Chapter 2
Discovery of Soft-Matter Quasicrystals and Their Properties

2.1 Soft-Matter Quasicrystals with 12- and 18-Fold Symmetries

During 2004, Zeng et al. [1] observed the quasicrystals with 12-fold symmetry in liquid crystals. Almost at the same time, in 2005 Takano [2] and in 2007 Hayashida et al. [3] discovered the similar structure in polymers. The quasicrystals of 12-fold symmetry were observed also in chalcogenides and organic dendrons.


Figure 2.1 shows the diffraction pattern of soft-matter quasicrystals with 12-fold symmetry.

In 2011, the 12- and 18-fold symmetry quasicrystals were discovered in colloids by Fischer et al. [5]; they observed the structures in PI_{30}-PEO_{120} of one of Poly (Isoprene-b-ethylene oxide) (PI_{n}-PEO_{m}) at room temperature, by using X-ray scattering and neutron scattering. The 18-fold symmetry quasicrystal is the first observed since 1982 in solid and soft-matter quasicrystals, whose diffraction pattern and Penrose tiling are shown in Figs. 2.2 and 2.3, respectively.

More recently, Cheng et al. [6] observed the 12-fold symmetry quasicrystals in giant surfactants, which is the first observation in this kind of soft matter.

Though the 12-fold symmetry quasicrystals in solids were discussed quite long time already, the 18-fold symmetry quasicrystals are studied for the first time to us, which have not been known previously. This is very new and interesting topics. The 12-fold symmetry quasicrystals in theory of solid quasicrystals are well known, but the 18-fold symmetry quasicrystals are totally a new phase to the researchers, which are unknown for most readers; we have only very few of understanding for the structure and properties.

In solid quasicrystals people know the formation of quasiperiodic structure lies in atom arrangement. In soft matter the formation of quasiperiodic structure mentioned above presents a quite different mechanism, e.g. self-assemble of spherical
building blocks by supramolecules, compounds, block copolymers, micrometre-sized colloidal grains and so on. Figure 2.4 shows some examples of compounds exhibiting conical conformations that self-assemble into spherical supramolecular dendrimers forming quasicrystals.

These discoveries present highly importance and extremely interesting. At first, under certain temperature and density, quasicrystal state in soft matter is stable; this promotes us to understand quasicrystals theoretically. It is well known that quasicrystal state in metallic alloys is formed under a rapid cooling condition, which is
quite different from that of soft-matter quasicrystals, because these two cases are in quite different thermodynamic environments. The discovery of 18-fold symmetry quasicrystals leads to appearance of new point groups and space groups, and promotes the development of symmetry theory and group theory. Of course, the appearance of these new quasicrystals enlarges the scope of the quasicrystal study. Finally, soft-matter quasicrystals may be a class of photon band gap material, present in the application or potential application meaning in electronics, device

Fig. 2.3 The Penrose tiling of quasicrystals with 18-fold symmetry in soft matter

Fig. 2.4 Seven examples of compounds exhibiting conical conformations that self-assembly into spherical supramolecular dendrimers forming quasicrystals
technology, etc. In addition, the self-assembly technique developing in the study is meaningful as well.

2.2 Characters of Soft-Matter Quasicrystals

Based on the experimental results the soft-matter quasicrystals are observed in different kinds of soft matter; their forms and structures are quite different to each other. This book is unable specially and in detail to study soft matter. Our object is only to study soft-matter quasicrystals, and for this purpose we have to understand a preliminary and necessary knowledge on soft matter. The nature of soft matter is an intermediate phase between ideal solid and simple fluid, or call it as a complex fluid or structured fluid, more exactly a complex liquid or structured liquid, which is one of soft condensed matters.

The soft-matter quasicrystals observed so far are two-dimensional. During the process of their formation it accomplished the chemical reactions and some phase transitions, such that crystal–quasicrystal transition, liquid crystal–quasicrystal transition, etc. In the formation process of quasicrystals coming from colloids, there is a connection with electricity, because the particles in colloids have charges. These complex physical–chemical effects are not discussed in our presentation on soft-matter quasicrystals yet.

