eta}
\]

2.6.4 System Sensitivity (Noise Equivalent Back Scattering Coefficient) [1]

Sensitivity of SAR System is an important parameter to measure the imaging capability for weak targets, and it is always be estimated by Noise Equivalent Sigma Zero(NESZ). That means that the power of echo scattered by the terrain, whose back scattering coefficient is NESZ, is equal to power of the SAR system noise.
Radar equation is the basis in radar system design, and the power of the transmission and the SNR of the end-to-end system can be estimated

\[
\frac{S}{N} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 (k_0 T_0 B_n F_n)^3},
\]

where \(P_t\) is transmit peak power, \(G\) is antenna gain, \(\sigma\) is radar cross section (RCS), \(k_0 = 1.38 \times 10^{-23} \text{ J/K}\) is the Boltzmann constant, \(T_0\) is absolute temperature, \(B_n\) is equivalent noise bandwidth that is approximately equal to system bandwidth \(B_r\), \(F_n\) is noise figure.

The SNR of SAR image can be improved by pulse compression in range and azimuth, the signal processing gain is \(G_{pr} \cdot G_{pa}\).

The processing gain in range \(G_{pr}\) is expressed as

\[
G_{pr} = \frac{B_r}{T_p},
\]

where \(T_p\) is the pulse length of the transmitting signal, and \(B_r\) is the band width of radar system. The azimuth focusing processing is equivalent to the coherent accumulation of pulses, and the coherent accumulation gain can be expressed as

\[
G_{pa} = T_S \cdot f_{PRF},
\]

where \(T_S\) is the synthetic aperture time and \(f_{PRF}\) is the pulse repetition frequency.

In Eq. (2.68), \(\sigma\) is radar cross-section area of the minimum resolution cell that can be given by

\[
\sigma = \sigma_0 \cdot \rho_g \rho_a,
\]

where \(\sigma_0\) is normalized backscattering coefficient, \(\rho_g\) is the ground range resolution, and \(\rho_a\) is azimuth resolution. Using Eqs. (2.69) and (2.71) in Eq. (2.68), the radar equation of SAR can be expressed as

\[
\frac{S}{N} = \frac{P_{av} G^2 \lambda^3 \sigma_0 \rho_g}{2(4\pi)^3 R^3 v_s (k_0 T_0 F_n) L_{oss}},
\]

where \(L_{oss}\) is system loss, \(P_{av} = P_t T_p \cdot \text{PRF}\) is average transmission power. According to Eq. (2.72), the SNR of radar output can be calculated under a certain average transmit power. Equation (2.72) can be rewritten as follows

\[
P_{av} = \frac{2(4\pi)^3 R^3 v_s (k_0 T_0 F_n) L_{oss}}{G^2 \lambda^3 \sigma_0 \rho_g} \cdot \frac{S}{N}
\]

Equation (2.73) gives the average transmits power for satisfying the requirement of the SNR at output of radar system.
Noise equivalent back scattering coefficient (NEσ⁰) is defined as

\[ \text{NE}\sigma^0 = \frac{\sigma_0}{S/N} = \frac{2(4\pi)^3R^3v_s(k_0T_0F_n)L_{\text{loss}}}{P_{av}G^2\lambda^3\rho_g} \] (2.74)

The SNR at output of radar system is the dB number that the actual back scattering coefficient σ⁰ is higher than the NEσ⁰.

2.6.5 Ambiguity [1, 8]

Since SAR operates in the pulse mode, ambiguity is inevitable as same as the pulse radar that means that the radar will receive useful echo signals and the ambiguity interference at the same time. The ambiguity signal is the echo signal from the outside of the image area, which is combined with the imaging signal to enter the radar receiver, and after signal processing it will degrade the image quality. Ambiguity in the spaceborne SAR is particularly prominent, and it is divided into the range ambiguity and the azimuth ambiguity.

Ambiguity ratio is defined as the ratio of the power between the ambiguity signal and the useful signal, and it is an index of the system’s ambiguity interference degree.

