Chapter 2
Design of LCL Filter

Abstract As the interface between renewable energy power generation system and the power grid, the grid-connected inverter is used to convert the dc power to the high-quality ac power and feed it into the power grid. In the grid-connected inverter, a filter is needed as the interface between the inverter and the power grid. Compared with the L filter, the LCL filter is considered to be a preferred choice for its cost-effective attenuation of switching frequency harmonics in the injected grid currents. To achieve high-quality grid current, the LCL filter should be properly designed. In this chapter, the widely used pulse-width modulation (PWM) schemes are introduced, including the bipolar sinusoidal pulse-width modulation (SPWM), unipolar SPWM and harmonic injection SPWM. The spectrums of the output PWM voltage with different SPWM are studied and compared. A design procedure for LCL filter based on the restriction standards of injected grid current is presented and verified by simulations.

Keywords Grid-connected inverter · LCL filter · Pulse-width modulation (PWM) · Total harmonics distortion (THD)

As the interface between renewable energy power generation system and the power grid, the grid-connected inverter is used to convert the dc power to the high-quality ac power and feed it into the power grid. In the grid-connected inverter, a filter is needed as the interface between the inverter and the power grid. Compared with the L filter, the LCL filter is considered to be a preferred choice for its cost-effective attenuation of switching frequency harmonics in the injected grid currents. To achieve high-quality grid current, the LCL filter should be properly designed. In this chapter, the widely used pulse-width modulation (PWM) schemes are introduced, including the bipolar sinusoidal pulse-width modulation (SPWM), unipolar SPWM and harmonic injection SPWM. The spectrums of the output PWM voltage with different SPWM are studied and compared. A design procedure for LCL filter based on the restriction standards of injected grid current is presented and verified by simulations.
2.1 PWM for Single-Phase Full-Bridge Grid-Connected Inverter

Figure 2.1 shows the topology of a single-phase full-bridge LCL-type grid-connected inverter, where switches \( Q_1 - Q_4 \) compose the two bridge legs, and inductors \( L_1, L_2 \) and capacitor \( C \) compose the LCL filter. Note that the two switches in the same bridge leg are switched in a complementary manner.

Generally, the bipolar SPWM and unipolar SPWM are usually used for single-phase full-bridge inverter. For convenience of illustration, the dc input voltage \( V_{\text{in}} \) is split into two ones equally, and the midpoint \( O \) is defined as the base potential.

2.1.1 Bipolar SPWM

Figure 2.2 shows the key waveforms of the bipolar SPWM for single-phase LCL-type grid-connected inverter, where, \( v_M \) is the sinusoidal modulation signal with the amplitude of \( V_M \), and \( v_{\text{tri}} \) is the triangular carrier with the amplitude of \( V_{\text{tri}} \). When \( v_M > v_{\text{tri}} \), \( Q_1 \) and \( Q_4 \) turn on, \( Q_2 \) and \( Q_3 \) turn off, resulting in \( v_{AO} = V_{\text{in}}/2 \) and \( v_{BO} = -V_{\text{in}}/2 \); When \( v_M < v_{\text{tri}} \), \( Q_1 \) and \( Q_4 \) turn off, \( Q_2, Q_3 \) turn on, resulting in \( v_{AO} = -V_{\text{in}}/2 \) and \( v_{BO} = V_{\text{in}}/2 \). The inverter bridge output voltage \( v_{\text{inv}} \) is the difference between \( v_{AO} \) and \( v_{BO} \), i.e., \( v_{\text{inv}} = v_{AO} - v_{BO} \). As shown in Fig. 2.2, \( v_{\text{inv}} \) has only two voltage levels, namely \( -V_{\text{in}} \) and \( +V_{\text{in}} \). So, this PWM scheme is often called as bipolar SPWM.

In the following, \( \omega_m \) and \( \omega_{\text{sw}} \) denote the angular frequencies of the modulation signal \( v_M \) and triangular carrier \( v_{\text{tri}} \), respectively, the initial phase of the modulation signal \( v_M \) is set to 0, and \( M_r \) denotes the ratio of \( V_M \) and \( V_{\text{tri}} \), i.e.,

\[
M_r = \frac{V_M}{V_{\text{tri}}} \quad (2.1)
\]

According to the Fourier transform theory, the time-varying signals \( v_{AO} \) and \( v_{BO} \) shown in Fig. 2.2 can be expressed as [1]
where, $J_n(x)$ is the Bessel function of the first kind [2], expressed as

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!} \left( \frac{x}{2} \right)^{2k+n}$$  \hspace{1cm} (2.3)$$

According to (2.2), the Fourier series expansion of the inverter bridge output voltage $v_{inv}$ with bipolar SPWM can be obtained, which is
\[ v_{\text{inv}}(t) = v_{A0}(t) - v_{BO}(t) \]

\[
= M_r V_{\text{in}} \sin \omega_o t + \frac{4V_{\text{in}}}{\pi} \sum_{m=1,3,5,...}^{\infty} \sum_{n=0,\pm2,\pm4,...}^{\pm\infty} \frac{J_n(mM_r\pi/2)}{m} \sin \frac{m\pi}{2} \cos(m\omega_{\text{sw}} t + n\omega_o t) \\
+ \frac{4V_{\text{in}}}{\pi} \sum_{m=2,4,6,...}^{\infty} \sum_{n=\pm1,\pm3,...}^{\pm\infty} \frac{J_n(mM_r\pi/2)}{m} \cos \frac{m\pi}{2} \sin(m\omega_{\text{sw}} t + n\omega_o t)
\]

(2.4)

where, the first term is the fundamental component, the second term is the sideband harmonics around odd multiples of the carrier frequency, and the third term is the sideband harmonics around even multiples of the carrier frequency. In the second and third terms, \( m \) is the carrier index variable, and \( n \) is the baseband index variable. \( m \) and \( n \) determine the harmonics distribution. When \( m \) is odd, \( |\sin(m\pi/2)| = 1 \); When \( m \) is even, \( |\cos(m\pi/2)| = 1 \). With a given \( V_{\text{in}} \), the amplitudes of the harmonics in \( v_{\text{inv}} \) are determined by \( |J_n(mM_r\pi/2)/m| \). Moreover, the harmonics in \( v_{\text{inv}} \) distribute only at the frequencies where \( m + n \) is odd.

According to (2.3), an example of \( |J_n(mM_r\pi/2)/m| \) with \( M_r = 0.9 \) and \( m = 1, 2 \) and \( 3 \) is depicted with the dots, as shown in Fig. 2.3, where the three dashed lines are plotted with Gamma Function \( \Gamma(k + n + 1) \), where the variable \( n \) uses a real number. As observed, the dot with the maximum value locates at the center frequency \( \omega_{\text{sw}} \), where \( m = 1, n = 0 \); the farther the sideband harmonic departs from the center frequency, the smaller its amplitude is. In contrast to the harmonics around the center frequency \( \omega_{\text{sw}} \), the amplitudes of the harmonics around twice and above the carrier frequency are much smaller. Thus, the dominant harmonics in \( v_{\text{inv}} \) are at around \( \omega_{\text{sw}} \), which needs to be attenuated by the \( LCL \) filter.

In conclusion, the spectrum of the inverter bridge output voltage, \( v_{\text{inv}} \), generated by the bipolar SPWM can be described as

![Fig. 2.3 Characteristic curves of Bessel function](image-url)
The harmonics in $v_{\text{inv}}$ distribute only at frequencies where $m + n$ is odd. When $m$ is odd, the harmonics distribute not only at $m$ times of the carrier frequency, but also at the sideband frequency when $n$ is even; When $m$ is even, the harmonics only distribute at the sideband frequency when $n$ is odd.

The dominant harmonics in $v_{\text{inv}}$ are at around the carrier frequency (e.g., $n = 0, \pm 2, \pm 4, \ldots$). The design of the LCL filter is determined by attenuating these dominant harmonics.

### 2.1.2 Unipolar SPWM

As mentioned above, with the bipolar SPWM, the voltage levels of $v_{\text{inv}}$ could only be $-V_{in}$ and $+V_{in}$. In fact, when $Q_1$ and $Q_3$ or $Q_2$ and $Q_4$ turn on simultaneously, $v_{\text{inv}}$ will be 0. The unipolar SPWM is such a kind of the modulation scheme that could make $v_{\text{inv}}$ be not only $+V_{in}$ and $-V_{in}$, but also 0.

Figure 2.4 shows the key waveforms of the unipolar SPWM for single-phase LCL-type grid-connected inverter, where $v_M$ is the sinusoidal modulation signal, and $v_{\text{tri}}$ and $-v_{\text{tri}}$ are the two sets of triangular carrier. Comparison of $v_M$ and $v_{\text{tri}}$ leads to the control signals for $Q_1$ and $Q_2$, and comparison of $v_M$ and $-v_{\text{tri}}$ leads to the control signals for $Q_3$ and $Q_4$. In detail, when $v_M > v_{\text{tri}}$, $Q_1$ turns on and $Q_2$ turns off, thus $v_{AO} = V_{in}/2$; When $v_M < v_{\text{tri}}$, $Q_1$ turns off and $Q_2$ turns on, thus $v_{AO} = -V_{in}/2$. Likewise, when $v_M > -v_{\text{tri}}$, $Q_4$ turns on and $Q_3$ turns off, thus $v_{BO} = -V_{in}/2$; When $v_M < -v_{\text{tri}}$, $Q_4$ turns off and $Q_3$ turns on, thus $v_{BO} = V_{in}/2$.