The main attention hereafter is on mechanical behaviour and continuous theory of soft-matter quasicrystals.

The related thermodynamics of soft-matter quasicrystals will be simply introduced. Lifshitz et al. [7, 8] studied thermodynamics; they attended the stability of the new phase, which is a very important problem, of course. For studying hydrodynamics of soft-matter quasicrystals, an equation of state is necessary. Fan and coworkers [9, 10] gave some preliminary discussions, but the model needs experimental verification. Some results concerning thermodynamics of soft matter will be discussed in brief in Chap. 4.

Due to lack of experimental data, numerical analysis can help us to obtain some results on mechanical and physical behaviour of the matter. For example, after our computation, we find the compressibility of soft-matter quasicrystals is quite large, e.g. \( \frac{\delta \rho}{\rho_0} = 10^{-4} - 10^{-3} \), \( \delta \rho = \rho - \rho_0 \), while for solid quasicrystals, \( \frac{\delta \rho}{\rho_0} = 10^{-13} \); in addition, the ratio \( \frac{p_{yy}}{\sigma_{yy}} \sim 1 \), i.e. the ratio between fluid stress over elastic stress is almost the same order of magnitude for soft-matter quasicrystals, while the ratio between viscous stress and elastic stress is about \( \frac{\sigma'_{yy}}{\sigma_{yy}} \sim 10^{-15} \) for solid quasicrystals, where \( \sigma_{yy} \) denotes the elastic stress, \( \sigma'_{yy} \) is the solid viscous stress, and \( p_{yy} \) is the fluid stress, refer to Cheng et al. [11, 12], which show the compressibility and fluid phonon are very important in soft-matter quasicrystals. The gigantic distinctions in the hydrodynamic behaviour between soft-matter quasicrystals and solid quasicrystals also reveal the great differences in
nature between these two kinds of matters, in fact. This is not differentiated only in quantity. Of course these computational results are needed to be verified by experiments.

In addition, some characters of general soft matter hold for soft-matter quasicrystals, e.g. the motion of soft-matter quasicrystals is in small Reynolds number, similar to that of the general soft matter.

2.3 Some Concepts Concerning Possible Hydrodynamics on Soft-Matter Quasicrystals

Solid quasicrystals are generated among metal alloys; soft-matter quasicrystals are generated in liquid crystals, colloids, polymers, surfactants, etc.; latter belong to some kinds of soft matter. These soft matters exist a quite long period; they belong to nontraditional materials rather than traditional ones, which we are not so familiar with them. Soft matter is the common title, introduced by de Gennes [13] in 1991, of liquid crystals, colloids, polymers, foams, emulsions, surfactants, biomacromolecules, etc.; they are neither ideal solid nor simple fluid, but present characters of both solid and fluid, belonging to an intermediate phase between isotropic fluid and ideal solid macroscopically. Sometimes, one calls them as anisotropic fluid or structured fluid [14–17]. In Chap. 1 we have mentioned in brief about these.

2.4 First and Second Kinds of Two-Dimensional Quasicrystals

Up to now, discovered soft-matter quasicrystals, i.e. 12- and 18-fold symmetry quasicrystals are two-dimensional quasicrystals, but there are distinctions between them from point of view of symmetry. The 12-fold symmetry quasicrystals are similar to those of 5-, 8- and 10-fold symmetry quasicrystals, they may be classified as first kind of two-dimensional quasicrystals according to quasiperiodic structure, while 18- and possible 7-, 9- and 14-fold symmetry quasicrystals belong to second kind of two-dimensional quasicrystals [18]. The detailed analysis will be given in Chaps. 10 and 11, but the second kind of two-dimensional quasicrystals has not well been studied including their symmetry; only a few of point groups are discussed. Based on the analysis, the first kind of two-dimensional quasicrystals has phonon and phason elementary excitations, while the second kind of two-dimensional quasicrystals has phonon, first and second phason elementary excitations, respectively. The concept of the second phasons was suggested by Hu et al. [19], in which they developed a hypothesis on six-dimensional embedding space, and the discussion was based on group representation theory. In Chaps. 7, 8 and 9, we will discuss the first kind of two-dimensional soft-matter quasicrystals,
and in Chaps. 10 and 11, we will discuss the second kind of two-dimensional soft-matter quasicrystals, respectively. Compared with the first kind of two-dimensional quasicrystals, the theory on the second one is developing, so there are only very few of results.

