

Chapter 2

Basic Principles of Light

Abstract The purpose of this chapter is to present an overview of the fundamental behavior of light. Having a good grasp of the basic principles of light is important for understanding how light interacts with biological matter, which is the basis of biophotonics. A challenging aspect of applying light to biological materials is that the optical properties of the materials generally vary with the light wavelength and can depend on factors such as the optical power per area irradiated, the temperature, the light exposure time, and light polarization. The following topics are included in this chapter: the characteristics of lightwaves, polarization, quantization and photon energy, reflection and refraction, and the concepts of interference and coherence.

Having a good grasp of the basic principles of light is important for understanding how light interacts with biological matter, which is the basis of biophotonics. As Chap. 6 describes, light impinging on a biological tissue can pass through or be absorbed, reflected, or scattered in the material. The degree of these interactions of light with tissue depends significantly on the characteristics of the light and on the optical properties of the tissue. A challenging aspect of applying light to biological materials is the fact that the optical properties of the material generally vary with the light wavelength. In addition, the optical properties of tissue can depend on factors such as the optical power per area irradiated, the temperature, the light exposure time, and light polarization. Chapter 6 and subsequent chapters present further details on these light-tissue interaction effects.

The purpose of this chapter is to present an overview of the fundamental behavior of light [1–11]. The theory of quantum optics describes light as having properties of both waves and particles. This phenomenon is called the *wave-particle duality*. The dual wave-particle picture describes the transportation of optical energy either by means of the classical electromagnetic wave concept of light or by means of quantized energy units or *photons*. The wave theory can be employed to understand fundamental optical concepts such as reflection, refraction, dispersion, absorption, luminescence, diffraction, birefringence, and scattering. However, the particle theory is needed to explain the processes of photon generation and absorption.

The following topics are included in this chapter: the characteristics of light-waves, polarization, quantization and photon energy, reflection and refraction, and the concepts of interference and coherence.

2.1 Lightwave Characteristics

The theory of classical electrodynamics describes light as electromagnetic waves that are transverse, that is, the wave motion is perpendicular to the direction in which the wave travels. In this *wave optics* or *physical optics* viewpoint, a series of successive spherical wave fronts (referred to as a *train of waves*) spaced at regular intervals called a *wavelength* can represent the electromagnetic waves radiated by a small optical source with the source at the center as shown in Fig. 2.1a. A *wave front* is defined as the locus of all points in the wave train that have the same phase (the term *phase* indicates the current position of a wave relative to a reference point). Generally, one draws wave fronts passing through either the maxima or the minima of the wave, such as the peak or trough of a sine wave, for example. Thus the wave fronts (also called *phase fronts*) are separated by one wavelength.

When the wavelength of the light is much smaller than the object (or opening) that it encounters, the wave fronts appear as straight lines to this object or opening. In this case, the lightwave can be represented as a plane wave and a light ray can indicate its direction of travel, that is, the ray is drawn perpendicular to the phase front, as shown in Fig. 2.1b. The light-ray concept allows large-scale optical effects

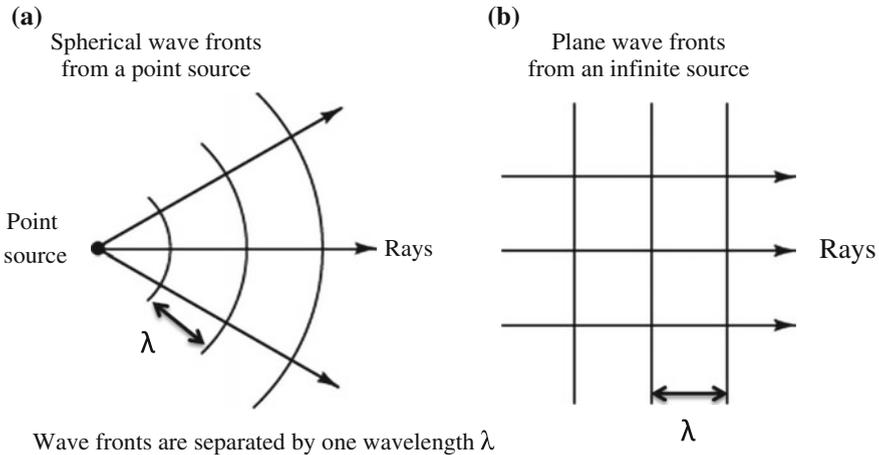
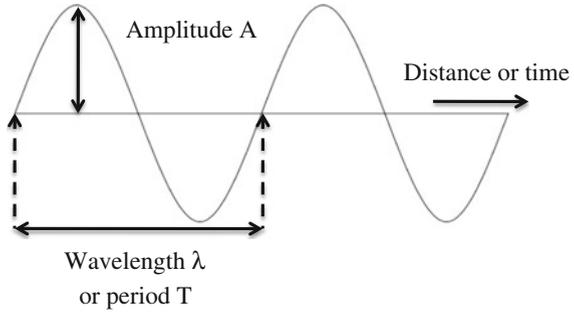