Since $v_{\text{inv}} = v_{AO} - v_{BO}$, the voltage levels of $v_{\text{inv}}$ could be $+V_{in}$, $-V_{in}$, and 0. In the positive period of $v_M$, the voltage levels of $v_{\text{inv}}$ could only be $+V_{in}$ and 0; while in the negative period of $v_M$, the voltage levels of $v_{\text{inv}}$ could only be $-V_{in}$ and 0. Therefore, this modulation scheme is called unipolar SPWM. Furthermore, the ripple frequency of $v_{\text{inv}}$ is twice the carrier frequency.

Since the control signal for $Q_1$ is obtained by comparing $v_M$ and $v_{\text{tri}}$, the Fourier series expansion of $v_{AO}$ is the same as (2.2). The control signal for $Q_4$ is obtained by comparing $v_M$ and $-v_{\text{tri}}$, and $-v_{\text{tri}}$ lags $v_{\text{tri}}$ with a phase of $\pi$, the Fourier series expansion of $v_{BO}$ can be obtained by replacing $\omega_{\text{in}}t$ in (2.2) with $\omega_{\text{in}}t - \pi$. Thus, $v_{BO}$ is expressed as

\[
v_{BO}(t) = -\frac{M_rV_{in}}{2}\sin\omega_{\text{in}}t
- \frac{2V_{in}}{\pi} \sum_{m=1,3,5,...}^{\infty} \sum_{n=0,\pm 2,\pm 4,...}^{\infty} \frac{J_n(mM_r\pi/2)}{m} \sin \frac{m\pi}{2}\cos(m(\omega_{\text{in}}t - \pi) + n\omega_{\text{in}}t)
- \frac{2V_{in}}{\pi} \sum_{m=2,4,6,...}^{\infty} \sum_{n=1,3,...}^{\infty} \frac{J_n(mM_r\pi/2)}{m} \cos \frac{m\pi}{2}\sin(m(\omega_{\text{in}}t - \pi) + n\omega_{\text{in}}t)
\]

\((2.5)\)
Equation (2.5) can be further simplified as

\[
v_{BO}(t) = -\frac{M_r V_{in}}{2} \sin \omega_c t + \frac{2V_{in}}{\pi} \sum_{m=1,3,5,...}^{\infty} \sum_{n=0,2,4,6,...}^{\infty} \frac{J_n(mM_r/2)}{m} \frac{m\pi}{2} \sin \left(\frac{m\pi}{2} \cos(m\omega_m + n\omega_s t)\right) \\
- \frac{2V_{in}}{\pi} \sum_{m=2,4,6,...}^{\infty} \sum_{n=\pm1,\pm3,\pm5,...}^{\infty} \frac{J_n(mM_r/2)}{m} \frac{m\pi}{2} \sin \left(\frac{m\pi}{2} \cos(m\omega_m + n\omega_s t)\right)
\]

(2.6)

According to (2.2) and (2.6), the Fourier series expansion of \(v_{inv}\) with the unipolar SPWM is expressed as

\[
v_{inv}(t) = v_{AO}(t) - v_{BO}(t) \\
= M_r V_{in} \sin \omega_c t + \frac{4V_{in}}{\pi} \sum_{m=2,4,6,...}^{\infty} \sum_{n=\pm1,\pm3,\pm5,...}^{\infty} \frac{J_n(mM_r/2)}{m} \frac{m\pi}{2} \sin \left(\frac{m\pi}{2} \cos(m\omega_m + n\omega_s t)\right)
\]

(2.7)

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Fig. 2.4 Unipolar SPWM for single-phase LCL-type grid-connected inverter
According to (2.7), the harmonic spectrum of $v_{\text{inv}}$ with the unipolar SPWM can be described as

1. The harmonics in $v_{\text{inv}}$ distribute only at the sideband frequencies where $m$ is even and $n$ is odd.
2. The dominant harmonics in $v_{\text{inv}}$ are at around twice the carrier frequency, which is the major consideration of filter design.

Comparing (2.4) and (2.7), it shows that the frequencies of the harmonics in $v_{\text{inv}}$ with the unipolar SPWM are twice that of those with the bipolar SPWM. This is because the ripple frequencies of $v_{\text{inv}}$ with the bipolar and unipolar SPWMs are one and two times of the carrier frequency, respectively, which can be found from Figs. 2.2 and 2.4.

### 2.2 PWM for Three-Phase Grid-Connected Inverter

Figure 2.5a shows the topology of a three-phase grid-connected inverter, where switches $Q_1$–$Q_6$ compose the three-phase legs, and three sets of inductors $L_1$, $L_2$, and capacitor $C$ compose the three-phase $LCL$ filter. Note that the three-phase capacitors in $LCL$ filter can be either delta- or star-connection. The capacitance needed in delta-connection is one-third of that in star-connection, and the capacitor
current and voltage stresses in delta-connection are $1/\sqrt{3}$ and $\sqrt{3}$ times of that in star-connection, respectively. In this book, star-connection is adopted. Similarly, $V_{in}$ is split into two ones equally for convenience of illustration, and the midpoint $O$ is defined as the base potential.

Figure 2.5b shows the equivalent circuit of the three-phase grid-connected inverter, where $v_{ao}$, $v_{bo}$, and $v_{co}$ are the three inverter bridge output voltages with respect to midpoint $O$; $i_{1x}$ ($x = a, b, c$) is the inverter-side inductor current; $v_{Cx}$ and $i_{Cx}$ are the filter capacitor voltage and current, respectively; $i_{2x}$ is the grid-side inductor current. From Fig. 2.5b, $v_{ao}$, $v_{bo}$ and $v_{co}$ can be expressed as

$$
v_{ao} = j\omega L_1 \cdot i_{1a} + v_{Ca} + v_{NO} \\
v_{bo} = j\omega L_1 \cdot i_{1b} + v_{Cb} + v_{NO} \\
v_{co} = j\omega L_1 \cdot i_{1c} + v_{Cc} + v_{NO}
$$

(2.8)

where $v_{NO}$ is the voltage across points $N$ and $O$.

The three-phase filter capacitor voltages can be expressed as

$$
v_{Ca} = i_{Ca}/(j\omega C) \\
v_{Cb} = i_{Cb}/(j\omega C) \\
v_{Cc} = i_{Cc}/(j\omega C)
$$

(2.9)

For three-phase three-wire system, $i_{1a} + i_{1b} + i_{1c} = 0$, $i_{Ca} + i_{Cb} + i_{Cc} = 0$. According to (2.8), the zero sequence component $v_{NO}$ is derived as

$$
v_{NO} = (v_{ao} + v_{bo} + v_{co})/3
$$

(2.10)

Similarly, $v_{NN'}$, the voltage across points $N$ and $N'$, can be obtained, expressed as

$$
v_{NN'} = (v_{ga} + v_{gb} + v_{gc})/3
$$

(2.11)

With PWM control, $v_{ao} + v_{bo} + v_{co} \neq 0$. So, according to (2.10), $v_{NO}$ is not equal to zero, which means that the potentials of $N$ and $O$ are not equal. When the three-phase grid voltages are balance, i.e., $v_{ga} + v_{gb} + v_{gc} = 0$, the potentials of $N$ and $N'$ are equal according to (2.11).

### 2.2.1 SPWM

Figure 2.6 shows the key waveforms of SPWM for three-phase grid-connected inverter, where $v_{tri}$ is the triangular carrier, and $v_{Ma}$, $v_{Mb}$, and $v_{Mc}$ are the three-phase sinusoidal modulation signals, expressed as
\[
\begin{align*}
V_{M} & = V_M \cdot \sin(\omega_o t) \\
V_{Mb} & = V_M \cdot \sin(\omega_o t - 2\pi/3) \\
V_{Mc} & = V_M \cdot \sin(\omega_o t + 2\pi/3)
\end{align*}
\]  
(2.12)

where \(V_M\) is the amplitude of the modulation signals, \(\omega_o\) is the angular frequency of the modulation signals, which is equal to the grid angular frequency.