2.6.5.1 Azimuth Ambiguity

Since the radar antenna pattern has got the side lobes, its Doppler spectrum is non-band limited, as the echo is sampled by a limited f_{PRF} in azimuth, it will cause the spectrum overlapping. It is the azimuth ambiguity. The formation process of the SAR azimuth signal is shown in Fig. 2.16.

The reason of the azimuth ambiguity is that the Doppler spectrum is sampled by the non-band limited PRF. According to the sampling theorem, the spectrum after sampled is extended along azimuth by the periodicity of PRF. Since the side lobes of the antenna azimuth pattern give rise to no strict non-band limited Doppler spectrum, the signal component outside effective processing bandwidth is folded into the signal processing bandwidth, and interferes the useful echo signal, that

![Diagram](image-url)
causes the azimuth ambiguity. Figure 2.17 shows a schematic diagram of azimuth ambiguity, and $B_p$ is Doppler processing bandwidth.

For convenient calculating, assuming that the terrain scattering characteristics of targets at different azimuth positions are uniform and the azimuth antenna pattern is the same in each range gate. The azimuth ambiguity-to-signal ratio (AASR) can be defined as

$$R_{AASR} = \frac{\sum_{m=-\infty}^{+\infty} \int_{-B_p/2}^{+B_p/2} G^2(f + m \cdot f_{PRF}) df}{\int_{-B_p/2}^{+B_p/2} G^2(f) df}, \quad (2.75)$$

where $f$ is Doppler frequency, and $G(f)$ is azimuth antenna pattern.

In terms of Eq. (2.75), $R_{AASR}$ can be improved by increasing PRF, but, in turn, increasing PRF will make the range ambiguity worse. Therefore, the choice of PRF is a tradeoff between the azimuth ambiguity and the range ambiguity.

2.6.5.2 Range Ambiguity

The generation mechanism of range ambiguity in SAR is the same as that of conventional pulse radar. Since the antenna has got the side-lobes in range, the echo signals from different ranges are received in the same sampling window, Fig. 2.18 shows a schematic diagram of range ambiguity. In a spaceborne SAR, since the range is far away, the platform flying velocity is fast and PRF is high, the echo is usually received after a number of PRTs as the pulse is transmitted. In this way, as soon as the radar receives the echo of the target in the swath (the second pulse), the echoes of the previous pulse (the first pulse) and the next pulse (the third pulse) may be received at the same time that is the origin of the range ambiguity. The echoes from the outside of the swath will interfere with the effective echo from the targets.
In order to further illustrate the range ambiguity, Fig. 2.19 shows the spatial position relationship between the swath and the ambiguity region. The swath is illuminated by the 3 dB beamwidth of the antenna pattern in range, and the first and the second ambiguity regions are given.

In order to evaluate the range ambiguity, the position of ambiguity points should be determined first. If the echo delay time of a target in the swath is $t_i$, the ambiguity signal can be in the following range:

$$R_{ij} = \frac{c}{2} \left( t_i + \frac{j}{f_{PRF}} \right) \quad j = \pm 1, \pm 2, \ldots, \pm n_h$$  \hspace{1cm} (2.76)
If $j$ is positive, the ambiguity signal comes from far range targets during the previous pulse. If $j$ is negative, the ambiguity signal comes from near range targets during the next pulse. When $j$ is equal to $n_h$, the ambiguity target is located at the horizon.

Figure 2.20 shows the geometry of the spaceborne SAR. The incident angle $\eta_{ij}$ can be calculated from the looking angle $\gamma_{ij}$ for the ambiguity point

$$\eta_{ij} = \sin^{-1}\left(\frac{R_e + h}{R_e}\sin\gamma_{ij}\right),$$

(2.77)

where $R_e$ is the earth radius, $h$ is the altitude of satellite. The looking angle $\gamma_{ij}$ is given by

$$\gamma_{ij} = \cos^{-1}\left[\frac{R_{ij} + (R_e + h)^2 - R_e^2}{2R_{ij}(R_e + h)}\right]$$

(2.78)

The range ambiguity-to-signal ratio ($R_{RASR}$) of the swath can be calculated by

$$R_{RASR} = \frac{\sum_{i=1}^{N} S_{ai}}{\sum_{i=1}^{N} S_i},$$

(2.79)

where $S_{ai}$ is the power of the ambiguity signal of the $i$th sample, and $S_i$ is the effective echo signal power at the same sampling point. Equation (2.80) gives the expression.

