Contents

Part I  Conservative Systems

1  Two Coupled Oscillators ........................................... 3
   1.1  Limiting Phase Trajectories of Two Weakly Coupled Identical Nonlinear Oscillators ........................................... 3
       1.1.1  The Model and Main Asymptotic Equations of Motion ........................................... 3
       1.1.2  Analytical Solution for LPT ........................................... 9
       1.1.3  Beating Close to LPTs ........................................... 16
   1.2  Effect of the Frequency Detuning Between the Oscillators ..... 16
       1.2.1  Equations of Motion and Explicit Approximate Solutions ........................................... 17
       1.2.2  Stationary States and LPTs ........................................... 19
       1.2.3  Critical Parameters ........................................... 23
   References ............................................................. 25

2  Two-Particle Systems Under Conditions of Sonic Vacuum ........ 27
   2.1  Weakly Coupled Oscillators Under Conditions of Local Sonic Vacuum .......................................................... 28
       2.1.1  Evidence of Energy Localization and Exchange in Coupled Oscillators in the State of Sonic Vacuum ........................................... 28
       2.1.2  Energy Localization ........................................... 29
       2.1.3  Complete Energy Exchanges (Strong Beating Response) ........................................... 30
       2.1.4  Asymptotic Analysis of Resonance Motion ........................................... 31
       2.1.5  Fixed Points and NNMs in the Neighborhood of Resonance ........................................... 32
       2.1.6  Limiting Phase Trajectories ........................................... 34
       2.1.7  Numerical Analysis of the Fundamental Model ........................................... 37
2.2 Non-local Sonic Vacuum ........................................ 43
  2.2.1 The Model .............................................. 43
  2.2.2 Two-Particle System (n = 2): Slow Transverse
          Oscillations ........................................ 47
  2.2.3 Slow Flow Reduction of the Dynamics ..................... 48
  2.2.4 Stationary and Non-stationary Dynamics .................. 52
  2.2.5 Analytical Approximations of the LPTs
          on the Two-Torus ...................................... 56
  2.2.6 Mixed Slow/Fast Axial Oscillations for n = 2 .......... 60
  2.2.7 Global Dynamics ...................................... 62
References .................................................................. 66

3 Emergence and Bifurcations of LPTs in the Chain of Three
Coupled Oscillators ................................................. 67
  3.1 “Hard” Nonlinearity .......................................... 67
    3.1.1 Bifurcations of Limiting Phase Trajectories
             and Routes to Chaos in the Anharmonic Chain
             of the Three Coupled Particles ...................... 67
    3.1.2 Nonlinear Normal Modes (NNMs) ....................... 69
    3.1.3 Emergence and Bifurcations of Limiting Phase
             Trajectories (LPTs) in the System of Three Coupled
             Oscillators ............................................... 71
    3.1.4 Numerical Results ...................................... 74
    3.1.5 Spatially Localized Pulsating Regimes ................. 77
  3.2 “Soft” Nonlinearity .......................................... 80

4 Quasi-One-Dimensional Nonlinear Lattices ...................... 85
  4.1 Finite Fermi–Pasta–Ulam Oscillatory Chain ................. 86
    4.1.1 The Model ............................................... 86
    4.1.2 Basic Asymptotic ....................................... 88
    4.1.3 From “Waves” to “Particles” .......................... 92
    4.1.4 Analytical Solution for the LPTs ....................... 100
  4.2 Klein–Gordon Lattice .......................................... 102
    4.2.1 The Model ............................................... 102
    4.2.2 Asymptotic Analysis .................................... 105
  4.3 Intense Energy Exchange and Localization in Periodic FPU
          Dimer Chains ............................................. 111
    4.3.1 The Model ............................................... 113
    4.3.2 Intensive Energy Exchanges: Linear Case
            (N = 1, a = 0) ........................................... 114
    4.3.3 Complete Energy Exchanges and Localization:
            Nonlinear Case (N = 1, a > 0) ......................... 119
4.3.4 Extension to the Higher Number of Light Particles
\( (N > 1, \alpha > 0) \) ........................................... 124

Appendix ..................................................... 136
References .................................................. 137

5 Localized Nonlinear Excitations and Inter-chain Energy Exchange .................................................. 141
5.1 Linear Chains with Weak Coupling ................................................................. 142
5.2 Nonlinear Chains ..................................................................................... 144
  5.2.1 Chains with Nonlinearity, Compatible with Coupling ................................ 147
References ..................................................... 152

Part II Extensions to Non-conservative Systems

6 Duffing Oscillators ................................................................................... 155
  6.1 Duffing Oscillator with Harmonic Forcing Near 1:1 Resonance ..................... 156
    6.1.1 Main Equations and Definitions ......................................................... 156
    6.1.2 Stationary States, LPTs, and Critical Parameters ................................. 158
    6.1.3 Non-smooth Approximations of Strongly Nonlinear Oscillatory Modes ................................................................. 162
    6.1.4 Analysis with Taking into Account the Energy Dissipation ................. 165
  6.2 Duffing Oscillator Subjected to Biharmonic Forcing Near the Primary Resonance ................................................................. 166
    6.2.1 Equations of Fast and Slow Motion ...................................................... 167
    6.2.2 LPTs of Slow Motion in a Non-dissipative System .................................. 168
    6.2.3 Super-Slow Dynamics ........................................................................ 169
    6.2.4 Relaxation Oscillations in a Lightly Damped System ......................... 171
  6.3 Super-Harmonic Resonance .................................................................... 177
    6.3.1 Equations of Motion ......................................................................... 177
    6.3.2 Super-Harmonic Resonance in the Non-dissipative System .................. 180
References ................................................................. 185

