Concrete is a quasi-brittle material and it possesses a low tensile strain capacity. Though it exhibits a good compressive strength and a significant crushing strain value, but the presence of the microcracks in the interfacial transition zone and the mortar matrix reduces its tensile strength virtually to a zero. When the compressive stress in the concrete specimen reaches the threshold limit (about 75% of the concrete compressive strength), the concrete undergoes dilation and exhibits a strain softening after reaching the peak/ultimate strength. However, the brittle characteristics of the concrete can be improved considerably by using and dispersing the short and discrete fibers, uniformly, throughout the concrete mass at the time of mixing. The concrete reinforced internally by means of the short and discrete steel fibers is called as steel fiber reinforced concrete (SFRC). It is an experimentally established fact that the concrete after the addition of steel fibers exhibits a better performance not only under static and quasi-statically applied loads, but also under fatigue, shock, and impulsive loadings (ACI 1993; Robins et al. 2002; Barros et al. 2005; Marco et al. 2009; Chalioris 2013; Singh 2015). Nevertheless, the random presence of the steel fibers, albeit uniformly, throughout the concrete mix and their relatively higher tensile strength in comparison to the hydrated cement paste in the concrete is a main contributor to the strength enhancement exhibited by the SFRC. The Chap. 1 described the microstructure of the concrete along with various factors that affect it and explains how it plays an important role in controlling the stress-strain response of the concrete under a tension and compressive type of loading. The stability of the concrete macrostructure and the consequential improvement in various concrete properties that the inclusion of the fibers in the mix would bring in was also described. The present chapter discusses various models developed to simulate the compression and tensile behavior of the SFRC. It also presents a forward analysis approach to determine the value of the fiber parameters (l/d and V_f) and the concrete grade required to achieve a desired value of the residual-tensile strength value of SFRC.
2.1 SFRC Constitutive Models

A constitutive model is a set of expressions that describe the response of a material when it is stressed, externally or internally. These expressions constitute an essential component of analysis models to predict the member response under different loading conditions. A number of constitutive models have been proposed by different researchers in the recent past. The most prominent among them are those proposed by Mohamed and Victor (1994), RILEM (2002), Barros et al. (2005), Soranakom and Mobasher (2007, 2009), Chalioris (2013). These models are used to develop various design methods for proportioning SFRC beams, but are convenient to use only in cases when limited tensile parameters are known as a prerequisite or at the start of the analysis. Because most of these models rely heavily on the tensile stress distributions determined through the experimental testing of the standard test specimens or some empirical expressions. The design procedures so developed always possess inherent flaws on account of the assumptions taken while developing the empirical relations or testing procedures. Generally, a fixed rectangular stress distribution for the compression block is used in some of the existing models that do not allow the designer to use a particular stress–strain distribution when the failure is reached before the ultimate compressive stress is reached in the concrete. However, the model proposed by Singh (2015) is more systematic and easier to use in the analysis and design of SFRC sections than the previous models. It requires various fiber parameters, such as aspect ratio (l/d), volume fraction (V_f), fiber shape and the characteristics compressive strength of the concrete to determine analytically the residual tensile strength of SFRC unlike previous models that requires input from experimental investigations to start the analysis. The following sections present the details of the compression and the tension constitutive material model of SFRC.

2.1.1 Compressive Model

Concrete being a heterogeneous material is not able to provide a resistance to the tensile forces, although it exhibits a significant strength in compression. Concrete’s inability to provide resistance to tensile forces is mainly due to the existence of a weak interfacial transition zone around coarse aggregates present in the concrete mass. The interfacial transition zone in concrete acts as a bridge between the coarse aggregates and the surrounding hydrated cement paste (mortar matrix) in concrete and is an only medium to transfer the stresses across different phases of the concrete. The presence of voids and other microcracks in the interfacial transition zone do not permit an effective stress transfer across its thickness and, thereby, leads to the reduction in the load carrying capacity of the concrete irrespective of the load type. However, a significant improvement in the post-peak compressive behavior is reported in the published literature when short fibers are randomly mixed in the
concrete mass, although the peak compressive strength of the concrete is marginally increased by the fiber inclusion in the mix (ACI 1993; Mohamed and Victor 1994; Robert et al. 1987; Lini et al. 1998; Marco et al. 2009; Kang 2011; Michels et al. 2013).

The contribution of fibers is most apparent in the post-peak region in the case of the SFRC, where the response is described by a relatively less steep decaying stress-strain response. Once the matrix cracks under induced tension, the debonding and pulling out of the fibers from the concrete dissipates energy, thereby, leading to a substantial increase in the post-cracking characteristics. Figure 2.1 shows a typical stress-strain response exhibited by SFRC specimen in compression when it is reinforced by the steel fibers with different aspect ratio.

Similar to the normal concrete, the SFRC exhibits a linear-elastic behavior up to a stress level of 30% of the ultimate concrete strength in compression. At this stage, the internal microstructure of the concrete remains more or less stable. However, when the stress level in the specimen is increased gradually to 50% of the ultimate concrete compressive strength, the stress-strain response starts deviating from the linearity. The initiation of the nonlinearity in the response of the concrete specimen mainly happens because of the internal extension of the microcracks present in the interfacial transition zone of the concrete to the adjoining regions.

After reaching a stress level of 50% of the ultimate concrete compressive strength, the steel fibers present in the concrete matrix get activated and delays the extension of the microcracks in the interfacial transition zone to the surrounding mortar matrix. This all happens because of the local reinforcing capability of the steel fibers which arrests the crack widening due to their anchorage into the hydrated cement paste adjoining to the coarse aggregates. However, its effect on the

![Fig. 2.1 A typical stress–strain response of SFRC under compressive](image)

---

**Fig. 2.1** A typical stress–strain response of SFRC under compressive—The local reinforcing capability of steel fibers present in the mortar matrix and the web-like structural network formed by these fibers around the surfaces of coarse particles in the mix significantly enhances the post-cracking characteristics of the concrete. The extent of improvement depends upon the fiber parameters considered in the mix viz: the fiber aspect ratio (l/d) and the volume fraction (V_f). Higher the value of the fiber parameters (V_f or l/d) adopted in the design more is the improvement in the post-peak characteristics of the SFRC.
shape of the compression stress-strain response remains insignificant and in the pre-cracking stage, therefore, the stress-strain response exhibited by the plain concrete can be used as such in the design, etc. without affecting the final results.

The effect of the steel fibers become significant and visible on the stress-strain response only after the stress level in the specimen is increased beyond the threshold level of a normal concrete i.e. 70 % of the ultimate concrete compressive strength. At this stage, the tensile strength of the fibers surrounding coarse aggregates in the concrete mobilizes and prevents the extension, and widening of the microcracks existing in the interfacial transition zone and their subsequent progress to the adjoining regions of the hardened concrete. This phenomenon considerably improves the post-cracking behavior of the SFRC.

It is reported that SFRC is able to sustain a compressive strain value of 0.005–0.009 on failure and the corresponding failure stress improves to about 0.95σ_{cu} from a value of 0.85σ_{cu} taken for a normal concrete, where σ_{cu} is concrete cylinder strength. This value of the concrete strength works out to be 0.75 times the cube compressive strength. The strength of concrete is a maximum load attained during the loading process under uniaxial compression conditions and it is determined as per the provisions of IS 516 (1959) wherein; 150 mm standard cubes are tested in the strain-controlled loading after 28 days of the water curing period. The prescribed uniform strain rate for the testing is 0.001 mm/mm per minute. As the concrete exhibits considerable variation in the test results, it must be reported as a characteristic (5-percentile) and mean values; normally, expressed as the concrete characteristics compressive strength (f_{ck}).

