Preface

Wave phenomena represent a fairly intriguing area of the contemporary applied mathematics and physics. Wave processes have numerous applications in hydrodynamics, electromagnetism, magnetohydrodynamics, biophysics and biomechanics, acoustics, etc. This monograph is devoted to two distinct aspects of wave dynamics: wave propagation and diffraction, with the main focus put on the wave diffraction.

Wave interaction with rough bottom surfaces (topography), offshore drilling platforms, and wave energy collectors is accompanied by the diffraction of waves. The diffraction theory lies at the interface between physics and applied mathematics. In the broad sense, wave diffraction means any deviation of the wave motion from the laws of geometrical optics. From a mathematical point of view, the purpose of the diffraction theory is to develop analytical and numerical methods for solving diffraction problems, to classify the corresponding solutions, and to investigate their properties.

It is possible to distinguish three stages in the development of the diffraction theory: (i) Fresnel (1818) formulated the Huygens–Fresnel principle that combines the geometric approach (Huygens 1690) and interference approach (Young 1800); (ii) Helmholtz (1859) gave a strict formulation of the Huygens principle and demonstrated that it results in an integral formula (as in the potential theory) that makes it possible to calculate the value of the field at some point in terms of the field values (including the field's normal derivative) on some auxiliary closed surface enclosing that point; (iii) Poincaré (1892) and Sommerfeld (1896) showed the diffraction problems to be ordinary boundary-value problems of mathematical physics. Sommerfeld (1912) also formulated the radiation conditions. Then Meixner (1948) established the boundary conditions on the edge.

The rigorous diffraction theory distinguishes three approaches: the method of surface currents, where the diffracted field is represented as a superposition of secondary spherical waves emitted by each element (the Huygens–Fresnel principle); Fourier method; method of separation of variables and Wiener–Hopf transformation. In this monograph, we apply various mathematical methods to the solution of typical problems in the theory of wave propagation and diffraction and analyse the results obtained.

Chapter 1 presents some of the methods that are useful for solving the problems of wave diffraction theory: method of separation of variables, method of power series, method of spline functions, and method of an auxiliary boundary. We also consider some algorithms for the numerical inversion of the Laplace transform, which is often used to solve the wave diffraction problems. Finally, we give a brief account of the method of multiple scales that is often used to study the propagation of transient waves.

Chapter 2 deals with spectral methods in the theory of wave propagation. The main focus is given to the Fourier methods in application to studying the Stokes (gravity) waves on the surface of an inviscid fluid. A spectral method for calculating the limiting Stokes wave with a corner at the crest is considered as well. We also briefly consider the evolution of narrow-band wave trains on the surface of an ideal finite-depth fluid. Finally, a two-parameter method for describing the nonlinear evolution of narrow-band wave trains is described by the example of the Klein–Gordon equation with a cubic nonlinearity. The problem is reduced to a high-order nonlinear Schrödinger equation for the complex amplitude of wave envelope. This equation is integrated numerically using a split-step Fourier technique to describe the evolution of quasi-solitons.

Chapter 3 presents some results of modelling the refraction of surface gravity waves in terms of the ray method that originates from geometrical optics. Wave refraction essentially depends on the bed topography. As a result, the convergence of wave energy (or its divergence) can be observed in some water areas. The ray method makes possible the discrete or analytical assignment of the bed topography. This approach can be realised in the form of a computational programme and allows the distribution of wave fronts, rays, and heights to be constructed and analysed for the case of the transition of regular waves from deep to shallow waters. The model is verified by comparing the results with exact analytical solutions and field observations. The asymptotic analysis of the nonlinear refraction theory extends the limits of applicability of the traditional theory and provides the prediction of ray bending at approaching the wave breaking conditions. Moreover, we study the anomalous refraction in caustics.

Chapter 4 is devoted to the diffraction of surface gravity waves. Main aspects of the wave diffraction theory are described. Specific aspects and methods used to solve the problems of the wave diffraction theory are described in brief. Wave diffraction by a partially submerged elliptical cylinder with elliptical front surface and by a circular submerged cylinder is considered. Ellipticity is demonstrated to have strong effect on the wave load and its extremums, depending on the wave number. Solutions to the problem of wave diffraction by a system of vertical cylinders are presented and analysed. In this case, the wave force does not attain its maximum on the front vertical cylinder because of the significant reconstruction of the diffracted wave fields in multiply connected regions. An exact solution to the problem of wave diffraction by an asymmetrically nonuniform cylindrical scatterer is derived in the case when the scatterer parameters depend on the two coordinates—

radial and angular. The scatterer inhomogeneity is demonstrated to affect the cross scattering. The method of auxiliary boundary is used to study the diffraction of waves by a vertical column of arbitrary revolution shape. The extremums of the wave force and overturning moment applied to a cone column are found as functions of the wave number. The numerical method of spline collocation is used to study the problem of diffraction of acoustic waves by an arbitrary body of revolution. The accuracy of the numerical solution is analysed. The problem of wave scattering by a truncated cone with smooth spherical ends is considered. The effect of wave incidence angle is studied.

Chapter 5 deals with the approach that is based on the repeated use of the method of images to solve the problems of stationary acoustic, electromagnetic, and elastic wave scattering and diffraction by cylindrical and spherical obstacles in a semi-infinite domain. The solution is written in terms of an infinite series of multiply diffracted fields. Explicit approximate asymptotic solutions are found and investigated for the case of distant scattered fields in the longwave approximation. The known solutions for point obstacles are obtained as special cases described by the first terms of the series.

Chapter 6 deals with some aspects of the initial-boundary-value problems of the initiation, generation, and propagation of tsunami waves. The generation of tsunami waves by bottom movements is considered. We formulate an appropriate initial-boundary-value problem and analyse the effect of the sharpness of vertical axisymmetric bottom disturbance and the disturbance duration on the generation of tsunami waves. The propagation of nonlinear waves on water and their evolution over a nonrigid elastic bottom are investigated. Some aspects and indeterminacy of the formulation of the initial-boundary-value problems dealing with the initiation and generation of tsunami waves are considered. We consider some typical types of tsunami waves that demonstrate the indeterminacy of their initiation in time because of the indeterminacy in the physical trigger mechanism of underwater earthquakes. Based on the three-dimensional formulation, evolution equations describing the propagation of nonlinear dispersive surface waves on water over a spatially inhomogeneous bottom are obtained with allowance for the bottom disturbances in time. We use the Laplace transform with respect to the time coordinate and the power series method with respect to the spatial coordinate to find a solution to the nonstationary problem of the diffraction of surface gravity waves by a radial bottom inhomogeneity that deviates from its initial position. The propagation and stability of nonlinear waves in a two-layer fluid with allowance for surface tension are analysed by the asymptotic method of multiscale expansions.

Some insights on the directions of further development of the wave diffraction theory are outlined in the conclusion.

Kyiv, Ukraine

Igor T. Selezov
Yuriy G. Kryvonos
Ivan S. Gandzha

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