7 Non-conventional Synchronization of Weakly Coupled Active Oscillators ........................................................................... 187
  7.1 Main Equations ....................................................................................... 188
    7.1.1 Coupled Active Oscillators .................................................................. 189
  7.2 NNMs and LPTs Symmetries ................................................................... 190
  7.3 Analysis of the Phase Plane and Analytical Solutions ............................ 190
References .................................................................................................. 194
# Limiting Phase Trajectories and the Emergence of Autoresonance in Anharmonic Oscillators

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Autoresonance in a SDOF Nonlinear Oscillator</td>
<td>197</td>
</tr>
<tr>
<td>8.1.1</td>
<td>Critical Parameters</td>
<td>198</td>
</tr>
<tr>
<td>8.1.2</td>
<td>Numerical Evidence of Capture into Resonance</td>
<td>202</td>
</tr>
<tr>
<td>8.2</td>
<td>Autoresonance Versus Localization in Weakly Coupled Oscillators</td>
<td>203</td>
</tr>
<tr>
<td>8.2.1</td>
<td>Energy Transfer in a System with Constant Excitation Frequency</td>
<td>204</td>
</tr>
<tr>
<td>8.2.2</td>
<td>Energy Localization and Transport in a System with a Slowly Varying Forcing Frequency</td>
<td>207</td>
</tr>
<tr>
<td>8.2.3</td>
<td>Energy Transfer in a System with Slow Changes of the Natural and Excitation Frequencies</td>
<td>209</td>
</tr>
<tr>
<td>8.3</td>
<td>Autoresonance in Nonlinear Chains</td>
<td>211</td>
</tr>
<tr>
<td>8.3.1</td>
<td>The Model</td>
<td>211</td>
</tr>
<tr>
<td>8.3.2</td>
<td>Quasi-steady States</td>
<td>214</td>
</tr>
<tr>
<td>8.3.3</td>
<td>Parametric Thresholds</td>
<td>215</td>
</tr>
<tr>
<td>8.3.4</td>
<td>Numerical Results</td>
<td>218</td>
</tr>
</tbody>
</table>

**References** | 222  

### Part III Applications

## Targeted Energy Transfer

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.1</td>
<td>The Model</td>
<td>227</td>
</tr>
<tr>
<td>9.2</td>
<td>Analytical Study</td>
<td>228</td>
</tr>
<tr>
<td>9.3</td>
<td>Selection of Resonance Terms and Principal Asymptotic Approximation</td>
<td>230</td>
</tr>
<tr>
<td>9.4</td>
<td>3 DOF Oscillators with the NES</td>
<td>235</td>
</tr>
<tr>
<td>9.5</td>
<td>Transient Dynamics of the Dissipative System</td>
<td>238</td>
</tr>
<tr>
<td>9.6</td>
<td>Reduction to a Model of the Single Oscillator</td>
<td>240</td>
</tr>
</tbody>
</table>

**References** | 243  

## Nonlinear Energy Channeling in the 2D, Locally Resonant, Systems

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>Unit Cell Model: High Energy Pulsations</td>
<td>245</td>
</tr>
<tr>
<td>10.1.1</td>
<td>The Model</td>
<td>246</td>
</tr>
<tr>
<td>10.1.2</td>
<td>Analytical Study</td>
<td>249</td>
</tr>
<tr>
<td>10.1.3</td>
<td>Numerical Verifications</td>
<td>263</td>
</tr>
<tr>
<td>10.1.4</td>
<td>Concluding Remarks</td>
<td>269</td>
</tr>
<tr>
<td>10.2</td>
<td>Unit Cell Model: Low Energy Excitation Regimes</td>
<td>270</td>
</tr>
<tr>
<td>10.2.1</td>
<td>Numerical Evidence of the Unidirectional Energy Channeling</td>
<td>270</td>
</tr>
<tr>
<td>10.2.2</td>
<td>Theoretical Study</td>
<td>271</td>
</tr>
</tbody>
</table>

**References** | 271  

---

8 Limiting Phase Trajectories and the Emergence of Autoresonance in Anharmonic Oscillators. ............................. 195
8.1 Autoresonance in a SDOF Nonlinear Oscillator .............................. 197
8.1.1 Critical Parameters ................................................. 198
8.1.2 Numerical Evidence of Capture into Resonance ................................................. 202
8.2 Autoresonance Versus Localization in Weakly Coupled Oscillators ................................................. 203
8.2.1 Energy Transfer in a System with Constant Excitation Frequency ................................................. 204
8.2.2 Energy Localization and Transport in a System with a Slowly Varying Forcing Frequency ................................................. 207
8.2.3 Energy Transfer in a System with Slow Changes of the Natural and Excitation Frequencies ................................................. 209
8.3 Autoresonance in Nonlinear Chains ................................................. 211
8.3.1 The Model ................................................. 211
8.3.2 Quasi-steady States ................................................. 214
8.3.3 Parametric Thresholds ................................................. 215
8.3.4 Numerical Results ................................................. 218