**Fig. 2.20** The geometry of spaceborne SAR
\[
\begin{align*}
S_i &= \frac{\sigma_0^i G_i^2}{R_i^i \sin \eta_i} \quad j=0 \\
S_a &= \sum_{j=-m_i}^{m_i} \frac{\sigma_0^i G_i^2}{m_i \sin \eta_i} \quad j \neq 0
\end{align*}
\]

\(G_{ij}\) is the antenna gain in the range dimension, \(\sigma_0^i\) is the normalized backscatter at the incident angle \(\eta_{ij}\), which is usually approximated by a negative exponential function \(\sigma_0^i = e^{-\eta_{ij}/\eta_0}\).

From the above analysis, it can be found that choosing those SAR parameters is closely related to the azimuth ambiguity and the range ambiguity. Since the azimuth ambiguity is caused by the undersampling of the echo in the azimuth, it will effectively reduce the azimuth ambiguity by increasing the sampling PRF. In addition, the Doppler bandwidth can be decreased by changing the antenna azimuth length or designing the appropriate operating orbit to reduce the azimuth ambiguity, although the azimuth ambiguity is reduced, but it is also at the expense of azimuth resolution.

The range ambiguity-to-signal ratio (RASR) means the ratio of the range ambiguity power to the effective echo power in range. Reducing PRF can increase the PRT, and reduce the ambiguity signal into the data record window, thereby improving the RASR. In contrast, increasing PRF will degrade the RASR. In addition, in order to reduce the RASR of system, it should choose the appropriate antenna pattern to reduce the power of side lobe signal.

### 2.6.6 Dynamic Range [5]

The dynamic range of the microwave imaging radar can be measured at different points of the receiving channel.

At the receiver input, the dynamic range is the ratio of the maximum received signal and thermal noise. As soon as the maximum received signal is obtained, the saturation noise and other nonlinear signals must be limited under a given threshold at the same time. This threshold is chosen to avoid degrading the image contrast so much that weak echo region and the shadows no longer are distinguished.

On the image itself, at the output of the signal processing, the dynamic range is the ratio between the maximum point target echo and the noise. The received signal is the average superposition of all points on the instantaneous observation area (IFOV), which is defined by the transmission pulse width and azimuth beamwidth. Due to this averaging effect, the maximum signal power received corresponds to a mean backscatter coefficient that is smaller than the power of the maximum backscatter coefficient in the observed area.
In receiver, the amplifier gain should be set so that the maximum signal is passed without noticeable saturation. The main source of nonlinearity and saturation are the analog to digital conversion (ADC). Due to the Gauss distribution statistics of the echo, the saturation level of the ADC has to be set much higher than the level of the maximum received signal.

### 2.6.6.1 Noise Introduced by ADC Converter

The probability density function (PDF) of noise introduced by ADC is

\[ p(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \]  

(2.81)

\( \sigma \) is standard deviation of input signals for I and Q CHANNEL?

Signal quantization noise ratio (\( R_{SQNR} \)) is

\[ R_{SQNR} = \frac{\sigma^2}{\int_{-\infty}^{+\infty} (x - q(x))^2 p(x) dx}, \]

where \( q \) is quantization function.

If the quantization order is \( Q \) and quantization bits is \( 2M \), \( R_{SQNR} \) is

\[ R_{SQNR} = \left\{ 1 - \sqrt{\frac{Q}{\pi \sigma \sqrt{2}}} \sum_{m=-M}^{M-1} \exp\left(-\frac{mQ}{\sigma \sqrt{2}}\right) + \frac{Q^2}{\sigma^2} \left( \left( M - \frac{1}{2} \right)^2 - \sum_{m=-M}^{M-1} m \cdot \text{erf}\left( \frac{mQ}{\sigma \sqrt{2}} \right) \right) \right\}^{-1} \]

(2.83)

Figure 2.21 shows the relationship between quantization bits and SQNR, where the number of quantization bits is a function of \( \sigma / V_{sat} \). ADC saturation level \( V_{sat} = MQ \). Table 2.2 summarizes the relationship among these parameters.