The design compressive stress of SFRC, therefore, can be adopted as 0.5f_{ck} by applying a partial safety factor of 1.5 to the stress value of 0.75f_{ck}, where the f_{ck} is the characteristic compressive strength of the concrete. The maximum value of the stress is reached at a strain approximately equal to 0.002 similar to the plain concrete and thereafter; an increase of the strain is accompanied by a drop of the stress. Nevertheless, the rate of drop of the compressive strength of SFRC in the post-peak range is relatively smaller than the plain concrete. Equation 2.1 gives the design compressive stress (f_{c}) of SFRC corresponding to any strain value ε, <0.004. The design value of the strain for SFRC in compression is 0.004.

\[
f_c = \begin{cases} 
0.5f_{ck} \left[2\left(\frac{\varepsilon}{0.002}\right) - \left(\frac{\varepsilon}{0.002}\right)^2\right], & \varepsilon < 0.002 \\
0.5f_{ck}, & 0.002 \leq \varepsilon \leq 0.004
\end{cases}
\]  

(2.1)

2.1.2 Tension Model

Concrete is not normally designed to resist direct tension due to its low tensile strain capacity. However, tensile stresses do develop in concrete members as a result of flexure and/or principle tensile stresses arising in the member from a multi-axial state of stress caused by external loading. For any structural member, therefore, it is
mandatory to possess adequate compressive as well as tensile strength to perform satisfactorily.

The low tensile strength of concrete is due to the presence of a weaker zone of separation—interfacial transition zone—mainly around coarse aggregates at their interface with the surrounding hydrated cement matrix. The microcracks in the interfacial transition zone propagate rapidly under tension and that happens at a much lower stress level in comparison to the case where the load is compressive in nature. Concrete cracks once the strain (longitudinal or lateral strain arising from Poisson’s effect) exceeds its limiting tensile strain capacity. Once that happens, the subsequent concrete failure is brittle in nature and it occurs abruptly within no times. The value of the limiting tensile strain in concrete is reported to range from 0.0001 to 0.0002. The value of the cracking tensile strength for the SFRC can be taken similar to the value \( \frac{0.7 \sqrt{f_{ck}}}{\text{MPa}} \) reported for the plain concrete, because the fibers in the SFRC activate only after the concrete cracks.

Unlike a plain concrete, the SFRC exhibits a significant residual-tensile strength because of the contribution of the fibers embedded in the hardened concrete, although its first cracking strength is almost same as that of a plain concrete. The SFRC member, however, during the loading process undergoes a considerable deformations and cracking before the failure. This all happens because of the fibers present uniformly in the concrete volume, almost in every possible direction that bridge the crack surfaces in the concrete member. But, this improved response exhibited by SFRC is greatly influenced by the amount of the fibers crossing the crack, their pullout behavior and the strength properties. The pullout behavior, which is an indirect measure of the fiber slip and the fiber elongation in the SFRC, is associated with the three identified bond mechanisms: namely, (1) adhesion, (2) friction, and (3) mechanical anchorage. It is an experimentally established fact that the fiber embedment length, the fiber orientation with respect to the loading direction and the matrix strength plays a significant role in defining the pullout response of the fibers embedded in the concrete (Kullaa 1992; Karayannis 2000; RILEM 2002; Gettu et al. 2005; Soranakom and Mobasher 2007, 2009; Hameed 2013; Michels et al. 2013). The fibers, usually, aligned in the direction of the loading are more effective in transferring the tensile force across the crack width than otherwise is the case. Fibers having an inclination of more than 60° contribute as little as 10 % of the case where these fibers are oriented along the loading axis.

This unique behavior exhibited by the SFRC permits an analyst to generate a stress-crack opening \( [\sigma-w] \) relationship and/or a stress-strain \( [\sigma-\varepsilon] \) response for a SFRC member over a wide spectrum of the strain values and/or displacements. An \( \sigma-w \) relationship basically describes the stresses (\( \sigma \)) carried by the steel fibers across a tension crack in the SFRC as a function of the crack-width/opening (\( w \)). This procedure is known as ‘fictitious crack approach’ and was first suggested by Hillerborg et al. (1976). The fiber parameters (\( V_f, l/d \) and the fiber shape) used in concrete mix play a significant role to the shape of an \( \sigma-w \) curve exhibited by SFRC. In addition, the casting process used to pour an SFRC into the moulds also has a considerable effect on orientation of the steel fibers in the member (Toutanji and Bayasi 1998; Swamy and Stavrides 1976; Stähli et al. 2007). In case, the fibers
get aligned along the tensile stress trajectories in the member during the pouring, these are more effective in transferring the stress across the cracked surfaces than otherwise would be the case. This generally happens near the surfaces of mould, where the steel fibers tend to align themselves parallel to the external surface of a mould. Studies on the fresh-state properties of SFRC in the past (Romualdi and Mandel 1964; Stroeven 1979; Soroushian and Lee 1990; Akkaya et al. 2000; Gettu et al. 2005; Dupont and Vandewalle 2005; Stähli et al. 2008) have confirmed this phenomenon, reported it as *wall-effect* in the published literature. Figure 2.2 schematically shows the wall-effect observed in the SFRC during the placement and compaction operations.

Two reasons have been identified for this preferential orientation of the steel fibers in the concrete—(a) the wall-effects that depend on the geometry of the formwork/mould; (b) the flow of concrete, which depends on the rheological properties of the material, the geometry of the formwork and the casting procedure. Because cementitious material flows only if a stress higher than a critical value is applied to ensure the movement of the concrete. The pumping force exerted to ensure the concrete movement tends to align the fibers in the direction of the flow. Even though the wall-effect has a local nature and it should not have a significant influence in the structural response of a big structural element, but the restrictions imposed by the rigid surfaces of the moulds can affect the post-cracking response of smaller specimens generally used in the laboratory testing. For example, Gettu et al. (2005), Torrents et al. (2012) have verified the wall-effect and observed a

![Fig. 2.2 Wall-effect](image-url)
preferential orientation of the steel fibers in the perpendicular plane to the casting direction caused by the vibration of the small beams (150 × 150 × 700 mm).

Therefore, the tensile response of SFRC expressed either in the form of $\sigma$–$w$ (see Fig. 2.3) or $\sigma$–$\varepsilon$ relationship (see Fig. 2.4) must be determined through an experimental testing that must take care of and consider all those factors that may affect the response at the time of their casting. It can be ensured by employing a pouring/casting method in the actual construction that is identical with the technique used while preparing the test specimens (prisms and tensile block). RILEM and other agencies has standardised the methods and procedure that require results of the uniaxial tensile test and the bending tests to determine the values of various key parameters to define a tensile-constitutive relationship of SFRC.

The stress-crack width [$\sigma$–$w$] response of SFRC specimen shown in Fig. 2.3 indicates that once the matrix fails in tension (end of the phase-1), it leads to the formation of a number of cracks in the concrete, though fine in crack-widths, but the fibers present there bridge the cracks and ensure transfer the tensile forces across the crack-width in the specimen. This composite action is responsible for sustenance of the load carrying capacity of SFRC, in the phase-2 of the response curve.

Figure 2.3 also depicts a typical phase-3 of the response exhibited by the SFRC that represents the strength contributed by the fibers alone in tension. Because of the wide cracks formed in this phase, the concrete did not contribute significantly towards the strength. Fibers crossing the crack try to stabilize the member by a

![Fig. 2.3 SFRC stress-crack width (σ–w) response in tension](image)

The local reinforcing capability of steel fibers present in the mortar matrix depends entirely on their pullout response. The uncracked response shown in the phase-1 in the figure is always elastic-linear in nature. The fibers in hardened concrete get activated only after the formation of the cracks in the member (phase-2). Once that happens, the concrete residual strength in this phase is coming from the composite action that exists between the concrete and the fibers. The crack-mouth-opening-displacement (CMOD) continuously increases with almost at a constant stress level exhibited by the SFRC member. The crack width is usually very fine in the phase-2 that increases at a rapid rate at the end of the phase, followed by a further increase in the crack widths, but that is generally accompanied by a stress reduction phenomenon. Usually, the crack width of already developed cracks in the specimen increases in the phase-2 with a formation of new ones. The phase-3 in the figure shows the contribution of the fibers alone that are anchored into the hydrated cement matrix in the concrete, in the load sharing at the end of the phase-2. The cracks then become wide enough to cause a drop in the strength exhibited by SFRC.
variety of bond mechanisms. During this process, a number of new cracks also form in the concrete and those already developed in the SFRC grow continuously and widen under the increasing load.