Obviously, the control signals for \(Q_1\) and \(Q_4\) are determined by comparing \(v_{Ma}\) and \(v_{tri}\), the control signals for \(Q_3\) and \(Q_6\) are determined by comparing \(v_{Mb}\) and \(v_{tri}\), and the control signals for \(Q_5\) and \(Q_2\) are determined by comparing \(v_{Mc}\) and \(v_{tri}\). Thus, the voltages of the midpoints of three-phase legs with respect to \(O\), \(v_{ao}, v_{bo}, v_{co}\), and \(v_{co}\), are obtained. \(v_{NO}\) can be determined according to (2.10). The output phase voltage \(v_{aN}\) is equal to \(v_{ao} - v_{No}\), and the output line voltage \(v_{ab}\) is equal to \(v_{ao} - v_{bo}\).
According to the modulation scheme, the expression of $v_{ao}$ is the same as (2.2). Since $v_{Mb}$ lags $v_{Ma}$ with a phase of $2\pi/3$ and $v_{Mc}$ leads $v_{Ma}$ with a phase of $2\pi/3$, by replacing $\omega_o t$ in (2.2) with $\omega_o t - 2\pi/3$ and $\omega_o t + 2\pi/3$, respectively, the expressions of $v_{bo}$ and $v_{co}$ can be obtained as

\[
v_{bo}(t) = \frac{M_r V_{in}}{2} \sin \left( \omega_o t - \frac{2\pi}{3} \right) + \frac{2V_{in}}{\pi} \sum_{m=1,3,\ldots}^\infty \sum_{n=0,\pm2,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \cos \left( m\omega_{sw} t + n \left( \omega_o t - \frac{2\pi}{3} \right) \right) + \frac{2V_{in}}{\pi} \sum_{m=2,4,\ldots}^\infty \sum_{n=\pm1,3,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \sin \left( m\omega_{sw} t + n \left( \omega_o t - \frac{2\pi}{3} \right) \right) \]

\[
v_{co}(t) = \frac{M_r V_{in}}{2} \sin \left( \omega_o t + \frac{2\pi}{3} \right) + \frac{2V_{in}}{\pi} \sum_{m=1,3,\ldots}^\infty \sum_{n=0,\pm2,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \cos \left( m\omega_{sw} t + n \left( \omega_o t + \frac{2\pi}{3} \right) \right) + \frac{2V_{in}}{\pi} \sum_{m=2,4,\ldots}^\infty \sum_{n=\pm1,3,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \sin \left( m\omega_{sw} t + n \left( \omega_o t + \frac{2\pi}{3} \right) \right) \]

(2.13)

(2.14)

Substituting (2.2), (2.13), and (2.14) into (2.10) yields

\[
v_{NO}(t) = \frac{2V_{in}}{3\pi} \sum_{m=1,3,\ldots}^\infty \sum_{n=0,\pm2,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \left( 1 + 2 \cos \frac{2n\pi}{3} \right) \sin \frac{m\pi}{2} \cos(m\omega_{sw} t + n\omega_o t) + \frac{2V_{in}}{3\pi} \sum_{m=2,4,\ldots}^\infty \sum_{n=\pm1,3,\ldots}^\infty J_n(mM_r \pi/2) m \frac{m\pi}{2} \left( 1 + 2 \cos \frac{2n\pi}{3} \right) \cos \frac{m\pi}{2} \sin(m\omega_{sw} t + n\omega_o t) \]

\[
= \frac{M_r V_{in}}{2} \sin \omega_o t + \frac{2V_{in}}{\pi} \sum_{m=1,3,\ldots}^\infty \sum_{n=0,\pm2,\ldots}^\infty 4 J_n(mM_r \pi/2) m \frac{3m\pi}{2} \sin \frac{2n\pi}{3} \cos(m\omega_{sw} t + n\omega_o t) + \frac{2V_{in}}{\pi} \sum_{m=2,4,\ldots}^\infty \sum_{n=\pm1,3,\ldots}^\infty 4 J_n(mM_r \pi/2) m \frac{3m\pi}{2} \cos \frac{2n\pi}{3} \sin(m\omega_{sw} t + n\omega_o t) \]

(2.15)

According to (2.2) and (2.15), the output phase voltage $v_{an}$ is obtained, which is

\[
v_{an}(t) = v_{ao}(t) - v_{NO}(t) \]

\[
= \frac{M_r V_{in}}{2} \sin \omega_o t + \frac{2V_{in}}{\pi} \sum_{m=1,3,\ldots}^\infty \sum_{n=0,\pm2,\ldots}^\infty 4 J_n(mM_r \pi/2) m \frac{3m\pi}{2} \sin \frac{2n\pi}{3} \cos(m\omega_{sw} t + n\omega_o t) + \frac{2V_{in}}{\pi} \sum_{m=2,4,\ldots}^\infty \sum_{n=\pm1,3,\ldots}^\infty 4 J_n(mM_r \pi/2) m \frac{3m\pi}{2} \cos \frac{2n\pi}{3} \sin(m\omega_{sw} t + n\omega_o t) \]

(2.16)
As seen in (2.16), for the harmonics in $v_{aN}$ at around odd times ($m = 1, 3, 5, \ldots$) of carrier frequency, when $n = 6k$ ($k$ is an integer), $\sin^2(n\pi/3) = 0$; when $n = 6k \pm 2$, $\sin^2(n\pi/3) = 3/4$. Similarly, for the harmonics in $v_{aN}$ at around even times ($m = 2, 4, 6, \ldots$) of carrier frequency, when $n = 3(2k - 1)$, $\sin^2(n\pi/3) = 0$; when $n = 6k \pm 1$, $\sin^2(n\pi/3) = 3/4$.

So, the harmonics spectrum of the output phase voltages of three-phase inverter controlled by SPWM can be described as

(1) The harmonics in the output phase voltages $v_{aN}$ ($x = a, b, c$) only distribute at frequencies where $m + n$ is odd. When $m$ is odd, the harmonics only distribute at the sideband frequencies where $n = 6k \pm 2$ ($k$ is an integer); when $m$ is even, the harmonics only distribute at the sideband frequencies where $n = 6k \pm 1$.

(2) The harmonics in the output phase voltages $v_{aN}$ at around the carrier frequency ($n = \pm 2, \pm 4, \ldots$) are the dominant harmonics, which is the major consideration of filter design.

According to (2.2) and (2.13), the output line voltage $v_{ab}$ can be obtained, expressed as

$$v_{ab}(t) = v_{ao}(t) - v_{bo}(t) = \frac{\sqrt{3}M_rV_{in}}{2} \sin\left(\omega_o t + \frac{\pi}{6}\right)$$
$$+ \frac{2V_{in}}{\pi} \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=0,2,4,\ldots}^{\infty} \frac{J_n(mM_r\pi/2)}{m} \sin\frac{m\pi}{2} 2 \sin\frac{n\pi}{3} \cos\left(m\omega_{ao}t + n\omega_o t + \frac{\pi}{2} - \frac{n\pi}{3}\right)$$
$$+ \frac{2V_{in}}{\pi} \sum_{m=2,4,6,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} \frac{J_n(mM_r\pi/2)}{m} \cos\frac{m\pi}{2} 2 \sin\frac{n\pi}{3} \sin\left(m\omega_{ao}t + n\omega_o t + \frac{\pi}{2} - \frac{n\pi}{3}\right)$$

(2.17)

By comparing (2.16) and (2.17), it can be observed that: (1) at the fundamental frequency, the amplitude of line voltage is $\sqrt{3}$ times of that of the phase voltage, and the line voltage leads to the phase voltage with a phase of $\pi/6$; (2) The harmonics of the output phase and line voltages $v_{aN}$ and $v_{ab}$ distribute at the same sideband frequencies, and the amplitudes of harmonics in line voltages are also $\sqrt{3}$ times of that of the harmonics in phase voltages, and it leads to the harmonics in the corresponding phase voltages with a phase of $\pi/2 - n\pi/3$.

### 2.2.2 Harmonic Injection SPWM Control

According to (2.17), when $0 \leq M_r \leq 1$, the maximum amplitude of output line voltage $v_{ab}$ is only $\sqrt{3}V_{in}/2$, i.e., 0.866$V_{in}$. It means that the dc voltage utilization of the three-phase inverter controlled by SPWM is only 0.866. However, according to (2.3) and (2.7), the dc voltage utilization of a single-phase full-bridge inverter is 1.
To make the dc voltage utilization of three-phase inverter attain 1, a third harmonic component $v_z$ as shown in Fig. 2.7 is injected to the three-phase sinusoidal modulation signals. It can be observed that the peak of $v_{Ma}$ and the valley of $v_z$ appear at the same time. As a result, the peak of the modulation signal $v_{Maz}$, which is the sum of $v_{Ma}$ and $v_z$, distributes not at but on both sides of the peak of $v_{Ma}$. When the amplitude of $v_{Maz}$ is equal to that of $v_{tri}$, the real amplitude of $v_{Ma}$ will be larger than that of $v_{tri}$. Define the modulation ratio of three-phase inverter is still the ratio of the amplitudes of $v_{Ma}$ and $v_{tri}$, then according to (2.1), the modulation ratio larger than 1 will be obtained.

Further study shows that when the amplitude of the injected third harmonic component $v_z$ is one-sixth of that of modulation sinusoidal signal $v_{Ma}$ [1], i.e.,

![Fig. 2.7 Third harmonic injection SPWM for three-phase LCL-type grid-connected inverter](image)

Fig. 2.7 Third harmonic injection SPWM for three-phase LCL-type grid-connected inverter
the dc voltage utilization of the three-phase inverter attains 1. A brief proof is presented as follows.