### 2.6.6.2 Dynamic Range of Input Signal

Due to the input signal fluctuation and the observed area is very broad for microwave imaging radar, the dynamic range of the receiver input signal is large.

**Total dynamic range at the receiver input**

Total dynamic range at the receiver input should be concerned with two parts: the dynamic range caused by range varied, and the echo variation caused by different backscattering coefficient into the observation area.

The dynamic range caused by range varied: according to the SAR range equation, the signal level of receiver is inverse proportion with \( R^3 \). For radar with range from 1 to 100 km, the variation of receiving signal levels will exceed 60 dB.
The dynamic range means the change of different back scattering coefficients due to the different frequency band, different incident angle, different polarization, and so on. The strong backscattering coefficient may be 5 dBm²/m², and a weak backscattering coefficient is −35 dBm²/m².

For a pulse compressing radar, transmitting pulse duration is larger, observe area is wider, and the signals are always attenuated after echo round-trip propagation from radar to targets, which caused the echo signal change greatly.

The total dynamic range at the receiver input may be 70 dB.

The relationship curve between quantization bits and SQNR

Table 2.2 The relationship between the quantization bits, optimal $Q/\sigma$, optimal $\sigma/V_{sat}$, and optimal SQNR

<table>
<thead>
<tr>
<th>Quantization bits</th>
<th>Optimal $Q/\sigma$</th>
<th>Optimal $\sigma/V_{sat}$ (dB)</th>
<th>Optimal SQNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.596 E+0</td>
<td>−4.06</td>
<td>4.40</td>
</tr>
<tr>
<td>2</td>
<td>9.957 E−1</td>
<td>−5.98</td>
<td>9.25</td>
</tr>
<tr>
<td>3</td>
<td>5.860 E−1</td>
<td>−7.40</td>
<td>14.27</td>
</tr>
<tr>
<td>4</td>
<td>3.352 E−1</td>
<td>−8.57</td>
<td>19.38</td>
</tr>
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<td>5</td>
<td>1.881 E−1</td>
<td>−9.57</td>
<td>24.57</td>
</tr>
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<td>6</td>
<td>1.041 E−1</td>
<td>−10.45</td>
<td>29.83</td>
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<td>8</td>
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<td>10</td>
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<td>51.55</td>
</tr>
<tr>
<td>11</td>
<td>4.650 E−3</td>
<td>−13.55</td>
<td>57.11</td>
</tr>
<tr>
<td>12</td>
<td>2.448 E−3</td>
<td>−14.00</td>
<td>62.71</td>
</tr>
</tbody>
</table>
Dynamic range in swath

The signal dynamic range in swath can be adjusted by automatic gain control (AGC) or time sensitive control (STC) to match with the dynamic range of ADC convert. The main factors affecting the dynamic range of the echo in the swath are included:

The backscattering coefficient changes from 20 to 35 dB.

The antenna gain in range dimension will change 2–6 dB from the center to the edge of the swath. And the signal attenuation in far range is different from that in near range that makes 0–8 dB dynamic range.

In general, the saturation level of the ADC converter is much higher than the maximum received signal in order to suppress the saturation noise. Therefore, it is needed to persist a certain interval between each code as the signal is encoded. The dynamic range of the ADC with 8–12 bits quantization will be 40–60 dB.

Instantaneous dynamic range

The receiver gain can be adapted to the average signal level received during a shorter period of time. This receiver parameter is the instantaneous dynamic range.

Since the variation of the signal envelope is spread over the entire pulse length and antenna azimuth beamwidth, the dynamic range of echo signals between two adjoining area is always small. For a part of the echo signal, the instantaneous dynamic range is much smaller than the whole dynamic range of the receiver.

The dynamic range of SAR image is usually 60–80 dB. Even if the point target is submerged in the echo of the IFOV, the ADC quantization does not eliminate the scattering information of the target. Similarly, the information of the small and strong scattering can still be retained after compression, but the thermal noise or clutter will be attenuated. Therefore, as long as the number of quantization bits is large enough, a suitable image dynamic range can be achieved.