During the third phase, however, the SFRC exhibits a reduction in the load carrying capacity. Generally, the fibers in the concrete start pulling-out from the matrix and/or fracturing depending upon the fiber-type, their aspect-ratio and the volume-fraction used in the mix. It eventually leads to the failure of the member once the applied load exceeds the tensile strength contributed by the fibers. It can occur in the fiber-pullout mode or fiber-fracture mode depending upon the fiber aspect ratio. The failure in the fiber pullout mode is always ductile in nature; whereas it occurs abruptly in case of a fiber fracture mode. It must therefore be avoided to occur by an appropriate selection of the fiber aspect ratio in the concrete mix.

The Fig. 2.4 shows a typical stress-strain (σ–ε) response exhibited by SFRC in tension. Like the stress crack-width response [σ–w], it also consists of three parts, denoting a contribution from the concrete alone (phase-1), initiation of a composite action; wherein, the mortar matrix and the steel fibers embedded in it are providing the requisite resistance (phase-2) and followed by the contribution from the steel fibres alone (phase-3).

In the phase-1, the tensile stress-strain (σ–ε) response exhibited by SFRC is linear-elastic in nature and generally, no cracking occurs in the specimen. The geometry of fibers usually has no influence on the load-carrying capacity at this stage. The fibers virtually remain silent till the formation of the first crack in the specimen, which normally formed at a strain value of 0.00014. The maximum value of the tensile stress in the specimen therefore is only related to the strength exhibited by the concrete. The Hooks law governs the stress-strain response of SFRC in this phase. The strain value corresponding to the strength exhibited by the material can be determined from the concrete strength and its elastic modulus. The end of the phase-1 signifies the onset of cracking and activation of the fibers in the SFRC.
Cracking initiates in the concrete, in the phase-2, which results in a sharp drop in the stress value. It is generally accompanied by the first major deviation from the linearity of the stress-strain response exhibited by SFRC member. As the crack-width is usually fine in this phase, a composite action still exists between the steel fibers and the concrete. However, the composite action quickly vanishes as the new cracks form in the section and those already existing there have started widening under the increasing strain. By the end of phase-2, the cracks are well established and wide enough (0.1–0.2 mm) to cause a complete loss of the contribution from the concrete part. Most of the fibers in the SFRC have exceeded their peak loads at this stage and they begin to slip at more or less constant loads.

Phase-3 starts with the formation of a system of well-defined cracks in the concrete. The fibers resisting the openings, caused by the crack formation, are doing so primarily through the fiber-pullout. Naturally, not all fibers that cross the crack contribute towards the strength, since many of them are not sufficiently anchored in the adjoining mortar mix. This mainly happens due to the random orientation of the fibers in the concrete. It is also expected that the residual-tensile strength exhibited by SFRC at this stage should theoretically remain constant if all the fibers that cross the crack surface were aligned in the load direction. But as a substantial number of fibers are inclined due to their random presence in the mix, they will not contribute in a similar way as the fibers would do, which are aligned in the direction of loading. With the increasing crack opening, however, the contribution from the inclined fiber will also increase as they straighten themselves under the tensile stress in the member. This process results in the drop of the effective stress across the crack during the middle stage of phase-3. Eventually, with increasing crack opening, most fibers will pull out with no residual force. This usually happens after a strain of 0.040.

A typical stress-crack opening relationship and stress-strain response exhibited by SFRC specimen is shown in Figs. 2.3 and 2.4, respectively, can be simplified for the analysis and design purpose of SFRC members. Three types of simplified relationships vis-à-vis Multilinear, Bilinear and Drop-constant relationship were suggested by different researchers in the past. These simplified versions of the relationship, help to simplify the analysis and obtain a satisfactory solution. Any one out of these simplified models can be selected depending upon the value of the limiting-strain adopted in the design.

2.1.2.1 Multi-linear Relationship

The experimental stress-strain ($\sigma$–$\varepsilon$) response of SFRC generally consists of first a descending part after the first crack forms in the SFRC, then a slow descending and finally, a descending that occurs at a rapid pace. Nevertheless, the response of the SFRC in the pre-cracking range is always elastic-linear in nature. Figure 2.5 shows a typical multilinear constitutive tension model for an SFRC; a simplified representation of actual stress-strain response shown in Fig. 2.4. In this model, a very realistic representation of the stress-crack opening relationships can be obtained and
it is best suited in cases where an study demands an analysis over a large range of strain values, lasting up to a value of 0.040, and for this to happen, the SFRC must contain fibers with a volume fraction of more than 1.25 % and having an aspect ratio sufficiently large to ensure a ductile behavior. The strain value corresponding to the point-B, shown in the Fig. 2.5, depends entirely on the fiber parameters used in the concrete mix and it must be evaluated from the experimental studies conducted on SFRC for a given set of the fiber parameters. The crack-width corresponding to the range shown from the point-C to the point-D in the Fig. 2.5 becomes large enough to cause the fiber pullout from the mortar matrix and it results in a rapid decline in the effective stress transfer across the cracks. Usually in the range C–D, very wide cracks appeared in the specimen.

2.1.2.2 Bilinear Relationship

A bilinear model is a further simplified version of the multilinear constitutive relationship given in the previous section. In this model, the range from the point-C to the point-D, shown in Fig. 2.5 that usually results in a rapid loss of the strength after the formation of wide cracks in the specimen, is ignored in the analysis. This model provides a reasonably good representation of the measured SFRC behavior, when the crack-width is a prime limit-state of the adopted design procedure. A typical bilinear model for SFRC is shown in Fig. 2.6. In this model, a total of four material parameters are required to describe the stress-strain response and the corresponding stress-crack opening relationship of the material. The value of the mobilized stress and the strain value corresponding to the point-B are determined through an experimental testing; whereas, the values corresponding to the point-C are usually dictated by the limit-state criterion adopted in the design with respect to the crack-width at ultimate state. The stress at this stage (point-C) denotes the design value of the residual-tensile strength of SFRC corresponding to a permitted crack opening. Similar to the model described in the previous section, the stress

![Fig. 2.5 SFRC multi-linear model](image-url)
value (at the point C) can be determined either by conducting the standard tests on the SFRC specimens or it can be determined from the material analytical models. One such model is described in the next section that considers the randomness of the steel fibers in the concrete and requires concrete compressive strength \( f_{ck} \), fiber aspect ratio \( l/d \) and fiber volume fraction \( V_f \) as an input parameter for estimation of the residual tensile strength of SFRC.

### 2.1.2.3 Drop-Constant Relationship

An even more simple representation of the stress-strain relationship for SFRC which has obvious advantages from a design point of view is shown in Fig. 2.7. It
is called as drop-constant relationship. Unlike the bilinear model, it needs only the value of the residual tensile strength of the SFRC corresponding to the point B or C shown in the model, which are virtually same on account of the assumption, along with the corresponding values of the strain. The strain value at the point-C in the model can be taken from the permitted value of the crack-width considered in the design. The strain value corresponding to the point A or B shown in the model is the limiting concrete strain that causes the formation of the first crack in the specimen when subjected to a tensile force.