Fig. 2.1 Representations of **a** spherical waves radiating from a point source and **b** plane waves and their associated rays

Fig. 2.2 A monochromatic wave consists of a sine wave of infinite extent with amplitude A , wavelength λ , and period T



such as reflection and refraction to be analyzed by the simple geometrical process of ray tracing. This view of optics is referred to as *ray optics* or *geometrical optics*. The concept of light rays is very useful because the rays show the direction of energy flow in the light beam.

To help understand wave optics, this section first discusses some fundamental characteristics of waveforms. For mathematical convenience in describing the characteristics of light, first it is assumed that the light comes from an ideal monochromatic (single-wavelength) source with the emitted lightwave being represented in the time domain by an infinitely long, single-frequency sinusoidal wave, as Fig. 2.2 shows. In a real lightwave or in an optical pulse of finite time duration, the waveform has an arbitrary time dependence and thus is polychromatic. However, although the waveform describing a polychromatic wave is nonharmonic in time, it may be represented by a superposition of monochromatic harmonic functions.

2.1.1 Monochromatic Waves

A monochromatic lightwave can be represented by a real waveform $u(\mathbf{r}, t)$ that varies harmonically in time

$$u(\mathbf{r}, t) = A(\mathbf{r}) \cos[\omega t + \varphi(\mathbf{r})] \tag{2.1}$$

where \mathbf{r} is a position vector so that

$A(\mathbf{r})$ is the wave amplitude at position \mathbf{r}

$\varphi(\mathbf{r})$ is the phase of the wave at position \mathbf{r}

$\omega = 2\pi\nu$ is the angular frequency measured in radians/s or s^{-1}

λ is the wavelength of the wave

$\nu = c/\lambda$ is the lightwave frequency measured in cycles/s or Hz

$T = 1/\nu = 2\pi/\omega$ is the wave period in units of seconds

As noted in Fig. 1.5, the frequencies of optical waves run from 1×10^{12} to 3×10^{16} Hz, which corresponds to maximum and minimum wavelengths of 300 μm and 10 nm, respectively, in the optical spectrum.

The waveform also can be expressed by the following complex function

$$U(\mathbf{r}, t) = A(\mathbf{r}) \exp\{i[\omega t + \varphi(\mathbf{r})]\} = U(\mathbf{r}) \exp(i\omega t) \quad (2.2)$$

where the time-independent factor $U(\mathbf{r}) = A(\mathbf{r}) \exp[i\varphi(\mathbf{r})]$ is the complex amplitude of the wave. The waveform $u(\mathbf{r}, t)$ is found by taking the real part of $U(\mathbf{r}, t)$

$$u(\mathbf{r}, t) = \text{Re}\{U(\mathbf{r}, t)\} = \frac{1}{2}[U(\mathbf{r}, t) + U^*(\mathbf{r}, t)] \quad (2.3)$$

where the symbol * denotes the complex conjugate.

The *optical intensity* $I(\mathbf{r}, t)$, which is defined as the optical power per unit area (for example, in units such as W/cm^2 or mW/mm^2), is equal to the average of the squared wavefunction. That is, by letting the operation $\langle x \rangle$ denote the average of a generic function x , then

$$I(\mathbf{r}, t) = \langle U^2(\mathbf{r}, t) \rangle = |U(\mathbf{r})|^2 \langle 1 + \cos(2[2\pi\nu t + \varphi(\mathbf{r})]) \rangle \quad (2.4)$$

When this average is taken over a time longer than the optical wave period T , the average of the cosine function goes to zero so that the optical intensity of a monochromatic wave becomes

$$I(\mathbf{r}) = |U(\mathbf{r})|^2 \quad (2.5)$$

Thus the intensity of a monochromatic wave is independent of time and is the square of the absolute value of its complex amplitude.