2.7 Antenna Area Limitation and System Quality Factor

For a spaceborne SAR system, on the one hand, the resolution should be as high as possible to obtain the rich target information, and convenient for target detection and identification. On the other hand, the imaging swath should be as wide as possible to shorten the revisit period. However, there is the ambiguity in range and azimuth in SAR system, the high resolution and wide swath incompatible, and both cannot be improved at the same time. In order to obtain high resolution and wide swath as soon as possible, a minimum antenna area has to be limited in the system design.
2.7.1 Limitation Of Minimum Antenna Area [8, 9]

Figure 2.22 shows the geometric relationship of an orthogonal side-looking SAR, where $D_a$ is the length of antenna, $H_a$ is the width of antenna, $V_s$ is the flying velocity of the satellite platform, $W_g$ is the swath on the ground, and $W_r$ is the swath in slant range.

\[
W_g = \frac{\beta_c R_0}{\cos \eta} = K_E \frac{\lambda R_0}{H_a \cos \eta},
\]  

(2.84)

where $\beta_c$ is the 3 dB beamwidth of the antenna in range, $K_E$ is the radiation pattern factor, which is relative to the antenna gain decrease dB at the edge of the swath. $\lambda$ is wavelength, and $R_0$ is the slant range from the center of antenna beam to target.

The SAR system is operating with pulse model, ambiguities in range and azimuth have to be considered in system design. The basic requirement for the range ambiguity is that the whole echo in the swath must be received and sampled within a pulse repetition time (PRT), it is to say. The time at which the antenna receives the echo as the reflection of a transmitting pulse at nearest slant range point within the imaging swath, has to be late than the time at which the antenna receives the echo as the reflection of the previous transmitting pulse, at farthest slant range point within the imaging swath. This can be expressed as

\[
\frac{2(R_f - R_n)}{c} = \frac{2W_r}{c} < f_{\text{PRT}} = \frac{1}{f_{\text{PRT}}},
\]  

(2.85)

where $c$ is the velocity of light, $R_n$ is the nearest slant range, and $R_f$ is the farthest slant range. In terms of Eq. (2.85), the upper bound of the allowable PRF is given by

\[
f_{\text{PRT}} < \frac{c}{2W_r} = \frac{c}{2(R_f - R_n)}
\]  

(2.86)

Fig. 2.22 Geometric relationship of radar imaging
The best azimuth resolution of orthogonal side-looking SAR is half of the antenna length.

\[ \rho_a \geq \frac{D_a}{2} \]  

(2.87)

The echo signal is sampled at a PRF in azimuth. In order to ensure that the echo spectrum do not overlap alias after sampled, the PRF must be greater than the Doppler bandwidth.

\[ f_{\text{PRT}} > \frac{v_s}{\rho_a} = 2K_AV_s/D_a, \]  

(2.88)

where \( K_A \) is the antenna pattern factor in azimuth, which is relative to the antenna gain decrease at the edge of the Doppler bandwidth.

From (2.85) to (2.88) just given above, the following can be obtained

\[ W_r < \frac{c}{2 \cdot f_{\text{PRT}}} < \frac{c}{2v_s} \cdot \rho_a \Rightarrow \frac{W_r}{\rho_a} < \frac{c}{2v_s} \]  

(2.89)

From Fig. 2.22, geometric relationship of radar imaging, \( W_r \) can be expressed as

\[ \begin{align*} 
W_r &= W_g \sin \eta \\
W_g \cos \eta &= KE \frac{\lambda R_0}{H_a} 
\end{align*} \]

\[ \Rightarrow \quad W_r = KE \frac{\lambda R_0}{H_a} \cdot \tan \eta \]  

(2.90)

Using Eq. (2.90) to rewrite Eq. (2.89), the following can be obtained

\[ \frac{W_r}{\rho_a} = \frac{2K_AKE \lambda R_0 \cdot \tan \eta}{D_a \cdot H_a} < \frac{c}{2v_s} \]  

(2.91)

The limitation of antenna area is given as follows:

\[ A_a = D_a \cdot H_a > \frac{4K_AKE \lambda R_0}{c} \cdot \tan \eta \]  

(2.92)

This is restricting for the minimum antenna area in spaceborne synthetic aperture radar system design.