The symmetry of the first kind of soft-matter quasicrystals can be drawn from the analysis of solid quasicrystals, i.e. the point groups are listed in Table 2.1, of course, the Laue classes are not needed here.

The symmetries on the soft-matter quasicrystals of the second kind have not well been studied, but Tang and Fan [20] put forward the point group classification and the group representation on the structure.

Based on the Schoenflies method, the point groups of 7-, 14-, 9- and 18-fold symmetry quasicrystals are listed in Table 2.2.

The theory of group representation including the character tables of the second kind of two-dimensional quasicrystals given in [20] is quite complicated. The introduction about the derivation needs a very large volume of space, which is not able to be conducted here. The key results concerning this are the determination of quadratic invariants of strain tensors of phonons, first and second phasons and their

### Table 2.1 Systems, Laue classes and point groups of the first kind of two-dimensional quasicrystals

<table>
<thead>
<tr>
<th>Systems</th>
<th>Laue classes (for solid)</th>
<th>Point groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-fold symmetry</td>
<td>11</td>
<td>5, 5</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>5m, 52, 5m</td>
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<tr>
<td>10-fold symmetry</td>
<td>13</td>
<td>10, 10/m</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>10mm, 1022, 10/m</td>
</tr>
<tr>
<td>8-fold symmetry</td>
<td>15</td>
<td>8, 8/m</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>8mm, 822, 8/m</td>
</tr>
<tr>
<td>12-fold symmetry</td>
<td>17</td>
<td>12, 12, 12/m</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>12mm, 1222, 12/mm</td>
</tr>
</tbody>
</table>

### Table 2.2 Point groups of the second kind of two-dimensional quasicrystals

<table>
<thead>
<tr>
<th>Axis</th>
<th>Plus $m_h$</th>
<th>Plus $m_v$</th>
<th>Plus $2_b$</th>
<th>Plus multi-operators</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>14</td>
<td>7m</td>
<td>72</td>
<td>14m2</td>
</tr>
<tr>
<td>7</td>
<td>7m</td>
<td>7m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>14/m</td>
<td>14mm</td>
<td>1422</td>
<td>14/mm</td>
</tr>
<tr>
<td>14</td>
<td>14m2</td>
<td>14m2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>9m</td>
<td>92</td>
<td>8m2</td>
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<tr>
<td>9</td>
<td>9m</td>
<td>9m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>18/m</td>
<td>18mm</td>
<td>1822</td>
<td>18/mm</td>
</tr>
<tr>
<td>18</td>
<td>18m2</td>
<td>18m2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
couplings (i.e. the determination of all independent nonzero components of physical modulus tensors of the material) so the constitutive equations, which will be presented in Chaps. 10 and 11, respectively.

2.5 Motivation of Our Discussion in the Book

The soft-matter quasicrystals including their formation mechanism are very interesting. However, we could not study the mechanism.

Soft-matter quasicrystals present some applications and potential applications; this suggests the study on their structures and properties. The structures and properties are very complex, and we aim to discuss only in macroscopic dynamics concerning the matter distribution, deformation and motion of the material, or say on their hydrodynamics. Due to a lack of experimental data, the discussion is only limited in computation with the assistant of mathematical physics and computational physics. A few of preliminary results may help reader to understand some macroscopic behaviour of soft-matter quasicrystals. Although we limit the macroscope of the discussion and do not concern their formation mechanism, which is connected to mesoscope still, for example, the equation of state used in the study is a mesoscope result. Of course, we do not further touch the mesocropic regime.

References

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