According to (2.12) and (2.18), the modulation signal $v_{Maz}$ is as follows:

$$v_{Maz} = V_M \cdot \sin \omega_o t + \frac{V_M}{6} \cdot \sin 3 \omega_o t = \frac{3V_M}{2} \cdot \sin \omega_o t - \frac{2V_M}{3} \sin^3 \omega_o t$$  \hspace{1cm} (2.19)$$

According to (2.19), it can be derived that the peak of $v_{Maz}$ locates at $\omega_o t = \pi/3$ or $2\pi/3$. If the amplitude of $v_{Maz}$ is set to equal to that of $v_{tri}$, $V_M/V_{tri}$ can reach 1.15, which indicates that the modulation ratio of the third harmonic injection SPWM can reach 1.15. When $M_r = 1.15$, according to (2.17), the amplitude of line voltage can attain $V_{in}$, which is the same as that of the single-phase full-bridge inverter. In other words, the dc voltage utilization attains 1.

From the Fourier transform theory, the expansions of $v_{ao}$ and $v_{bo}$ in Fig. 2.7 can be obtained, which are

\begin{align*}
v_{ao}(t) &= \frac{M_r V_{in}}{2} \sin \omega_o t + \frac{M_r V_{in}}{12} \sin 3\omega_o t + \sum_{m=1,2,3,...}^{\infty} \sum_{n=0,\pm 1,\pm 2,...}^{\infty} A_{mn} \cos(m\omega_{sw}t + n\omega_o t) \\
v_{bo}(t) &= \frac{M_r V_{in}}{2} \sin \left(\omega_o t - \frac{2\pi}{3}\right) + \frac{M_r V_{in}}{12} \sin 3\omega_o t \\
&\quad + \sum_{m=1,2,3,...}^{\infty} \sum_{n=0,\pm 1,\pm 2,...}^{\infty} A_{mn} \cos \left(m\omega_{sw}t + n\left(\omega_o t - \frac{2\pi}{3}\right)\right)
\end{align*}

(2.20)

(2.21)

where $A_{mn}$ is the amplitude of harmonics, expressed as [1]

\begin{align*}
A_{mn} &= \frac{2V_{in}}{m\pi} \left[ J_0(mM_r\pi/12)J_k(mM_r\pi/2) \sin[(m+k)\pi/2]|_{k=|n|} \\
&\quad + J_0(mM_r\pi/2)J_h(mM_r\pi/12) \sin[(m+h)\pi/2]|_{h=|n|} \\
&\quad + \sum J_k(mM_r\pi/2)J_h(mM_r\pi/12) \sin[(m+k+h)\pi/2]|_{k+h=|n|} \\
&\quad + \sum J_k(mM_r\pi/2)J_h(mM_r\pi/12) \sin[(m+k+h)\pi/2]|_{3h-k=|n|}\right]
\end{align*}

(2.22)
Same as the derivation of output phase voltage $v_{aN}$ with SPWM in Sect. 2.2.1, the expression of $v_{aN}$ with the third harmonic injection SPWM can be derived, expressed as

$$v_{aN}(t) = \frac{M_r V_n}{2} \sin \omega_o t + \sum_{m=1,2,3,...}^{\infty} \sum_{n=0,\pm1,\pm2,...}^{\pm\infty} \frac{4}{3} \sin^2 \frac{n\pi}{3} \cdot A_{mn} \cos(m\omega_{sw} t + n\omega_o t) \quad (2.23)$$

By comparing (2.16) and (2.23), the harmonics spectrum of the output phase voltages of three-phase inverter with the third harmonic injection SPWM can be concluded as follows:

(1) The harmonics in the output phase voltages $v_{xN}$ ($x = a, b, c$) only distribute at the frequencies where $m + n$ is odd. When $m$ is odd, the harmonics only distribute at even sideband frequencies where $n = 6k \pm 2$ ($k$ is an integer); when $m$ is even, the harmonics only distribute at odd sideband frequencies where $n = 6k \pm 1$.

(2) The harmonics in $v_{aN}$ at around the carrier frequency ($n = \pm2, \pm4, \ldots$) are the dominant harmonics, which is the major consideration of filter design.

According to (2.20) and (2.21), the output line voltage $v_{ab}$ can be obtained as

$$v_{ab}(t) = v_{ao}(t) - v_{bo}(t)$$

$$= \frac{\sqrt{3} M_r V_n}{2} \sin \left( \omega_o t + \frac{\pi}{6} \right)$$

$$+ \sum_{m=1,2,3,...}^{\infty} \sum_{n=0,\pm1,\pm2,...}^{\pm\infty} 2 \sin \frac{n\pi}{3} \cdot A_{mn} \cos \left( m\omega_{sw} t + n\omega_o t + \frac{\pi}{2} - \frac{n\pi}{3} \right) \quad (2.24)$$

Besides (2.18), the harmonic $v_z$ injected to the modulation sinusoidal signal can be generated from the envelope magnitude of $v_{Ma}$, $v_{Mb}$, and $v_{Mc}$ [1], which means that the maximum magnitude of $|v_{Ma}|$, $|v_{Mb}|$, and $|v_{Mc}|$ is selected, as shown in Fig. 2.8. In detail, within $\omega_o t \in [0, \pi/6) \cup [5\pi/6, 7\pi/6) \cup [11\pi/6, 2\pi)$, $|v_{Ma}|$ is the largest one, so $v_z$ is extracted from $v_{Ma}$; Likewise, within $\omega_o t \in [\pi/6, \pi/2) \cup [7\pi/6, 3\pi/2)$, $|v_{Mb}|$ is the largest one, then $v_z$ is extracted from $v_{Mb}$; Within $\omega_o t \in [\pi/2, 5\pi/6) \cup [3\pi/2, 11\pi/6)$, $|v_{Mc}|$ is the largest one, and $v_z$ is extracted from $v_{Mc}$. Due to $v_{Ma} + v_{Mb} + v_{Mc} = 0$, $v_z$ can be expressed as follows

$$v_z = -k(\max\{v_{Ma}, v_{Mb}, v_{Mc}\} + \min\{v_{Ma}, v_{Mb}, v_{Mc}\}) \quad (2.25)$$

It also can be proved that the peak of the modulation signal $v_{Ma,z}$ locates at $\omega_o t = \pi/3$ or $2\pi/3$. When $k$ in (2.25) equals to 0.5, the dc voltage utilization can also
attain 1. The result of harmonic injection SPWM shown in Fig. 2.8 is equivalent to the space vector modulation (SVM) [1]. Since the zero sequence component extracts directly from the modulation sinusoidal signals, the realization of the three-phase modulation signals shown in (2.25) is simple and widely used.

The amplitude $A_{mn}$ of output harmonics voltage controlled by the harmonic injection SPWM shown in Fig. 2.8 is expressed as [2]

![Fig. 2.8 Harmonic injection SPWM for three-phase LCL-type grid-connected inverter that is equivalent to SVM](image-url)
\[
A_{mn} = \frac{4V_{in}}{m\pi^2} \left[ \frac{\pi}{6} \sin\left(\frac{(m+n)\pi}{2}\right) J_n\left(\frac{3mM_r\pi}{4}\right) + 2\cos\frac{n\pi}{6} J_n\left(\frac{\sqrt{3}mM_r\pi}{4}\right) \right] + \frac{1}{n} \sin\frac{m\pi}{2} \cos\frac{n\pi}{2} \sin\frac{n\pi}{6} \left[ J_0\left(\frac{3mM_r\pi}{4}\right) - J_0\left(\frac{\sqrt{3}mM_r\pi}{4}\right) \right]_{n\neq 0} + \sum_{k=1}^{\infty} \sum_{k\neq n}^{\infty} \left\{ \frac{1}{n+k} \sin\frac{(m+k)\pi}{2} \cos\frac{(n+k)\pi}{2} \sin\frac{(n+k)\pi}{6} \left[ J_k\left(\frac{3mM_r\pi}{4}\right) + 2\cos\frac{(2n+3k)\pi}{6} J_k\left(\frac{\sqrt{3}mM_r\pi}{4}\right) \right] \right\} + \sum_{k=1}^{\infty} \sum_{k\neq n}^{\infty} \left\{ \frac{1}{n-k} \sin\frac{(m+k)\pi}{2} \cos\frac{(n-k)\pi}{2} \sin\frac{(n-k)\pi}{6} \left[ J_k\left(\frac{3mM_r\pi}{4}\right) + 2\cos\frac{(2n-3k)\pi}{6} J_k\left(\frac{\sqrt{3}mM_r\pi}{4}\right) \right] \right\}
\]

(2.26)

### 2.3 LCL Filter Design

The PWM output voltage of the grid-connected inverters contains abundant of switching harmonic components, which results in the harmonic current injecting into the grid. Therefore, a filter is required to interface between the inverter bridge and the power grid. The LCL filter is usually employed since it has better ability of suppressing high frequency harmonics than the L filter. This section will focus on the design of the LCL filter.