From (2.92), it can be seen that once the slant range \( R_0 \), incident angle \( \eta \), wavelength \( \lambda \), and the platform velocity \( v_s \) are confirmed, the limitation of minimum antenna area is ensured. For the steady velocity of the satellite, if the orbit altitude, the incident angle and wavelength of the radar are known, then the limitation of antenna area can be calculated. At present, the system performance has to be a tradeoff between the swath width and the azimuth resolution, and scanning imaging mode and spotlight imaging mode are two typical examples for it.
2.7.2 Quality Factor \([10–15]\)

Swath width and azimuth resolution are two key parameters of spaceborne SAR performances. The SAR quality factor \(M_f\) can be defined as the ratio of swath width to azimuth resolution.

\[
M_f = \frac{W_g}{\rho_a}
\]  

(2.93)

The quality factor \(M_f\) depends on swath width in range and Doppler bandwidth of azimuth antenna radiation under the limitation of ambiguity ratio.

When the range ambiguity is considered

\[
W_g < \frac{c}{2 \cdot f_{\text{PRT}_{\text{max}}} \cdot m_R \cdot \sin \eta_{\text{max}}},
\]  

(2.94)

where \(W_g\) is swath, \(f_{\text{PRT}_{\text{max}}}\) is the maximum PRF under the meeting of the RASR, and \(\eta_{\text{max}}\) the maximum incident angle. \(m_R\) is related to the RASR, which is numerically equal to the ratio of PRT to the echo window.

When the azimuth ambiguity is considered

\[
f_{\text{PRT}_{\text{min}}} > m_D \cdot B_a
\]  

(2.95)

\[
\rho_a = \frac{v_s}{\xi B_a}
\]  

(2.96)

\[
\xi = \frac{R_e + H}{R_e},
\]  

(2.97)

where \(B_a\) is the azimuth Doppler bandwidth, \(\rho_a\) is azimuth resolution, \(R_e\) is the earth radius, \(H\) is the orbit altitude, and \(v_s\) is the platform flying velocity. \(m_D\) is related to the AASR, which is numerically equal to the ratio of PRF to the Doppler bandwidth.

When range ambiguity and azimuth ambiguities are comprehensively considered, \(M_f\) can be represented by the expression as follows:

\[
M_f < \frac{c}{2v_0} \cdot \xi^{1.5} \cdot \frac{f_{\text{PRT}_{\text{min}}}}{f_{\text{PRT}_{\text{max}}}} \cdot \frac{1}{\sin \eta_{\text{max}} \cdot m_R m_D},
\]  

(2.98)

where \(v_0 = \sqrt{GM/R_e}\), gravitational constant \(GM\) is equal to \(3.986005 \times 10^{14}\) \(\text{m}^3/\text{s}^2\). The flying velocity of the satellite \(v_s = v_0 \cdot \xi^{-0.5}\). Since \(\xi\) and \(\eta_{\text{max}}\) depend on the satellite–earth geometry model, \(m_R\) and \(m_D\) depend on the ambiguity ratio. The quality factor is mainly ensured by \(c/2v_0\) as long as the orbit altitude, the maximum incident angle, and the ambiguity ratio are confirmed. For spaceborne SAR, \(c/2v_0\) is equal to approximately 20,000, and for the airborne system, the value
is generally about 500,000. Therefore, the quality factor of the spaceborne SAR will be restricted greatly.