Example 2.1 Consider a spherical wave given by

$$U(r) = \frac{A_0}{r} \exp(-i2\pi r/\lambda)$$

where $A_0 = 1.5 \text{ W}^{1/2}$ is a constant and r is measured in cm. What is the intensity of this wave?

Solution: From Eq. (2.5),

$$I(r) = |U(r)|^2 = (A_0/r)^2 = \left[(1.5 \text{ W}^{1/2})/r \text{ (in cm)} \right]^2 = 2.25 \text{ W}/\text{cm}^2$$

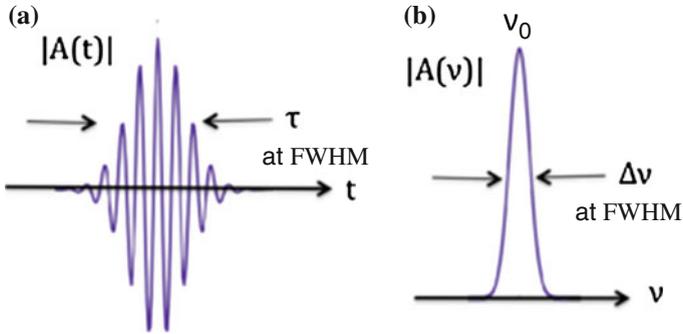


Fig. 2.3 The **a** temporal and **b** spectral characteristics of a pulsed plane wave

2.1.2 Pulsed Plane Waves

Many biophotonics procedures use pulses of light to measure or analyze some biological function. As noted above, these light pulses are polychromatic functions that can be represented by a pulsed plane wave, such as

$$U(\mathbf{r}, t) = A\left(t - \frac{Z}{c}\right) \exp\left[i2\pi\nu_0\left(t - \frac{Z}{c}\right)\right] \quad (2.6)$$

In this case, the complex envelope A is a time-varying function and the parameter ν_0 is the central optical frequency of the pulse. Figure 2.3 shows the temporal and spectral characteristics of a pulsed plane wave. The complex envelope $A(t)$ typically is of finite duration τ and varies slowly in time compared to an optical cycle. Thus its Fourier transform $A(\nu)$ has a *spectral width* $\Delta\nu$, which is inversely proportional to the *temporal width* τ at the full-width half-maximum (FWHM) point and is much smaller than the central optical frequency ν_0 . The temporal and spectral widths usually are defined as the root-mean-square (rms) widths of the power distributions in the time and frequency domains, respectively.

2.2 Polarization

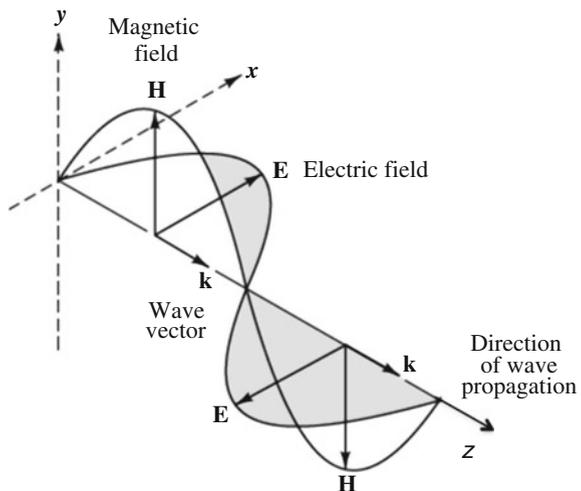
Light emitted by the sun or by an incandescent lamp is created by electromagnetic waves that vibrate in a variety of directions. This type of light is called *unpolarized light*. Lightwaves in which the vibrations occur in a single plane are known as *polarized light*. The process of *polarization* deals with the transformation of unpolarized light into polarized light. The polarization characteristics of lightwaves are important when describing the behavior of polarization-sensitive devices such as optical filters, light signal modulators, Faraday rotators, and light beam splitters.