It is important to note that the ultimate flexural capacity of an SFRC member remains practically unaffected, if instead of multilinear or bilinear model, a drop-constant model is adopted in the analysis. This is because of the fact that before the strain in the concrete section reaches the limiting concrete tensile strain especially during the first-time loading; a small part of the concrete existing below and close to the section neutral axis remains uncracked and effective in the flexural member. The magnitude of the resulting tensile force contributed by the small uncracked portion of the concrete and the corresponding internal moment is negligibly small and it can be safely neglected in the design calculations without affecting the accuracy of the results. In case, the loading is done on a previously loaded beam, it is possible that prior overloading may have caused the tensile cracks to penetrate deep enough to effectively eliminate the little contribution coming from the previously uncracked concrete portion of the section. Hence, the assumption to consider a constant value of the residual design tensile strength after the formation of the first crack is satisfactory and appears to be realistic. It also helps, simultaneously, to simplify the calculations and final expressions considerably. This is the reason that design engineers usually prefer material constitutive models that results in conservative estimation of the member capacity in a short time.

### 2.2 Analytical Tensile-Constitutive Model Based upon Drop-Constant Relationship

Consider SFRC containing short and discrete steel fibers dispersed randomly in the concrete mix at the time of mixing. It is assumed that these are present uniformly throughout the concrete mix. Figure 2.8a depicts one such steel fiber embedded in the concrete. It is in equilibrium under the induced pullout force (F). The length and diameter of the fiber is \( l \) and \( d \), respectively; thereby giving a fiber aspect ratio of \( l/d \). The ultimate tensile capacity of an SFRC can be estimated by multiplying the number of fibers (\( n \)) crossing the unit area of the crack and the average bond resistance (\( \tau_b \)) being mobilized on the surface of the fiber.

Volume-fraction (\( V_f \)) is used to quantify the number of fibers in the SFRC and it is normally expressed as a percentage of the concrete volume. This parameter is used to prescribe dosages of the steel fibers to be mixed in the concrete to get a desired performance, both in the fresh as well as the hardened state. Equation 2.2
gives an equivalent number of fibers (n) crossing a unit area of the crack, where \( d \) is the diameter of the fiber.

\[
n = \frac{4V_f}{\pi d^2} \quad (2.2)
\]

Figure 2.8b shows a group of fibers crossing a vertical flexural crack. The fibers are assumed to be aligned perpendicular to a typical flexural crack and are contained within a quadrilateral domain, which represents all the possible fiber arrangements across the crack. Out of all such fibers in the domain, the fiber-1 lying at mid height of the quadrilateral domain with a pull-out length of \( l/2 \) will be most effective in transmitting tensile force across the crack; whereas, the fibers lying above or below this height will have a higher probability of being pulled-out from the concrete as they have an inadequate value of embedment length, which varies from zero (shown at the top/bottom face, e.g. Fiber-2) and is equal to \( l/2 \) for the fibers laying at the mid height of the domain (e.g., Fiber-1).

The force \((F_{tp})\) required to pull-out a fiber (e.g. like a Fiber-1), with an average value of the embedment length of \( l/2 \) from the concrete can be estimated from Eq. 2.3. In this equation, the average mobilized interfacial shear stress \((\tau_b)\), in MPa, at the surface of a plain straight fiber can be taken from a lower bound empirical equation: \(0.75\sqrt{f_{ck}}\), where \( f_{ck} \) is concrete characteristic compressive strength, taken in MPa. The shape of the fiber has a significant effect on this strength value (Šalna and Marčiukaitis 2003; Shaurya 2015) and, accordingly, it should be multiplied by a suitable factor depending upon the fiber shape.

\[
F_{tp} = \tau_b \pi d \frac{l}{2} \quad (2.3)
\]

The total pull-out tensile strength \((\sigma_{tp})\) contributed by all such fibers in the direction of the load and those crossing the unit crack area can be determined by...
multiplying Eqs. 2.2 and 2.3. The final expression of the residual tensile strength mobilized in the fiber pullout failure mode is given in Eq. 2.4.

$$
\sigma_{tp} = \frac{F_{tp}}{l} = 2\gamma_o \gamma_d \gamma_l \left( \tau_b V_f \frac{l}{d} \right)
$$

(2.4)

In Eq. 2.4, the constants $\gamma_o$, $\gamma_d$ and $\gamma_l$ denote the effect of fiber orientation with respect to the loading axis; fiber dispersion along the length and the width of the flexural member caused during the concrete compaction and placement operations; length factor $\gamma_l$ takes into account the nonlinear distribution of the bond stress along the length of the fiber, respectively. It is an experimentally established fact that the effect of the fiber orientation on the pull-out strength is insignificant, if the fibers are aligned at an orientation-angle up to 30° with respect to the loading. A reduction as high as 60% in the strength was observed; when the orientation of the fibers is kept more than 40° and up to 60° for various fiber lengths. The fibers with an orientation angle of more than 60° will contribute as little as 10% (Robert et al. 1987; Gettu et al. 2005; Kang 2011; Soranakom and Mobasher 2007, 2009; Michels et al. 2013). The fiber pull-out behaviour becomes increasingly more influenced by the mechanical properties of the steel fibers embedded in concrete than the strength of the matrix as the fiber orientation angle increases with respect to the loading axis.

The weighted average value of $\gamma_o$ was found to be 0.8 based upon the strength reduction reported in the past investigations. Probability analysis indicates that the value of $\gamma_d$ can be taken as 0.5 because the fibers have equal probability to align themselves along the two plan dimensions of the member during the placement and compaction of the concrete mix. The value of the length factor $\gamma_l$ can be taken as 0.5 for the fiber length $l$ less than its critical length $l_c$; otherwise this value is $(1 - 0.5 l/l_c)$ (Karayannis 2000). Therefore, the total pullout tensile strength ($\sigma_{tp}$) of plain straight fibers crossing the unit area of a crack of in the SFRC member can be simplified to Eq. 2.5 and it must be multiplied by a suitable fiber-shape factor depending upon the fiber types. The shape factor for the hooked-end fibers and the wavy-fibers are reported as two and three, respectively (Malik 2014; Shaurya 2015; Nadda 2015).

$$
\sigma_{tp} = 0.3V_f \frac{l}{d} \sqrt{f_{ck}}
$$

(2.5)

The second failure mode in which an SFRC specimen could fail is a fiber-fracturing mode; wherein the fibers embedded in concrete get fractured at collapse. This type of failure usually occurs when fibers are long enough to shift the failure from a pullout mode to the fracture mode. In such cases, the pullout resistance of the fibers become more than their yield strength because of their longer embedment length, thereby resulting in a high bond strength for a given fiber diameter. The residual-tensile strength ($\sigma_{tf}$) for such cases can be estimated by multiplying the design yield strength ($f_y$) of the steel fiber (diameter, $d$) with their total cross-section area crossing the crack. Equation 2.6 gives a force over the unit crack area ($\sigma_{tf}$) required to cause yielding of fibers embedded in the concrete and
their subsequent fracturing that happens suddenly soon after the fiber yielding; the steel fibers possess a low percentage elongation. A partial safety factor of 1.15 is used in Eq. 2.6 to take care of the variation of strength properties of material from their nominal values.

$$\sigma_{\text{fr}} = 0.87f_y \left( n\pi \frac{d^2}{4} \right) = 0.87f_y V_f$$

(2.6)

The length of the fibers embedded in concrete at which the bond stress mobilized on the surface of steel fibers becomes equal to their yield strength is called as critical length. The value of the fiber critical aspect-ratio \((l/d)_c\) can be determined by equating Eqs. 2.5 and 2.6. If the actual fiber aspect-ratio \((l/d)\) in an SFRC is kept less than its critical value \((l/d)_c\), the failure always occurs in the fiber pullout mode; otherwise, it will happen in a fiber-fracture mode. Equation 2.7 gives a simplified expression for the critical fiber aspect ratio. The value given in Eq. 2.6 should be multiplied with a factor of 0.2 \((=\eta_o \eta_d \eta_i)\) to consider the effect of the fiber orientation and its distribution in the SFRC before it is equated with Eq. 2.5; it is given below:

$$0.3 \sqrt{f_{\text{ck}}} V_f \frac{l}{d} = 0.2 \times 0.87f_y V_f$$

$$\Rightarrow \left( \frac{l}{d} \right)_c = \frac{0.174f_y}{0.3\sqrt{f_{\text{ck}}}} = \frac{0.58f_y}{\sqrt{f_{\text{ck}}}}$$