The single-phase full-bridge inverter, as shown in Fig. 2.1, could be simplified to the equivalent circuit as shown in Fig. 2.9a. Likewise, when the three-phase grid voltages are balanced, the voltage potentials of node \( N \) and \( N' \) are identical. As a result, the three-phase circuit, as shown in Fig. 2.5b, can be decoupled and each phase could be simplified to the equivalent circuit as shown in Fig. 2.9b, where \( x = a, b, c \). As seen, the structures of the equivalent circuits of the single-phase and three-phase LCL filters are the same, so the design procedures of them are almost uniform, except that the harmonic spectrum of the imposed PWM voltages are different. In the following, the grid voltage \( v_g \) is assumed a pure sinusoidal waveform.

**Fig. 2.9** Equivalent circuits of single-phase and three-phase LCL-type grid-connected inverters
2.3.1 Design of the Inverter-Side Inductor

From Figs. 2.1 and 2.5a, it can be observed that the current flowing through the filter inductor $L_1$ and the switches are the same. The larger the inductor current ripple is, the larger the inductor losses and higher current stress of the switches are. As a result, the conduction and switching losses will increase. Thus, the inductor current ripple should be limited.

2.3.1.1 Single-Phase Full Bridge Grid-Connected Inverter

1. Bipolar SPWM

Figure 2.10a gives the key waveforms of the single-phase full-bridge inverter with bipolar SPWM, where $i_{1,f}$ is the fundamental component in the inverter-side inductor current, and $T_{sw}$ is the carrier period.

When $v_M > v_{tri}$, switches $Q_1$ and $Q_4$ turn on simultaneously, and the bridge output voltage $v_{inv} = V_{in}$. The voltage applied on inductor $L_1$ is

$$L_1 \frac{di_1}{dt} = V_{in} - v_C$$  (2.27)

where $v_c$ is the filter capacitor voltage. Within one carrier period, $v_C$ can be regarded to be constant, and $V_{in} > v_C$. So, the inductor current $i_1$ increases linearly, and the increment is
\[ \Delta i_{1(+)} = \frac{V_{in} - v_C}{L_1} T_{(+)} \]  

(2.28)

where \( T_{(+)} = t_{12} \) is the time interval when \( Q_1 \) and \( Q_4 \) conduct simultaneously.

When \( v_M < v_{tri} \), \( Q_2 \) and \( Q_3 \) turn on simultaneously, and \( v_{inv} = -V_{in} \). The voltage across inductor \( L_1 \) is

\[ L_1 \frac{di_1}{dt} = -V_{in} - v_C \]  

(2.29)

Similarly, \( i_1 \) decreases linearly, and the decrement is

\[ \Delta i_{1(-)} = \frac{V_{in} + v_C}{L_1} T_{(-)} \]  

(2.30)

where \( T_{(-)} = t_{23} \) is the time interval when \( Q_2 \) and \( Q_3 \) conduct simultaneously.

The equation for solving the intersection points of \( v_M \) and \( v_{tri} \) is transcendental, so regular sampling SPWM is usually used to calculate \( T_{(+)} \). In detail, a horizontal line is drawn across point \( S \), as shown in Fig. 2.10, and it would intersect the triangle carrier at \( t_{11} \) and \( t_{21} \). Considering the fundamental frequency is much lower than the carrier frequency, it is reasonable to have \( T_{(+)} = t_{12} \approx t'_{12} \). Then, \( T_{(+)} \) can be calculated, which is

\[ T_{(+)} = \frac{v_M + V_{tri}}{2V_{tri}} T_{sw} = \frac{1}{2} T_{sw}(M_r \sin \omega_o t + 1) \]  

(2.31)

Likewise, \( T_{(-)} \) can be expressed as

\[ T_{(-)} = T_{sw} - T_{(+)} = \frac{1}{2} T_{sw}(1 - M_r \sin \omega_o t) \]  

(2.32)

Generally, the fundamental component in the voltage across inductors \( L_1 \) and \( L_2 \) are small, so the filter capacitor voltage \( v_C \) can be approximated to the grid voltage \( v_g \) and it equals to the fundamental component of the bridge output voltage \( v_{inv} \), i.e.,

\[ v_C \approx v_g = M_r V_{in} \sin \omega_o t \]  

(2.33)

Substituting (2.32) and (2.33) into (2.28) and (2.30), respectively, \( \Delta i_{1(+)} \) and \( \Delta i_{1(-)} \) can be derived as

\[ \Delta i_{1(+)} = \frac{V_{in} T_{sw}}{2L_1} \left( 1 - M_r^2 \sin^2 \omega_o t \right) \]  

(2.34)

As seen in (2.34), either the maximum increment or decrement of the current of inductor \( L_1 \) (denoted as \( \Delta i_{1,\text{max}} \)) within a carrier period appears at \( \sin \omega_o t = 0 \), i.e.,

\[ \Delta i_{1,\text{max}} = V_{in} T_{sw}/(2L_1) \]. Defining the ripple coefficient as \( \lambda_{c,L1} = \Delta i_{1,\text{max}}/I_1 \), where
$I_1$ is the rated RMS value of the fundamental component of $i_1$, the minimum inductance of $L_1$ can be obtained as

$$L_{1 \text{ min}} = \frac{V_{in} T_{sw}}{2 \lambda_{c \_L1} I_1} \quad (2.35)$$

In practice, $\lambda_{c \_L1}$ is set to be 20–30% [2].

The maximum value of $L_1$ could be determined from the fundamental voltage of $L_1$, which is defined as $v_{L1 \_f}$. The smaller $v_{L1 \_f}$ is, the lower the dc-link voltage is required. Defining the ratio of RMS values of $v_{L1 \_f}$ and $v_C$ as $\lambda_{v \_L1}$, the maximum value of $L_1$ can be obtained, which is

$$L_{1 \text{ max}} = \frac{\lambda_{v \_L1} V_C}{\omega_o I_1} \approx \frac{\lambda_{v \_L1} V_g}{\omega_o I_1} \quad (2.36)$$

where $V_g$ is the RMS value of the grid voltage, and $\lambda_{v \_L1}$ is usually set to be about 5%.

2. Unipolar SPWM

Figure 2.10b gives the key waveforms of the single-phase full-bridge inverter with unipolar SPWM. When $v_M > v_{tri}$ and $v_M > -v_{tri}$, switches $Q_1$ and $Q_4$ turn on simultaneously, and $v_{inv} = V_{in}$. As a result, $i_1$ increases linearly. From Fig. 2.10b, the ratio of $T_{(+)}$ and $T_{sw}/2$ can be obtained, which is

$$\frac{T_{(+)}}{T_{sw}/2} = \frac{v_M}{V_{tri}} = M_r \sin \omega_o t \quad (2.37)$$

Substituting (2.33) and (2.37) into (2.28), the increment $\Delta i_{1(\+)}$ can be derived as

$$\Delta i_{1(\+)} = \frac{V_{in} T_{sw}}{2L_1} (1 - M_r \sin \omega_o t) M_r \sin \omega_o t \quad (2.38)$$

Similarly, the decrement $\Delta i_{1(\-)\_}$ when both $Q_2$ and $Q_3$ turn on can be calculated, which is the same as (2.38).

As seen in (2.38), the maximum increment and decrement of $i_1$ appear when $\sin \omega_o t = 1/(2M_r)$, and $\Delta i_{1 \_\text{max}} = V_{in} T_{sw}/(8L_1)$. Then, the minimum of $L_1$ with unipolar SPWM is

$$L_1 = \frac{V_{in} T_{sw}}{8 \lambda_{c \_L1} I_1} \quad (2.39)$$

By Comparing of (2.35) and (2.39), it can be seen that the required $L_1$ with unipolar SPWM is only one-fourth of that with bipolar SPWM when that the permitted maximum increment (or decrement) of inductor current are identical. The reasons are: (1) the equivalent carrier frequency with unipolar SPWM is twice that with bipolar SPWM; (2) the bridge output voltage $v_{inv}$ switches between $V_{in}$ and
−Vin when bipolar SPWM is used, while it is switched between Vin and 0, or 0 and −Vin when unipolar SPWM is used.

### 2.3.1.2 Three-Phase Grid-Connected Inverter

Similar to the single-phase grid-connected inverter, the inverter-side inductor \( L_1 \) of the three-phase grid-connected inverter is also determined by the maximum current ripple. The fundamental voltage of \( L_1 \) is also ignored here, and the filter capacitor voltage \( v_{Cx} \) is approximated to the fundamental voltage of the inverter bridge output voltage \( v_{xN} \), i.e., \( v_{Ca} \approx (M_r V_{in}/2) \sin \omega_o t \). However, differed from the single-phase full-bridge inverter, the three-phase inverter bridge output voltage \( v_{xN} \) can output five levels, i.e., 0, ±Vin/3, and ±2Vin/3. As a result, the current ripple of \( i_{1x} \) (\( x = a, b, c \)) is more complex. In the following, a detailed analysis about the current ripple of \( i_{1x} \) will be presented. Since the voltages and currents are periodic, only the key waveforms in a quarter of one cycle, i.e., \( \omega_o t \in [0, \pi/2] \) is given, as shown in Fig. 2.11.