References ................................................. 222

---

9 Targeted Energy Transfer ................................................. 227
9.1 The Model ................................................. 227
9.2 Analytical Study ................................................. 228
9.3 Selection of Resonance Terms and Principal Asymptotic Approximation ................................................. 230
9.4 3 DOF Oscillators with the NES ................................................. 235
9.5 Transient Dynamics of the Dissipative System ................................................. 238
9.6 Reduction to a Model of the Single Oscillator ................................................. 240

References ................................................. 243

---

10 Nonlinear Energy Channeling in the 2D, Locally Resonant, Systems ................................................. 245
10.1 Unit Cell Model: High Energy Pulsations ................................................. 245
10.1.1 The Model ................................................. 246
10.1.2 Analytical Study ................................................. 249
10.1.3 Numerical Verifications ................................................. 263
10.1.4 Concluding Remarks ................................................. 269
10.2 Unit Cell Model: Low Energy Excitation Regimes ................................................. 270
10.2.1 Numerical Evidence of the Unidirectional Energy Channeling ................................................. 270
10.2.2 Theoretical Study ................................................. 271

References ................................................. 271
## 11 Nonlinear Targeted Energy Transfer and Macroscopic Analogue of the Quantum Landau-Zener Effect in Coupled Granular Chains

11.1 Introduction ........................................ 293
11.2 System Description ................................... 295
11.3 Recurrent Energy Exchange Phenomena in the System of Coupled Granular Chains .............................. 296
11.4 Nonlinear Targeted Energy Transfer and Energy Exchange: Analysis ....................................................... 301
  11.4.1 Localization of Energy by Complete Decoupling ..... 302
11.5 Targeted Energy Transfer Through the Landau-Zener Tunneling Effect in Space. ................................. 303
  11.5.1 Nonlinear Targeted Energy Transfer and Irreversible Energy Exchange: Simulation .................... 314
11.6 Conclusions ........................................ 319
Appendix ............................................... 319
References............................................... 323

## 12 Forced Pendulum ........................................ 327

12.1 The Model ......................................... 328
12.2 Nonstationary Dynamics and Dynamical Transitions ......... 330
12.3 Poincaré Sections and Onset of Chaotic Motion ............. 332
References............................................... 335

## 13 Classical Analog of Linear and Quasi-Linear Quantum Tunneling ....................................................... 337

13.1 Two Weakly Coupled Linear Oscillators .................. 338
13.2 Approximate Analysis of Energy Transfer in the Linear System ............................................................... 340
13.3 Classical Analog of Quasi-Linear Quantum Tunneling .................. 343
13.4 Moderately and Strongly Nonlinear Adiabatic Tunneling .......................... 346
  13.4.1 Moderately Nonlinear Regimes ................... 346
13.5 Strongly Nonlinear Regimes ............................ 352
References............................................... 353

## 14 Strongly Nonlinear Lattices ............................ 355

14.1 The Large-Amplitude Oscillations in the Discrete Finite Frenkel–Kontorova Model .......................... 355
14.2 Large-Amplitude Nonlinear Normal Modes of the Discrete Sine-Lattices ....................................................... 362
14.3 Is Energy Localization Possible in the Conditions of Acoustic Vacuum? 374
14.3.1 The Model and Equations of Motion 375
14.3.2 Two-Mode Approximation 376
14.3.3 Cluster Variables 379
14.3.4 Equations in Angular Variables in Cluster Variant 380
14.3.5 Phase Plane 381
14.3.6 Analytical solution for LPT 382
14.3.7 Poincare Sections 384
Appendix 1: Timescale Separation 385
Appendix 2: Projection onto Two Modes—Formulas 387
References 389

15 Nonlinear Vibrations of the Carbon Nanotubes 391
15.1 Nonlinear Optical Vibrations of Single-Walled Carbon Nanotubes 392
15.1.1 The Model 393
15.1.2 Radial Breathing Mode 394
15.1.3 Circumferential Flexure Mode 403
15.2 Coupling Shell- and Beam-Type Oscillations of Single-Walled Carbon Nanotubes 414
15.2.1 The Model 414
15.2.2 Stationary Solutions 417
15.2.3 Multi-scale Expansion 419
15.2.4 Analysis of the Steady States Solutions and Non-stationary Dynamics 422
References 431

Conclusions 435
Nonstationary Resonant Dynamics of Oscillatory Chains and Nanostructures
Manevitch, L.I.; Kovaleva, A.; Smirnov, V.; Starostevsky, Y.
2018, XXII, 436 p. 194 illus., 116 illus. in color., Hardcover
ISBN: 978-981-10-4665-0