(2.7)

The design tensile strength \(\sigma_t\) of a concrete with randomly distributed short and discrete plain-straight steel fibers can be taken from the Eq. 2.8, corresponding to any strain value \(\varepsilon < \varepsilon_t\) (=limiting value of the tensile strain considered in the design of SFRC member based upon the permitted crack-width). This value can be adopted from an experimental stress-crack opening relationship of SFRC. In the absence of such data, however, this value can be taken as 0.015. The failure of SFRC can take place by pullout of the fibers or by fiber fracturing depending on whether the fiber aspect ratio is kept more than the critical value given in Eq. 2.7 or not. The value of the corresponding design residual-tensile strength \(\sigma_{\text{tr}}\) of SFRC should be taken as a least of \(\sigma_{tp}\) and \(\sigma_{tf}\), respectively, as is given in Eq. 2.8.

$$\sigma_t = \begin{cases} 5000\varepsilon \sqrt{f_{\text{ck}}}, & \varepsilon \leq 0.00014 \\ 0.3 \sqrt{f_{\text{ck}}} V_f \frac{l}{d}, & 0.00014 < \varepsilon \leq \varepsilon_t (=0.015) \end{cases} \quad \text{For } \frac{l}{d} \leq \left( \frac{l}{d} \right)_c$$

$$\sigma_t = \begin{cases} 5000\varepsilon \sqrt{f_{\text{ck}}}, & \varepsilon \leq 0.00014 \\ 0.87f_y V_f, & 0.00014 < \varepsilon \leq \varepsilon_t (=0.015) \end{cases} \quad \text{For } \frac{l}{d} \geq \left( \frac{l}{d} \right)_c$$

(2.8)

Figure 2.9 shows a normalised tension-constitutive material model for SFRC. It is based upon the constitutive model, given in Eq. 2.8 and considers both types of the material failures depending upon the value of the fiber aspect ratio taken in the
production of SFRC. It is important to note that the fiber parameters \( (l/d \text{ and } V_f) \) play an important role in fixing the failure mode of SFRC. It can fail by exhibiting either a strain-hardening response or a strain-softening response in its post-cracking range.

The Fig. 2.9, and the constitutive model given in Eq. 2.8, indicate that the contribution from the steel fibers activates in an SFRC member only after the tensile strain in the concrete reaches its limiting value and afterwards, the stress drops or rises suddenly to a new value depending upon the fiber parameters: namely, the fiber aspect ratio \( (l/d) \), volume fraction \( (V_f) \), their shape and the concrete compressive strength \( (f_{ck}) \) adopted in the design of an SFRC member. The strength exhibited by SFRC in its post-cracking range at some prescribed value of the crack opening \( (w) \) is termed as a residual-tensile strength \( (\sigma_t) \). Normally, the value of the permitted crack opening \( (w) \) is decided based upon the results of the standard bending tests. The next section presents details of such procedures recommended by various design guidelines. If this value of the strength in its post-cracking stage is reported to be more than the first crack strength, the SFRC is termed as strain-hardening material; otherwise, it will act as a strain-softening material. Equations 2.9 and 2.10 give expressions for estimating the values of the fiber-parameters necessary to ensure a SFRC post-cracking response in the strain-softening or the strain-hardening mode for a given set of \( f_y \) and \( f_{ck} \) values considered in the design.

The values of the fiber-parameters \( (V_f \text{ and } l/d) \) that define the transition of the post-cracking behavior of SFRC from a ‘strain-softening’ to the ‘strain-hardening’
can be determined by equating the strength values given by Eq. 2.8 with the cracking tensile strength \( \sigma_{cr} \). It is given in Eq. 2.9 for an SFRC prepared using short, plain fibers, but it must be divided by the corresponding fiber-shape factor of two and three for the hooked-end and the wavy fibers, respectively. If the value of the volume fraction \( V_f \) or the fiber aspect ratio \( l/d \) to be adopted in the concrete mix is kept less than the value given in the Eq. 2.9, the SFRC will exhibit a strain-softening in its post-cracking range; otherwise, it will lead to the strain hardening.

\[
0.3 \sqrt{f_{ck}} V_f \frac{l}{d} = 0.7 \sqrt{f_{ck}} \quad \text{For} \quad \frac{l}{d} < \left( \frac{1}{d} \right)_c \\
\Rightarrow V_f \left( \frac{l}{d} \right) = 2.33
\]

Equations 2.7 and 2.9 can be combined to get a value of the volume fraction \( V_f \) that would lead to strain-hardening response of SFRC. If the fiber aspect-ratio \( l/d \) in the concrete mix is kept less than its critical value (as given in Eq. 2.7), a value of the volume fraction \( V_f \) taken more than the value, given in Eq. 2.10, always give a strain-hardening response in the material post cracking range.

\[
V_f \geq 4.0 \frac{\sqrt{f_{ck}}}{f_y}
\]

Equations 2.9 and 2.10 indicates that if the fiber aspect ratio \( l/d \) in the SFRC is kept less than its critical value \( l/d \); then, the selected value of the fiber aspect ratio \( l/d \) and the volume fraction \( V_f \) used in the concrete mix only controls the post-cracking behaviour of the SFRC. Otherwise, the material properties, such as the characteristic compressive strength \( f_{ck} \) of concrete and the yield strength \( f_y \) of the steel fibers plays their part in controlling the residual tensile strength \( \sigma_t \) and the post cracking response of SFRC. Nevertheless, the SFRC in the latter case will exhibit a brittle failure with the specimen failing in the fiber-fracturing mode and it must be avoided in practice as the impending failure is silent in nature and occurs suddenly without giving any warning and time to escape.

### 2.3 Experimental Characterization of the Tensile Response

Most of the design guidelines use bending tests to characterize the post-cracking response of the SFRC and in determining the other key parameters needed to define its constitutive-tensile model. Table 2.1 gives a brief of different loading-setups and the specimen sizes recommended by several guidelines in this regards.

The uniaxial test is the most direct method to obtain a stress-crack opening \([\sigma-w]\) relationship of the SFRC, particularly for deciding the value of the safe stress
to be considered in the design of SFRC members at the service load and to check the adequacy of the section at ultimate load. It is an accepted practice to obtain the characteristic and the design tensile strengths, independently, through analytical models, code prescriptions or the standard testing methods intended for a plain concrete.

The bending tests are accepted procedure to develop a stress-strain ($\sigma$–$\varepsilon$) relationship of the material from a load-displacement response exhibited by the standard SFRC flexural member. This process is known as inverse analysis [Tlemat et al. (2006)]. In contrast to the direct tension tests, this process is easier to use as they require measured load-displacement or moment-curvature responses obtainable with minimal testing complexities than those associated with the procedures requiring results from the direct tensile tests. The four-point or three-point bending tests are prescribed by most of the guidelines to obtain a load-displacement response of the specimens.