These types of components typically incorporate polarization-sensitive birefringent crystalline materials such as calcite, lithium niobate, rutile, and yttrium vanadate.

Light consists of *transverse electromagnetic waves* that have both electric field (\mathbf{E} field) and magnetic field (\mathbf{H} field) components. The directions of the vibrating electric and magnetic fields in a transverse wave are perpendicular to each other and are orthogonal (at right angles) to the direction of propagation of the wave, as Fig. 2.4 shows. The waves are moving in the direction indicated by the *wave vector* \mathbf{k} . The magnitude of the wave vector \mathbf{k} is $k = 2\pi/\lambda$, which is known as the *wave propagation constant* with λ being the wavelength of the light. Based on Maxwell's equations, it can be shown that \mathbf{E} and \mathbf{H} are both perpendicular to the direction of propagation. This condition defines a *plane wave*; that is, the vibrations in the electric field are parallel to each other at all points in the wave. Thus, the electric field forms a plane called the *plane of vibration*. An identical situation holds for the magnetic field component, so that all vibrations in the magnetic field wave lie in another plane. Furthermore, \mathbf{E} and \mathbf{H} are mutually perpendicular, so that \mathbf{E} , \mathbf{H} , and \mathbf{k} form a set of orthogonal vectors.

An ordinary lightwave is made up of many transverse waves that vibrate in a variety of directions (i.e., in more than one plane), which is called *unpolarized light*. However any arbitrary direction of vibration of a specific transverse wave can be represented as a combination of two orthogonal plane polarization components. As described in Sect. 2.4 and in Chap. 3, this concept is important when examining the reflection and refraction of lightwaves at the interface of two different media, and when examining the propagation of light along an optical fiber. In the case when all the electric field planes of the different transverse waves are aligned parallel to each other, then the lightwave is *linearly polarized*. This describes the simplest type of polarization.

Fig. 2.4 Electric and magnetic field distributions in a train of plane electromagnetic waves at a given instant in time



2.2.1 Linear Polarization

Using Eq. (2.2), a train of electric or magnetic field waves designated by \mathbf{A} can be represented in the general form

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{e}_j A_0 \exp[i(\omega t - \mathbf{k} \cdot \mathbf{r})] \quad (2.7)$$

where $\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z$ represents a general position vector, $\mathbf{k} = k_x\mathbf{e}_x + k_y\mathbf{e}_y + k_z\mathbf{e}_z$ is the wave propagation vector, \mathbf{e}_j is a unit vector lying parallel to an axis designated by j (where $j = x, y, \text{ or } z$), and k_j is the magnitude of the wave vector along the j axis. The parameter A_0 is the maximum amplitude of the wave and $\omega = 2\pi\nu$, where ν is the frequency of the light.

The components of the actual (measurable) electromagnetic field are obtained by taking the real part of Eq. (2.7). For example, if $\mathbf{k} = k_z\mathbf{e}_z$, and if \mathbf{A} denotes the electric field \mathbf{E} with the coordinate axes chosen such that $\mathbf{e}_j = \mathbf{e}_x$, then the measurable electric field is

$$\mathbf{E}_x(z, t) = \text{Re}(\mathbf{E}) = \mathbf{e}_x E_{0x} \cos(\omega t - kz) = \mathbf{e}_x E_x(z) \quad (2.8)$$

which represents a plane wave that varies harmonically as it travels in the z direction. Here E_{0x} is the maximum wave amplitude along the x axis and $E_x(z) = E_{0x} \cos(\omega t - kz)$ is the amplitude at a given value of z in the xz plane. The reason for using the exponential form is that it is more easily handled mathematically than equivalent expressions given in terms of sine and cosine. In addition, the rationale for using harmonic functions is that any waveform can be expressed in terms of sinusoidal waves using Fourier techniques.