![Fig. 2.11 Inverter-side inductor current of three-phase inverter](image-url)
From Figs. 2.6, 2.7 and 2.8, it can be observed that no matter SPWM or harmonic injection SPWM is used, the three-phase filter capacitor voltages satisfy the relation $v_{MC} > v_{MA} > v_{MB}$ within $\omega_{c}\delta \in [0, \pi/6]$. Moreover, $v_{MA}$ increases monotonously and reaches its maximum value at $\omega_{c}\delta = \pi/6$. Since $v_{CA}$ is proportional to $v_{MA}$ in the linear modulation region, $v_{CA} > v_{MA}$ is also true, and $v_{CA}$ increases monotonously and reaches its maximum value at $\omega_{c}\delta = \pi/6$. Thus, the maximum value of $v_{CA}$ equals to $(M_r V_{in}/2) \sin(\pi/6) = M_r V_{in}/4$. When SPWM or harmonic injection SPWM is used, the maximum values of $v_{CA}$ are $V_{in}/4$ and $1.15V_{in}/4$, respectively. Obviously, $v_{CA} < V_{in}/3$ is always true within $\omega_{c}\delta = [0, \pi/6]$. When $\omega_{c}\delta \in [\pi/6, \pi/2]$, $i_{1a}$ can be divided into six sections in one carrier period, i.e., $[t_0, t_6]$, as shown in Fig. 2.11a, and three cases can be found in the six sections.

**Case 1**: when $t \in [t_0, t_1] \cup [t_2, t_3]$, $v_{AN} = V_{in}/3$. Since $v_{CA} < V_{in}/3$, $i_{1a}$ increases linearly;  
**Case 2**: when $t \in [t_1, t_2] \cup [t_4, t_5]$, $v_{AN} = 0$. Since $v_{CA} > 0$, $i_{1a}$ decreases linearly;  
**Case 3**: when $t \in [t_3, t_4] \cup [t_5, t_6]$, $v_{AN} = -V_{in}/3$. Since $v_{CA} < V_{in}/3$, $i_{1a}$ decreases linearly.

When $\omega_{c}\delta \in [\pi/6, \pi/2]$, $v_{CA} > v_{CC} > v_{CB}$ is true, and $v_{CA}$ increases monotonously and reaches its maximum value at $\omega_{c}\delta = \pi/2$. The maximum value of $v_{CA}$ is $M_r V_{in}/2$. When SPWM or harmonic injection SPWM is used, the maximum values of $v_{CA}$ are $V_{in}/2$ and $1.15V_{in}/2$, respectively. Obviously, $v_{CA} < 2V_{in}/3$ is always true within $\omega_{c}\delta \in (\pi/6, \pi/2)$. Similarly, when $\omega_{c}\delta \in (\pi/6, \pi/2)$, $i_{1a}$ can also be divided into six sections in one carrier period, i.e., $[t_0, t_6]$, as shown in Fig. 2.11b, c, and three cases can also be found in the six sections.

**Case 1**: when $t \in [t_0, t_1] \cup [t_4, t_5]$, $v_{AN} = 2V_{in}/3$. Since $v_{CA} < 2V_{in}/3$, $i_{1a}$ increases linearly;  
**Case 2**: when $t \in [t_1, t_2] \cup [t_3, t_4]$, $v_{AN} = V_{in}/3$. If $v_{CA} < V_{in}/3$, $i_{1a}$ increases linearly, as shown in Fig. 2.11b. If $v_{CA} > V_{in}/3$, $i_{1a}$ decreases linearly, as shown in Fig. 2.11c;  
**Case 3**: when $t \in [t_2, t_3] \cup [t_5, t_6]$, $v_{AN} = 0$. Since $v_{CA} > 0$, $i_{1a}$ decreases linearly.

Defining $\omega_{c}\delta$ when $v_{CA} = V_{in}/3$ as $\phi$, yields

$$\frac{M_r V_{in}}{2} \sin \phi = \frac{V_{in}}{3}$$

(2.40)

Then, $\phi$ can be calculated as

$$\phi = \arcsin \left( \frac{2}{3M_r} \right)$$

(2.41)

According to (2.41), it can be obtained that only when $M_r \geq 2/3$, $v_{CA}$ will be possible to be larger than $V_{in}/3$, thus the case shown in Fig. 2.11c appears; and when $M_r < 2/3$, $v_{CA}$ will be never larger than $V_{in}/3$, thus the case shown in Fig. 2.11c does not appear.
As seen from Fig. 2.11a, $i_{1a}$ continues decreasing within $[t_3, t_6]$. As seen from Fig. 2.11b, $i_{1a}$ continues increasing within $[t_0, t_2]$ or $[t_3, t_5]$, and decreases within $[t_2, t_3]$ or $[t_5, t_6]$. As seen from Fig. 2.11c, $i_{1a}$ increases within $[t_0, t_1]$ or $[t_4, t_5]$ and continues decreasing within $[t_1, t_4]$ or $[t_5, t_6]$. As mentioned above, the maximum increment and decrement of the inverter-side inductor current is identical. In the following, only the decrements of $i_{1a}$ within $[t_3, t_6]$ shown in Fig. 2.11a, within $[t_2, t_3]$ or $[t_5, t_6]$ shown in Fig. 2.11b, and within $[t_1, t_4]$ or $[t_5, t_6]$ shown in Fig. 2.11c, will be derived. Based on these decrements, the lower limit of the inverter-side inductor can be obtained.

According to Fig. 2.11a, the decrement of $i_{1a}$ within $[t_3, t_6]$ can be expressed as

$$
\Delta i_{1a(1)} = \frac{-V_{in}/3 - v_{Ca}}{L_1} t_{34} + \frac{0 - v_{Ca}}{L_1} t_{45} + \frac{-V_{in}/3 - v_{Ca}}{L_1} t_{56} \\
= \frac{V_{in}}{3L_1} (t_{36} - t_{45}) + \frac{v_{Ca}}{L_1} t_{36}
$$

According to Fig. 2.11b, the decrements of $i_{1a}$ within $[t_2, t_3]$ and $[t_5, t_6]$ can be, respectively, expressed as

$$
\Delta i_{1a(2)} = \frac{v_{Ca}}{L_1} t_{23}
$$

$$
\Delta i_{1a(3)} = \frac{v_{Ca}}{L_1} t_{56}
$$

According to Fig. 2.11c, the decrement of $i_{1a}$ within $[t_1, t_4]$ can be expressed as

$$
\Delta i_{1a(4)} = \frac{V_{in}/3 - v_{Ca}}{L_1} t_{12} + \frac{0 - v_{Ca}}{L_1} t_{23} + \frac{V_{in}/3 - v_{Ca}}{L_1} t_{34} \\
= \frac{V_{in}}{3L_1} (t_{14} - t_{23}) - \frac{v_{Ca}}{L_1} t_{14}
$$

And the expression of the decrement of $i_{1a}$ within $[t_5, t_6]$ is the same as (2.44). If the SPWM is used, the following relations can be obtained from Fig. 2.11.

$$
\left\{\begin{array}{l}
t_{36} = T_{sw} \cdot (V_{tri} - v_{Ma})/2V_{tri} \\
t_{45} = T_{sw} \cdot (V_{tri} - v_{Mc})/2V_{tri}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
t_{23} = T_{sw} \cdot (V_{tri} + v_{Mb})/2V_{tri} \\
t_{56} = T_{sw} \cdot (V_{tri} - v_{Ma})/2V_{tri}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
t_{14} = T_{sw} \cdot (V_{tri} + v_{Mc})/2V_{tri} \\
t_{23} = T_{sw} \cdot (V_{tri} + v_{Mb})/2V_{tri}
\end{array}\right.
$$
If the harmonic injection SPWM is used, $v_{Ma}$, $v_{Mb}$, and $v_{Mc}$ in (2.46)–(2.48) should be replaced by $v_{Maz}$, $v_{Mbz}$, and $v_{Mcz}$, respectively.