Idealised tensile stress-strain constitutive relationships are suggested for the SFRC by most of the guidelines that uses the results from a deformation-controlled beam-bending tests, conducted on a standard specimens to determine the peak and post-cracking stresses. In the suggested procedures, generally, the strains corresponding to the stresses were empirically estimated as fixed values. Nevertheless, this type of the procedure is not free from the inherent flaws; the accuracy of the tensile stress-strain response depends greatly upon the precision with which an analyst determines the load at the initiation of the crack on a measured load-deflection response in addition to the assumptions made for the calculation of the post-cracking strength. Moreover, the results from these bending tests are highly dependable on the beam size, the casting method and the loading-setup. It is important to note that because of so many factors influencing the flexural response, the results from these tests shows a large scatter. This may be attributed to the

<table>
<thead>
<tr>
<th>Test</th>
<th>Specimen size (mm)</th>
<th>Loading-setup</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-point bending test</td>
<td>$600 \times 150 \times 150$</td>
<td>RILEM TC 162-TDF (2002) and EN 14651:2005</td>
<td></td>
</tr>
<tr>
<td>4-point bending test</td>
<td>$600 \times 150 \times 150$</td>
<td>CNR-DT 204 (2006)</td>
<td></td>
</tr>
<tr>
<td>Uniaxial tensile test</td>
<td>$150 \Phi \times 150$</td>
<td>RILEM TC 162-TDF (2002)</td>
<td></td>
</tr>
</tbody>
</table>
amount of fibers and their distribution in the cracked section and to minimize it, the use of a three-point bending test conducted on a standard specimen with a central notch is recommended by most of the standard testing guidelines. The central notch at the point of the maximum bending moment is recommended to fix the position of the failure plane in an SFRC Beam specimen. The following sections describe in detail different test procedures prescribed by various guidelines.

### 2.3.1 RILEM TC162-TDF (2002) Procedure

This guideline prescribes a three-point bending test conducted using a simply supported notched-beam to obtain various key-parameters of the SFRC tensile stress-strain relationships. The beam specimen of $150 \times 150 \times 600$ mm (an effective span of 500 mm) with a central notch is recommended for this purpose. The unnotched depth of the beam specimen should be kept as $125 \text{mm} \pm 1 \text{mm}$ with a 5 mm wide or less notch at its mid span. The dimensions of the specimen shall not vary by more than 2 mm on all sides. It is also recommended that the difference in the overall dimensions on opposite sides of the specimen shall not be greater than 3 mm. The device for measuring the specimen dimensions should have an accuracy of 0.1 mm.

The guidelines did not recommend the use of the prescribed beam size for a beam having steel fibers longer than 60 mm and made using aggregates larger than 32 mm. It is required that the concrete should be compacted by means of external vibration only; however, in the case of a self-compacting steel fiber concrete, the mould shall be filled in a single pour and levelled off without any compaction. The concrete specimens shall be cured in the moulds for 24 h after the casting, at $(27 \pm 2) ^{\circ} C$, either under polyethylene sheeting or at not less than 95 % relative humidity. The specimens are then demoulded between 24 and 48 h after the casting and cured for a further period until preparation for testing. Testing shall normally be performed at 28 days (or at any other specified age of the specimen).

The steel rollers with a diameter of $30 \text{mm} \pm 1 \text{mm}$ shall be used to support the beam at its ends and also, at the load-point on its top face. These shall be capable of rotating freely around their axis. The device used for measuring the load and the mid-span deflection, and crack-mouth-opening displacement (CMOD) in the test setup must have a least count of 0.1 kN and 0.01 mm, respectively. The notch CMOD measuring system should be installed along the longitudinal axis at the mid-width of the test specimen, so that the distance between the bottom face of the specimen and the axis of the measuring system is 5 mm or less.

The loading should be applied so that the measured midspan-deflection of the specimen increases at a constant rate of 0.2 mm/min until the specified final deflection is reached that normally is taken as 3 mm. The value of the load and the beam midspan-deflection should be recorded continuously. It is important to note that if the crack forms outside the notch, the test has to be rejected and it should be performed on a fresh beam specimen. When the test is executed by means of
CMOD control, the machine shall be operated in such a manner that the CMOD increases at a constant rate of 50 μm/min for CMOD from 0 to 0.1 mm, followed by a constant rate of 0.2 mm/min until the end of the test. The test results are obtained in the form of load-crack opening response of the specimens. Later on, this test data can be converted into the stress-strain response of SFRC using a set of the empirical relations. A typical response of the SFRC in the form of a load-crack opening displacement and the recommended format of the stress-strain response is shown in Fig. 2.10.

The residual tensile strengths $F_{Ri}$, shown in Fig. 2.10a, are key-parameters characterising the post-cracking behavior of SFRC. The value of these parameters can be determined by following the testing procedure described above and using Eq. 2.11. The load $F_L$, is a load corresponding to the limit of proportionality exhibited by the test specimen. It can be taken as a load corresponding to the 0.5 mm midspan-deflection ($\delta$) or a CMOD value of 0.05 mm. It is important to note that the load-deflection response of the member is linear-elastic up to the load $F_L$ and it may or may not be accompanied by the crack formation; therefore either of these two values can be adopted as the limit of proportionality.

Similar to the value of $F_L$, the residual-tensile strengths, $F_{R1}$ and $F_{R4}$, respectively, are fixed as a load value corresponding to the specified value of the CMOD or the mid-span deflection measured in the prescribed bending test. $F_{R1}$ is a load value corresponding to the CMOD of 0.5 mm or a deflection value of 0.46 mm. The value of this residual stress can be less or higher than the value determined corresponding to the $F_L$. The residual stress value, $F_{R4}$ is measured at a CMOD of 3.5 mm or a

![Fig. 2.10 SFRC tension constitutive model recommended by RELIM—
a load-crack opening displacement response; b a stress-strain response. The load-CMOD response of the material can be obtained following the procedure prescribed by EN 14651:2005](image-url)
deflection value of 3.00 mm. Generally, this value of the residual-stress represents the strength of SFRC specimen corresponding to the ultimate state.

The guideline has proposed a strain value of 0.025 at the ultimate state by taking the characteristic length $l_{cs}$ equal to the height of the neutral axis (=140 mm) in the specimen, taken at the point where the CMOD is measured. It gives a strain value of $(3.5/140) = 0.025$ for a CMOD of 3.5 mm. This is the inherent flaw in the model as the depth of the neutral axis depends on the fiber parameters, such as the fiber-aspect ratio ($l/d$), the volume fraction ($V_f$) and the concrete compressive strength. However, because of the simplicity and familiarity of the testing procedure, it is used in practice more commonly than any other method.

$$\sigma_{R_i} = \frac{3F_{R_i}L}{2bh_1^2}$$

(2.11)

In Eq. 2.11, L is an effective span of the beam specimen ($B \times D$). The parameter $h_1$ denotes a distance from the top of the notch provided in the beam specimen to its top face. The parameter $F_{R_i}$ is a load recorded at CMOD$_i$ or midspan-deflection ($\delta_{R_i}$) during the beam testing (see Fig. 2.10). The values of stresses $\sigma_i$ shown in Fig. 2.10 can be determined from a set of expressions given in Eqs. 2.12–2.15.

$$\sigma_1 = 0.7Y\sqrt{f_{ck}}(1.6 - D)\text{MPa}; \quad \text{Where, } D \text{ (in ‘m’) is the depth of the beam specimen and the value of constant}$$

$(Y) = 0.75 \quad \text{for } f_{ck} < 45 \text{ and it is } 0.85 \quad \text{for } f_{ck} > 50.$

(2.12)

$$\varepsilon_1 = \frac{\sigma_1}{5000\sqrt{f_{ck}}}$$

(2.13)

$$\sigma_2 = 0.45f_{R1}k_h\text{MPa}; \quad \varepsilon_2 = \varepsilon_1 + 0.0001$$

(2.14)

$$\sigma_3 = 0.37f_{R4}k_h\text{MPa}; \quad \varepsilon_3 = 0.025$$

(2.15)

In Eqs. 2.14 and 2.15, $k_h$ is a beam-depth factor and it reflects the effect of the specimen size on the SFRC strength properties. The value of the factor can be calculated from Eq. 2.16. Nevertheless, the equation is applicable for a beam-depth of 125–600 mm only. This apparent specimen size-effect is not yet fully understood as of now and further investigations are going on in order to identify whether this is arising due to a discrepancy of the material properties between different batches, or to the size-effect intrinsic to the procedure, or a combination of these parameters. Accordingly, it is strictly recommended in the guidelines that the depth of the beam specimen must lie in the prescribed range.

$$k_h = 1 - 0.6 \frac{D \text{ (in cm)} - 12.5}{47.5}$$

(2.16)

It is important to note that the maximum crack-width in the test specimen at the ultimate limit state is restricted to 3.5 mm so as to ensure a sufficient anchorage
capacity for the steel fibers present in the concrete. However, if crack-widths larger than 3.5 mm are expected to form at collapse or are used in the design, the residual-tensile strength corresponding to that crack-width and the value measured during the bending test has to be used to calculate the value of the stress $\sigma_3$. It is also recommended by the design guideline that this value, which replaces $F_{R4}$ in Eq. 2.15, should not be lower than 1 MPa; if it is coming less than 1 MPa, the mix proportions of SFRC should be altered to get the value more than that required.