Under the critical design conditions, the antenna is uniformly weighted, and Doppler bandwidth is corresponding to the 3 dB beamwidth of the antenna. When the AASR is \(-20\, \text{dB}\), \(m_D\) approximately is equal to 1.17. When the AASR is \(-25\, \text{dB}\), \(m_D\) approximately is equal to 1.26. If the swath is defined as the area illuminated by 1 dB beamwidth of the antenna, \(m_R\) is equal to 2 when the RASR is \(-22\, \text{dB}\). So system quality factor under critical design conditions is defined as

\[
M_f = K_{\text{amb}} \cdot \frac{\zeta^{1.5}}{\sin \eta_{\text{max}}},
\]

where \(K_{\text{amb}} = \frac{c^2}{2v_0} \cdot \frac{f_{\text{FTR min}}}{f_{\text{FTR max}}} \cdot \frac{1}{m_{RmD}},\) it depends on the AASR and RASR. When the AASR is \(-25\, \text{dB}\) and the RASR is \(-22\, \text{dB}\), \(K_{\text{amb}}\) equals 6900. When the AASR is \(-20\, \text{dB}\) and the RASR is \(-22\, \text{dB}\), \(K_{\text{amb}}\) equals 7300. Therefore under critical design conditions, the quality factor approximately is equal to \(10^4\) (\(45^\circ < \eta_{\text{max}} < 65^\circ\)). In general, it means that the ratio of swath width to azimuth resolution approximately equals \(10^4\) for spaceborne SAR with the maximum incidence angle from \(45^\circ\) to \(65^\circ\).

The minimum effective area, length, and height of the antenna under the critical design conditions are given as follows.

The effective height of the antenna \(H_a\) on vertical to sight line direction should meet the requirement of the swath \(W_g\).

\[
\beta_e = \frac{K_E \lambda}{H_a}
\]

\[
W_g = K_E \frac{\lambda}{H_a} \cdot \frac{R_{\text{max}}}{\cos \eta_{\text{max}}},
\]

where \(\beta_e\) is the 1 dB beamwidth of the antenna, \(K_E\) is the factor of range radiation pattern, and \(R_{\text{max}}\) is the slant range at the maximum incident angle.

The effective length of the antenna \(D_a\) can be derived by the following formulas.

\[
B_a = \frac{2v_s}{\lambda} \beta_a
\]

\[
\beta_a = K_A \frac{\lambda}{D_a}
\]

\[
\rho_a = \frac{v_s}{\zeta B_a} = \frac{\lambda}{2\zeta \beta_a} = \frac{D_a}{2\zeta K_A},
\]

where \(\beta_a\) is the beamwidth of the Doppler bandwidth, \(K_A\) is the factor of azimuth radiation pattern. The minimum effective area of the antenna is given by
Using Eq. (2.98) to express Eq. (2.105)

\[ A = D_a H_a = \frac{2K_E K_A \xi \lambda R_{\text{max}}}{M_f \cos \eta_{\text{max}}} \]  

(2.105)

Under the critical design condition, if the swath width is defined as the area illuminated by the 1 dB beamwidth of the antenna in range, \( K_E \) is equal to 0.56. If the Doppler bandwidth is defined as the bandwidth by the 3 dB beamwidth of the antenna in azimuth, \( K_A \) is equal to 0.9. And then the minimum effective area, length, and height of the antenna under the critical design are given as follows.

\[ A = \frac{\xi^{-0.5} \lambda R_{\text{max}} \tan \eta_{\text{max}}}{K_{\text{amb}}} \]  

(2.107)

\[ D_a = 1.8 \xi \rho_a \]  

(2.108)

\[ H_a = \frac{0.55 \xi^{-1.5} \lambda R_{\text{max}} \tan \eta_{\text{max}}}{K_{\text{amb}} \rho_a} \]  

(2.109)

In terms of Eq. (2.107), the antenna area can be estimated under the condition of the ambiguity, which can be called the least effective area of the antenna in the design of the SAR system. But Eq. (2.92) gives the minimum area estimation without considering the ambiguity signal, which can be called the minimum area of the antenna. Comparison of Eq. (2.107) with Eq. (2.92), the difference between them is a \( m_R \ m_D \) factor that is just used to limit the ratio of ambiguity to signal.

From the previous analysis, it can be known that swath width and azimuth resolution of spaceborne SAR contradict each other. Once the system geometry and ambiguity requirements are specified, the upper limitation of the system quality factor is ensured if one wants to obtain wide swath and high azimuth resolution at the same time, considering the new ways. One idea is to obtain a wider Doppler bandwidth than the conventional radar for the enhancement of azimuth resolution without increasing the PRF, meanwhile the echo is sampled properly in range.

**References**

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