When the SPWM is used, $v_{Ma}$, $v_{Mb}$, and $v_{Mc}$ given in (2.12) and $M_r = V_M/V_{tri}$ are substituted into (2.46), $t_{36}$ and $t_{45}$ can be calculated. Then, by substituting $t_{36}$, $t_{45}$, and $v_{Ca} \approx (M_rV_{in}/2)\sin \omega_o t$ into (2.42), $\Delta i_{1a(1)}$ will be obtained. On the base of $M_rV_{in}T_{sw}/(2L_1)$, the normalized $\Delta i_{1a(1)}$ is finally expressed as

$$
\Delta i_{1\text{ SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(1)}}{M_rV_{in}T_{sw}/(2L_1)} = \left| \frac{1}{6} \sin \omega_o t + \frac{1}{3} \sin \left( \omega_o t + \frac{2\pi}{3} \right) - \frac{M_r}{2} \sin^2 \omega_o t \right| 
$$

(2.49)

Similarly, according to (2.12), (2.43)–(2.45), (2.47) and (2.48), the normalized $\Delta i_{1a(2)}$, $\Delta i_{1a(3)}$, and $\Delta i_{1a(4)}$ can be derived, expressed as

$$
\Delta i_{2\text{ SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(2)}}{M_rV_{in}T_{sw}/(2L_1)} = \left| \sin \omega_o t \left[ \frac{1}{2} + \frac{M_r}{2} \sin \left( \omega_o t - \frac{2\pi}{3} \right) \right] \right| 
$$

(2.50)

$$
\Delta i_{3\text{ SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(3)}}{M_rV_{in}T_{sw}/(2L_1)} = \left| \sin \omega_o t \left( \frac{1}{2} - \frac{M_r}{2} \sin \omega_o t \right) \right| 
$$

(2.51)

$$
\Delta i_{4\text{ SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(4)}}{M_rV_{in}T_{sw}/(2L_1)} 
\begin{align*}
&= \left[ \frac{2}{3} \sin \left( \omega_o t + \frac{2\pi}{3} \right) - \frac{1}{6} \sin \omega_o t - \frac{M_r}{2} \sin \omega_o t \sin \left( \omega_o t + \frac{2\pi}{3} \right) \right] \\
&= \left| \frac{2}{3} \sin \left( \omega_o t + \frac{2\pi}{3} \right) - \frac{1}{6} \sin \omega_o t - \frac{M_r}{2} \sin \omega_o t \sin \left( \omega_o t + \frac{2\pi}{3} \right) \right| 
\end{align*}
$$

(2.52)

Same as the above calculation procedure for the SPWM, when the harmonic injection SPWM is used, the normalized $\Delta i_{1a(1)}$, $\Delta i_{1a(2)}$, $\Delta i_{1a(3)}$, and $\Delta i_{1a(4)}$ can be derived, expressed as

$$
\Delta i_{1\text{ HI-SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(1)}}{M_rV_{in}T_{sw}/(2L_1)} = \left| \frac{1}{6} \sin \omega_o t + \frac{1}{3} \sin \left( \omega_o t + \frac{2\pi}{3} \right) - \frac{3M_r}{4} \sin^2 \omega_o t \right| 
$$

(2.53)

$$
\Delta i_{2\text{ HI-SPWM}}(\omega_o t) \triangleq \frac{\Delta i_{1a(2)}}{M_rV_{in}T_{sw}/(2L_1)} 
\begin{align*}
&= \left| \sin \omega_o t \left[ \frac{1}{2} + \frac{M_r}{2} \sin \left( \omega_o t - \frac{2\pi}{3} \right) + \frac{M_r}{4} \sin \left( \omega_o t + \frac{2\pi}{3} \right) \right] \right| 
\end{align*}
$$

(2.54)
Note that the harmonic injection SPWM is equivalent to SVM, the discussion of the inverter-side inductor current ripple with SVM is not repeated here.

Since \( \sin \omega_o t = \sin(\omega_o t - 2\pi/3) - \sin(\omega_o t + 2\pi/3) \), by substituting it into (2.55), it is easy to find that \( \Delta i_{2\_HI-SPWM} = \Delta i_{3\_HI-SPWM} \). According to (2.49)–(2.52), the curves of \( \Delta i_{1\_SPWM}(\omega_o t) \), \( \Delta i_{2\_SPWM}(\omega_o t) \), \( \Delta i_{3\_SPWM}(\omega_o t) \), and \( \Delta i_{4\_SPWM}(\omega_o t) \) are depicted, as shown in Fig. 2.12a. According to (2.53)–(2.56), the curves of \( \Delta i_{1\_HI-SPWM}(\omega_o t) \), \( \Delta i_{2\_HI-SPWM}(\omega_o t) \), \( \Delta i_{3\_HI-SPWM}(\omega_o t) \), and \( \Delta i_{4\_HI-SPWM}(\omega_o t) \) are depicted, as shown in Fig. 2.12b. From Fig. 2.12, the maximum value of the inverter-side inductor current ripple can be obtained. Thus, when the current ripple coefficients \( \hat{c}_{L1} \) are given, the lower limits of \( L_1 \) can be determined. In addition, the maximum value of \( L_1 \) can also be calculated from (2.36). According to the lower and upper limits of \( L_1 \), the value of \( L_1 \) can be properly selected.

![Fig. 2.12](image-url) Curves of inverter-side inductor current ripple
2.3.2 Filter Capacitor Design

The filter capacitor will lead to reactive power. The larger the capacitance is, the higher the reactive power is introduced, and also the larger the current flows through inductor $L_1$ and the power switches [3]. Thus, the conduction loss of the switches will increase. Defining $\lambda_C$ as the ratio of the reactive power introduced by the filter capacitor to the rated output active power of the grid-connected inverter, the maximum value of filter capacitor could be expressed as

$$C = \lambda_C \cdot \frac{P_o}{\omega_o V_s^2} \quad (2.57)$$

where $P_o$ is the rated output active power of single-phase full-bridge inverter or the rated output active power of one phase for three-phase full-bridge inverter. In practice, $\lambda_C$ is usually recommended to be about 5% [4].

2.3.3 Grid-Side Inductor Design

According to Fig. 2.9a, the transfer function of the grid current $i_2$ to the inverter bridge output voltage $v_{inv}$ can be obtained, which is

$$G_{LCL}(s) \triangleq \frac{i_2(s)}{v_{inv}(s)} = \frac{1}{L_1L_2Cs^3 + (L_1 + L_2)s} = \frac{1}{(L_1 + L_2)s} \cdot \frac{\omega_r^2}{s^2 + \omega_r^2} \quad (2.58)$$

where $\omega_r$ is the resonance angular frequency, which is

$$\omega_r = \sqrt{\frac{L_1 + L_2}{L_1L_2C}} \quad (2.59)$$

The expression of $G_{LCL}(s)$ for three-phase grid-connected inverter is the same as (2.58).

After the inverter-side inductor and the filter capacitor are determined, the grid-side inductor $L_2$ could be designed according to the harmonic restriction standards such as IEEE Std. 929-2000 and IEEE Std. 1547-2003 [5, 6]. Table 2.1 lists the current harmonic restriction, including the limits on individual harmonics and the limit on the total harmonics distortion (THD) of the injected grid current. If the specifications of the grid-connected inverter is given, the spectrum of $v_{inv}$ can be calculated from (2.4) or (2.7), from which the angular frequency $\omega_h$ and amplitude $|v_{inv}(j\omega_h)|$ of the dominant harmonics can be obtained. Substituting the obtained $\omega_h$ and $|v_{inv}(j\omega_h)|$ into (2.59), yields...
\[
\frac{|i_2(j\omega_h)|}{|v_{\text{inv}}(j\omega_h)|} = \frac{1}{L_1 L_2 C(j\omega_h)^3 + j\omega_h(L_1 + L_2)}
\] (2.60)

According to the spectrum of the inverter bridge output voltage \(v_{\text{inv}}\), the angular frequency \(\omega_h\) and harmonic order \(h\) of the dominant harmonics can be determined. Then, according to (2.60), Table 2.1, and the expected harmonics proportion \(\lambda_h\), the minimum value of \(L_2\) can be obtained, which is

\[
L_2 = \frac{1}{L_1 C \omega_h^2 - 1} \cdot \left( L_1 + \frac{|V_{\text{inv}}(j\omega_h)|}{\omega_h \lambda_h I_2} \right)
\] (2.61)

where \(V_{\text{inv}}(j\omega_h)\) and \(I_2\) are the RMS value of the inverter bridge output voltage and the rated injected grid current, respectively. If three-phase grid-connected inverter is used, \(V_{\text{inv}}(j\omega_h)\) in (2.60) and (2.61) is replaced by \(V_{aN}(j\omega_h)\).

After \(L_1, C\) and \(L_2\) are determined, the simulation or experimental validations is conducted to check whether the individual harmonics and the THD of the grid current satisfy the restriction shown in Table 2.1 or not.

### 2.4 Design Examples for LCL Filter

To validate the above design methods, two prototypes are designed, where single-phase full-bridge grid-connected inverter is controlled by the unipolar SPWM, and three-phase grid-connected inverter is controlled by the harmonic injection SPWM. The specifications of the single-phase full-bridge grid-connected inverter are as follows: the dc input voltage is 360 V, the rated power is 6 kW, the carrier frequency is 10 kHz, and the grid voltage is 220 V/50 Hz. The specifications of the three-phase grid-connected inverter are as follows: the dc input voltage is 700 V, the rated power is 20 kW, the carrier frequency is 10 kHz, and the grid voltage is 380 V/50 Hz.
### 2.4.1 Single-Phase LCL Filter

Setting the inductor current ripple coefficient \( \lambda_{c-L1} \) to 30%, and substituting the corresponding parameters into (2.39), the minimum value of \( L_1 \) is calculated as 550 \( \mu \)H. Defining the ratio of the RMS value of the fundamental voltage of \( L_1 \) to that of the capacitor voltage as \( \lambda_{v-L1} \), and assuming \( \lambda_{v-L1} = 5\% \), the maximum value of \( L_1 \) is calculated from (2.36), which is 1.28 mH. Finally, \( L_1 = 600 \) \( \mu \)H is chosen.