### 2.3.2 CNR-DT 204 (2006) Guidelines

The Italian National Research Committee published a set of design guidelines (CNR-DT 204) in the year 2006. This prescribes a set of rules for proportioning and construction of SFRC members. Unlike the RELIM guidelines, it has given two different types of material models describing the tensile behavior of SFRC member: namely, (a) the linear-elastic model and (b) the plastic-rigid model, as shown in Fig. 2.11. The linear post-cracking behavior differentiates between the strain-hardening and strain-softening materials. In case of strain-hardening response, the slope of the curve in the post-cracking range (after onset of the strain $\varepsilon_1$) increases with an increase in the strain-value, while it reduces in the strain-hardening response. These models are expressed in terms of $\sigma$-$\varepsilon$ and $\sigma$-$w$ relationships.

The linear-elastic model can be used to proportion SFRC beams, both at the ultimate limit state and the serviceability limit state of the design procedure, but the plastic-rigid model is recommended to be used only in case of the ultimate limit state. Unlike the RELIM guidelines, use of the four-point bending tests and uniaxial tensile test is prescribed to obtain the $\sigma$-$\varepsilon$ and $\sigma$-$w$ relationships. The tests should be carried out as per the provisions of the Italian standards UNI 11039 (2003) and UNI 11188 (2004), respectively.

The post-cracking strength of SFRC is defined on the basis of either the point-values ($f_i$) taken corresponding to a specified nominal value of the crack-opening ($w_i$), or on the mean-values ($f_{eqi}$) calculated for assigned intervals of the crack-opening. Unlike the RELIM guidelines, the crack opening is taken as the displacement between the two points at the notch-tip ‘CTOD’ caused after the

---

**Fig. 2.11** SFRC stress-strain ($\sigma$-$\varepsilon$) response recommended by the CNR-DT 204—

- **(a)** A linear-elastic response;
- **(b)** A plastic response
formation of a crack instead of the crack-mouth opening displacement (CMOD) considered by the RILEM (2002). The selection of the residual-tensile strength on the basis of the crack-width is important as the SFRC loses its load carrying capacity significantly with an increase of the crack opening. A high value of the crack opening in an SFRC member leads to a complete loss of the fiber anchorage, thereby resulting in a drop of the strength values with the increasing crack-width. The guidelines permit the analyst to select the design stress depending upon the crack-width taken either from the fiber anchorage considerations or the serviceability criterion. The Eqs. 2.17 and 2.18 gives the mathematical expressions derived using the equilibrium conditions of a rectangular section under bending to determine the key parameters of the linear-elastic model (see, Fig. 2.11a).

\[
\sigma_1 = 0.45 f_{eq1} \\
\sigma_2 = k \left[ \sigma_1 - \left( \frac{w_u}{w_{i2}} \right) \left( \sigma_1 - 0.5 f_{eq2} + 0.2 f_{eq1} \right) \right] \geq 0
\]

In Eqs. 2.17 and 2.18, \( f_{eq1} \) and \( f_{eq2} \) are the SFRC post-cracking equivalent strengths considered in the serviceability and the ultimate limit states, respectively. The parameter \( w_{i2} \) is the mean value of the crack opening (in the intervals \( \text{CTOD}_1 \) and \( \text{CTOD}_2 \)) where \( f_{eq2} \) is evaluated from the \( \sigma-w \) response of SFRC. The parameter \( w_u \) denotes the crack-width at the ultimate state, generally taken as 3 mm for a structural element in bending, and 1.5 mm when it is subjected to tension. The values of \( f_{eq1} \) and \( f_{eq2} \) can be determined from the test results (Fig. 2.12 depicts a typical curve) of the four-point bending test conducted in accordance with the UNI 11039 (2003). It is important to note that the \( \text{CTOD}_1 \) is a permitted value of the crack-width corresponding to the serviceability limit state; whereas the \( \text{CTOD}_3 \) is a value of the crack-width at the ultimate limit state. The value of the \( \text{CTOD}_2 \) can be selected arbitrarily depending upon the desired performance level. In Eq. 2.18, \( k \) is

---

Fig. 2.12 \( \sigma-w \) response—It can be obtained using the 4-point bending test or axial tension testing of SFRC standard specimens, conducted in accordance of UNI 11039 (2003) and UNI 11188 (2004), respectively.
a coefficient taken equal to 0.7 for the specimens fully subjected to the tensile stresses and it is equal to 1 in all other cases of loading.

In case of the plastic model (Fig. 2.11b), Eq. 2.19 can be used to determine the stress \( f_{tu} \) at the ultimate state. This expression is derived using the equivalence of the moment capacity of the SFRC rectangular section considering the plastic-tensile response with a peak stress-value \( f_{tu} \) and an imaginary rectangular section with a linear-elastic stress distribution. The stress \( f_{eq2} \) is the peak value of the stress is taken in the elastic stress distribution.

\[
\sigma = f_{tu} = \frac{f_{eq2}}{3}
\]  

(2.19)

Figure 2.12 shows a typical \( \sigma-w \) response of the SFRC, obtained experimentally to evaluate different key-parameters of the material constitutive model. It is important to note the linearity of the \( \sigma-w \) response, especially in its post-cracking range. This characteristic of the response exhibited by the SFRC in the post-cracking range of the stress-crack width \( (\sigma-w) \) curve is used to derive and simplify the expression of the post-peak strength \( (\sigma_2) \), given in Eq. 2.18.

It is important to note that the parameter \( f_{eq} \) used in the CNR-DT-204 is better to use over the parameter \( f_R \) adopted by the RILEM (2002) guidelines for developing the tensile constitutive model of SFRC. Some studies in the past, such as Barros et al. (2005) are supporting the use of the parameter \( f_{eq} \) instead of \( f_R \). This is because of the fact that the former parameter represents the energy-absorption capacity of the material up to a certain deflection exhibited by the SFRC member; whereas, the parameter \( f_R \) corresponds to the stress associated with the force at a certain deflection level, which is more susceptible to the local irregularities of the load-displacement curve than the energy absorption capacity which is a measured of the area of the load-displacement response curve. As the procedure adopted by the CNR-DT-204 is based upon the equivalent strength values, it is essential to evaluate the post-cracking strength of the material by using the recommended value of the limiting crack-width, which has a significant effect on the toughness. The wider cracks could be highly detrimental to the strength and may also cause a significant reduction of the material toughness.

In case of a strain-hardening response, the SFRC section exhibits a set of multiple cracking that develops with the increasing load and the average strain-value may be obtained directly from the experimental tests. Nevertheless, the ultimate strain value for this type of the material response is limited to 0.010; whereas, in case of a strain-softening response, there appears only one main crack in the section usually at the point of the maximum moment in the section.

The ultimate strain for a strain-softening response is related to the crack-width appeared in the section at the ultimate state \( (w_u) \) and the equivalence should be carried out by means of a characteristic length \( (l_{cs}) \). The value of \( l_{cs} \) should be determined from the minimum of the average crack spacing \( (s_{rm}) \) observed in the
tests and the height of the neutral-axis (h₂). However, the ultimate tensile strain in the section for this case is limited to 0.020 and there is also an upper limit [w_u = ε_u · l_cs ≤ 3.0 mm] on the value of the crack-width (w_u) formed in the specimen at the ultimate load.

2.3.3 fib Model Code (2010)

The fib (Fédération Internationale du Béton), similar to the CRN-DT 204 guidelines, has also recommended two different types of constitutive relationships to model for the tensile behavior of SFRC in its updated version (MC2010) of the CEB-FIP Model Code 90; namely, the linear-postcracking behavior and the plastic-rigid behavior. Figure 2.11 depicts these two types of the SFRC material model. Interestingly, SFRC can exhibit a strain-softening or strain-hardening response depending upon the fiber parameters taken in the design of SFRC mix. The key parameters (ffts and fftu) in both the models are defined by means of the residual flexural-tensile strengths, determined from a set of the test results, conducted under the three-point load bending conditions in accordance with the provisions of EN 14651 (2005).

The parameter ffts represents the residual-tensile strength of SFRC. It is defined as the post-cracking strength of the material corresponding to the crack-opening displacement (w) considered in the serviceability limit state; whereas, fftu is value of the ultimate value of the residual-tensile strength corresponding to the maximum permitted value of the crack-opening displacement (w_u) of SFRC in the design. For the rigid-plastic model, the value of (w_u) is prescribed as 2.5 mm and that for the linear-elastic model, it depends on the ductility level adopted in the design, but it should not exceed 2.5 mm. The expressions to determine the parameters ffts and fftu are given in Eqs. 2.20 and 2.21.

\[
f_{fts} = 0.45f_{R1} \tag{2.20}
\]

\[
f_{ftu} = k \left[ f_{fts} - \left( \frac{w_u}{CMOD_3} \right) (f_{fts} - 0.5f_{R3} + 0.2f_{R1}) \right] \tag{2.21}
\]

In Eq. 2.21, k is a fiber orientation-factor considered in the design. This factor equals one for an isotropic fiber distribution. However, the value can be taken lower or higher than one, if favourable or unfavourable effects are experimentally verified on the real-scale structural elements. Equation 2.11 is used to compute the value of the parameters (f_R1 and f_R3) taken in the Eqs. 2.20 and 2.21.

The fib Model Code has specified a strain-level for the identification of strain-hardening or strain-softening response exhibited by SFRC in its post-cracking range. It will be a strain-hardening, if it occurs at an ultimate tensile strain value of 0.010; otherwise, the material is considered as of strain-softening nature. For a SFRC exhibiting a strain-softening response, the σ-ε diagram is
defined by identifying the crack-width \((w)\) appearing in the section and the corresponding characteristic length \((l_{cs})\) of the structural element. Therefore, the corresponding value of the strain can be expressed as \(\varepsilon = w/l_{cs}\). The value of \(l_{cs}\) is a minimum of the average crack spacing \((S_{rm})\) or the distance between the neutral axis and the tensile face of the cross section \((h_2)\).

In case of the beam section with conventional rebars; the parameter \(h_2\) is evaluated in the cracked-phase assuming no tensile strength of the SFRC, similar to the conventional RC sections, and for a load configuration corresponding to the serviceability state of crack-width and spacing. However, the value of \(h_2\) is assumed equal to the height of the beam-section in case of a SFRC beam section without conventional tensile reinforcement. Therefore, the ultimate crack-width \(w_u\) be calculated as \(w_u = l_{cs} \varepsilon_u\). The ultimate strain \(\varepsilon_u\) equals 0.020 for variable strain distribution along the cross section and it is 0.010 for uniform tensile strain distribution along the cross-section.

For a SFRC exhibiting a strain-hardening response, however, the parameter \((l_{cs})\) is not necessary. The \(\sigma-\varepsilon\) diagram is defined by assuming the stain at ultimate \((\varepsilon_u)\) equal to 0.020 for a variable-strain distribution along the depth of the section (e.g., beams) and by taking it equals to 0.010 for the uniform tensile-strain distribution along the cross section (e.g. axial tension case). It is important to remark that the ultimate crack-width \(w_u\) required in estimating the parameter \(f_{tu}\) may be calculated using relationship: \(w_u = l_{cs} \varepsilon_u\).

**Example 2.1**

Standard prisms of 150 × 150 × 750 mm were cast using SFRC. It contains short steel hooked-end fibers (0.5 mm diameter and 30 mm long) with a fiber volume fraction of 1%. The sample size was 6 units and these were tested under a standard 3-point loading conditions, with an effective span of 500 mm, following a standard testing procedure. Table 2.2 gives the results from the test taken at various prescribed CMOD values. The results from the standard 150 mm concrete cubes indicate that the SFRC possesses a 28 days compressive strength of 42.58 MPa.

Based upon the test data, the tension constitutive model of the SFRC can be derived using the applicable equations, given in the previous sections:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>The stress values (MPa) corresponding to the CMOD of 0.05 mm, 0.5 mm, 2.5 mm, 3.5 mm</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_L)</td>
<td>(f_{R1})</td>
</tr>
<tr>
<td>1</td>
<td>9.14</td>
<td>13.53</td>
</tr>
<tr>
<td>2</td>
<td>7.20</td>
<td>10.03</td>
</tr>
<tr>
<td>3</td>
<td>7.09</td>
<td>10.60</td>
</tr>
<tr>
<td>4</td>
<td>7.55</td>
<td>11.79</td>
</tr>
<tr>
<td>5</td>
<td>7.49</td>
<td>10.11</td>
</tr>
<tr>
<td>6</td>
<td>6.76</td>
<td>9.48</td>
</tr>
<tr>
<td>Average value</td>
<td>7.51</td>
<td>10.92</td>
</tr>
</tbody>
</table>
2.3 Experimental Characterization of the Tensile Response

- RELIM model (Eqs. 2.12–2.15)

The value of the shape factor, $k_h = 0.9684$

$\sigma_1 = 4.97$ MPa, $\epsilon_1 = 0.000152$

$\sigma_2 = 4.76$ MPa, $\epsilon_2 = 0.000252$

$\sigma_3 = 2.42$ MPa, $\epsilon_3 = 0.025$

- fib model (Eqs. 2.20–2.21)

$f_{ts} = 4.914$ MPa, $\epsilon_1 = 0.00015$

$f_{tu} = 1.276$ MPa, $\epsilon_2 = 0.025$

- Ultimate tensile strength (Eq. 2.8)

$$\sigma_u = 2 \times 0.3 \times V_f \times \sqrt{\frac{f_{ck}}{d}} = 2.35 \text{ MPa}, \quad \epsilon_u = 0.015$$

- CNR-DT 204 (2006) (Eqs. 2.17–2.18)

$$f_{eq1} = \frac{10.92 + 7.51}{2} = 9.215 \text{ MPa} \quad \text{for } w_{11} = \frac{0.05 + 0.5}{2} = 0.275 \text{ mm}$$

$$f_{eq2} = \frac{6.92 + 7.51}{2} = 4.835 \text{ MPa} \quad \text{for } w_{21} = \frac{2.5 + 3.5}{2} = 3.0 \text{ mm}$$

$f_{ts} = 4.146$ MPa, $\epsilon_1 = 0.00015$

$f_{tu} = 0.5945$ MPa, $\epsilon_2 = 0.020$

Knowing the key points of the stress-strain response (see, Figs. 2.9, 2.10 and 2.11), the flexural capacity of SFRC member can be computed using the equations of statics (see, Chap. 3 for details).

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