Setting \( \lambda_C = 3\% \) and substituting \( P_o = 6 \) kW, \( V_g = 220 \) V, and \( f_o = 50 \) Hz into (2.58), yields \( C < 12 \) \( \mu \)F. Here, \( C = 10 \) \( \mu \)F is chosen.

Assuming that the output power factor (PF) of the grid-connected inverter equals 1, the fundamental RMS value of \( i_{L1} \) could be calculated, i.e.,

\[
I_1 = \sqrt{I_C^2 + I_2^2} = \sqrt{(\omega_0 C \cdot V_g)^2 + I_2^2} = 27.28 \text{ (A)}.
\]

According to the dc input voltage and the magnitude of grid voltage, the modulation ratio can be obtained, which is \( M_r = 311/360 = 0.86 \). By substituting \( M_r = 0.86 \) into (2.7), the spectrum of the bridge output voltage \( v_{inv} \) can be depicted, as shown in Fig. 2.13. As seen, the dominant harmonics locate at \( f_h = 19.95 \text{ kHz} \) and 20.05 kHz, and the corresponding \( |V_{inv}(j2\pi f_h)/V_{in}| = 28\% \). As long as these dominant harmonics in \( i_2 \) are attenuated to satisfy the aforementioned standards, the other harmonics in \( i_2 \) will naturally satisfy the standards. Since the orders of the dominant harmonics are higher than 33, the required current harmonic proportion \( \lambda_h \) should be less than 0.3%. Setting \( \lambda_h = 0.2\% \), and substituting \( I_1 = 27.28 \) A, \( |V_{inv}(j2\pi f_h)/V_{in}| = 28\% \), \( V_{in} = 360 \) V, \( L_1 = 600 \) \( \mu \)H, \( C = 10 \) \( \mu \)F, \( \omega_h = 2\pi \times 19,950 \), and \( \lambda_h = 0.2\% \) into (2.61) leads to \( L_2 = 164 \) \( \mu \)H. Finally, \( L_2 = 150 \) \( \mu \)H is selected. The final single-phase LCL filter parameters are listed in Table 2.2.

Figure 2.14 shows the simulation results. In Fig. 2.14a, the waveforms from top to bottom are the inverter-side inductor current \( i_1 \), the grid current \( i_2 \), the capacitor current \( i_C \) and its fundamental component, respectively. In Fig. 2.14b, the waveforms from top to bottom are the inverter bridge output voltage \( v_{inv} \), the spectrums

![Fig. 2.13 Calculated spectrum of \( v_{inv} \) in single-phase LCL-type grid-connected inverter with unipolar SPWM](image-url)
of \(v_{\text{inv}}\) and \(i_2\), respectively. The maximum current ripple of \(i_1\) is 7.73 A, and the RMS value of \(i_1\) is 27.28 A. As a result, \(\Delta i_{1,\text{max}}/I_1 = 28\%\). As seen from Fig. 2.14b, the maximum harmonic magnitude of \(v_{\text{inv}}\) is 100 V, and it appears at 19.95 kHz. The magnitude of the harmonic is 27.8\% of \(V_{\text{in}}\), which is agreement with the calculated results shown in Fig. 2.13. Through the LCL filter, the magnitude of the current harmonic in \(i_2\) at 19.95 kHz is suppressed below 0.06 A, which is 0.15\% of the rated injected grid current; and the THD of \(i_2\) is 0.8\%. Clearly, both the single harmonic and THD satisfy the restriction standards, which validate the effectiveness of the design procedure for single-phase LCL filter.

### 2.4.2 Three-Phase LCL Filter

According to (2.23), the modulation ratio can be obtained as \(M_r = 220\sqrt{2}/350 = 0.888\). As observed from Fig. 2.12b, the maximum current ripple of the inverter-side inductor appears at \(\omega_{\text{o}}t = 0\). Setting the inductor current ripple coefficient \(\lambda_{c,L1} = 30\%\), and according to Eq. (2.49), the minimum value of
$L_1$ is calculated as 988 $\mu$H. Assuming $\lambda_{v,L1} = 5\%$, the maximum value of $L_1$ is calculated from (2.36), which is 1.16 mH. So, $L_1 = 1$ mH is selected.

Setting $k_C = 5\%$ and substituting $P_o = 20/3$ kW, $V_g = 220$ V, and $f_o = 50$ Hz into (2.58) yields $C < 22$ $\mu$F. Here, $C = 20$ $\mu$F is selected.

Assuming $PF = 1$, the fundamental RMS value of $i_{L1}$ could be calculated as $I_1 = 30.31$ A. Substituting $M_r = 0.888$ into (2.26), the spectrum of the output phase voltage $v_{aN}$ is depicted, as shown in Fig. 2.15. As seen, the dominant harmonics locate at $f_h = 9.9$ kHz and 10.1 kHz, where $|V_{aN}(j2\pi f_h)/V_{in}| = 17.6\%$. Likewise, as long as these dominant harmonics in $i_2$ are attenuated to satisfy the aforementioned standards, the other harmonics in $i_2$ will naturally satisfy the standards. Since the orders of these dominant harmonics are higher than 33, so the required $\lambda_h$ should be less than 0.3%. Here, setting $\lambda_h = 0.15\%$, and substituting $I_1 = 30.31$ A, $|V_{inv}(j2\pi f_h)/V_{in}| = 17.6\%$, $V_{in} = 360$ V, $L_1 = 1$ mH, $C = 20$ $\mu$F, $\omega_h = 2\pi \times 9900$ and $\lambda_h = 0.15\%$ into (2.61), produces $L_2 = 301$ $\mu$H. Finally, $L_2 = 300$ $\mu$H is selected. The final three-phase $LCL$ filter parameters are listed in Table 2.3.

Figure 2.16 shows the simulation results with the prototype parameters of Table 2.3. In Fig. 2.16a, the waveforms from top to bottom are the inverter-side inductor current $i_{1a}$, the injected grid current $i_{2a}$, the capacitor current $i_{Ca}$ and its fundamental component, respectively. In Fig. 2.16b, the waveforms from top to bottom are the output phase voltage $v_{aN}$, the spectrums of $v_{aN}$ and $i_{2a}$, respectively. The maximum current ripple of $i_{1a}$ is 9.5 A, and the RMS value of $i_1$ is 30.31 A. As a result, $\Delta i_{1,max}/I_1 = 31.4\%$. As seen from Fig. 2.16b, the maximum harmonic magnitude in $v_{aN}$ appears at 9.9 kHz and it is about 60 V, which is 17.1% of $V_{in}/2$ and in agreement with the calculated results shown in Fig. 2.15. Through the $LCL$

![Fig. 2.15 Calculated spectrum of $v_{aN}$ when harmonic injection SPWM is used](image)

| Table 2.3 Parameters for three-phase $LCL$-type full-bridge grid-connected inverter |
|---|---|---|---|---|---|
| Parameter          | Symbol | Value | Parameter          | Symbol | Value |
| Input voltage      | $V_{in}$ | 700 V | Switching frequency | $f_{sw}$ | 10 kHz |
| Grid voltage       | $V_{gab}$ | 380 V | Inverter-side inductor | $L_1$ | 1 mH |
| Output power       | $P_o$ | 20 kW | Filter capacitor     | $C$ | 20 $\mu$F |
| Fundamental wave frequency | $f_o$ | 50 Hz | Grid-side inductor   | $L_2$ | 300 $\mu$H |
filter, the current harmonic magnitude of \( i_2 \) at 9.9 kHz is suppressed below 0.05 A, which accounts for 0.13% of the rated injected grid current. The THD of \( i_2 \) is 0.8%. Both the single harmonics and THD satisfy the restriction standards, which validate the design procedure for three-phase LCL filter.

### 2.5 Summary

In this chapter, the design procedure of LCL filter is presented. The Fourier series expansions of the inverter bridge output voltage of single- and three-phase LCL-type grid-connected inverter with different PWM schemes are derived for the purpose of determining the dominant harmonics which needs to be suppressed. The harmonic spectrum shows that for single-phase inverter, the dominant harmonics with the bipolar SPWM distribute around the carrier frequency, whereas those with the unipolar SPWM distribute around twice the carrier frequency. For the three-phase inverter, the dominant harmonics with both the SPWM and the harmonic injection SPWM distribute around the carrier frequency. Considering the permitted current ripple of the inverter-side inductor, the allowable reactive power introduced by the filter capacitor, and the maximum harmonic limit of the grid current, the filter parameters can be determined. The design procedure for the LCL filter is given as follows:

1. By limiting the maximum inductor current ripple in one cycle and the fundamental voltage on the inductors, the lower and upper limits of the inverter-side inductor is obtained, from which, a proper inverter-side inductor can be selected.
According to the maximum reactive power introduced by the filter capacitor, the upper limit of the filter capacitor can be obtained.

By limiting the single harmonic of the grid current in accord with the restriction standards, the minimum value of the grid-side inductor can be determined, from which, the proper grid-side inductor can be selected.

The LCL filter design procedure is verified by simulations.

References

Control Techniques for LCL-Type Grid-Connected Inverters
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