The plane wave example given by Eq. (2.8) has its electric field vector always pointing in the \mathbf{e}_x direction, so it is linearly polarized with polarization vector \mathbf{e}_x . A general state of polarization is described by considering another linearly polarized wave that is independent of the first wave and orthogonal to it. Let this wave be

$$\mathbf{E}_y(z, t) = \mathbf{e}_y E_{0y} \cos(\omega t - kz + \delta) = \mathbf{e}_y E_y(z) \quad (2.9)$$

where δ is the relative phase difference between the waves. Similar to Eq. (2.8), E_{0y} is the maximum amplitude of the wave along the y axis and $E_y(z) = E_{0y} \cos(\omega t - kz + \delta)$ is the amplitude at a given value of z in the yz plane. The resultant wave is

$$\mathbf{E}(z, t) = \mathbf{E}_x(z, t) + \mathbf{E}_y(z, t) \quad (2.10)$$

If δ is zero or an integer multiple of 2π , the waves are in phase. Equation (2.10) is then also a linearly polarized wave with a polarization vector making an angle

$$\theta = \arcsin \frac{E_{0y}}{E_{0x}} \quad (2.11)$$

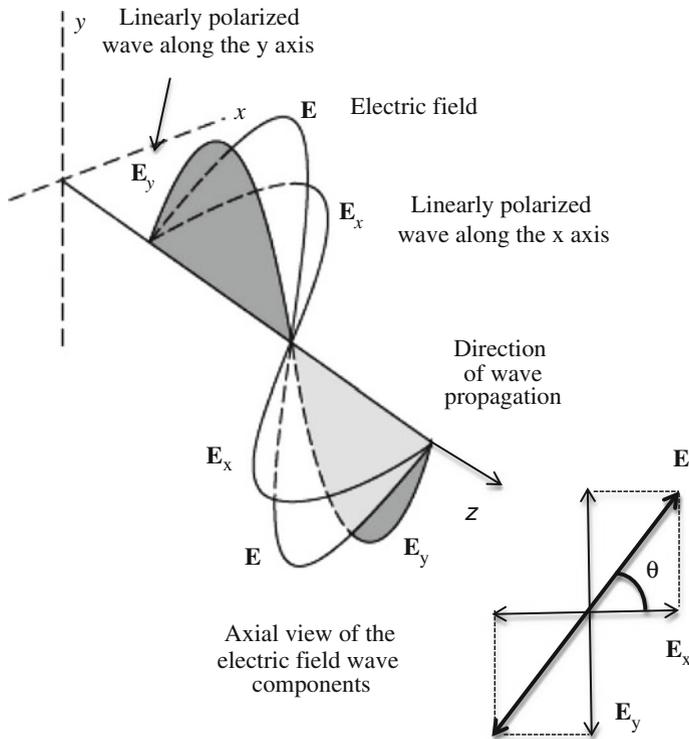


Fig. 2.5 Addition of two linearly polarized waves having a zero relative phase between them with respect to \mathbf{e}_x and having a magnitude

$$E = \left(E_{0x}^2 + E_{0y}^2 \right)^{1/2} \quad (2.12)$$

This case is shown schematically in Fig. 2.5. Conversely, just as any two orthogonal plane waves can be combined into a linearly polarized wave, an arbitrary linearly polarized wave can be resolved into two independent orthogonal plane waves that are in phase. For example, the wave \mathbf{E} in Fig. 2.5 can be resolved into the two orthogonal plane waves \mathbf{E}_x and \mathbf{E}_y .

Example 2.2 The general form of an electromagnetic wave is

$$y = (\text{amplitude in } \mu\text{m}) \times \cos(\omega t - kz) = A \cos[2\pi(\nu t - z/\lambda)]$$

Find the (a) amplitude, (b) the wavelength, (c) the angular frequency, and (d) the displacement at time $t = 0$ and $z = 4 \mu\text{m}$ of a plane electromagnetic wave specified by the equation $y = 12\cos[2\pi(3t - 1.2z)]$.

Solution: From the above general electromagnetic wave equation

- (a) Amplitude = $12 \mu\text{m}$
- (b) Wavelength: $1/\lambda = 1.2 \mu\text{m}^{-1}$ so that $\lambda = 833 \text{ nm}$
- (c) The angular frequency is $\omega = 2\pi\nu = 2\pi(3) = 6\pi$
- (d) At time $t = 0$ and $z = 4 \mu\text{m}$, the displacement is

$$y = 12 \cos[2\pi(-1.2 \mu\text{m}^{-1})(4 \mu\text{m})] = 12 \cos[2\pi(-4.8)] = 10.38 \mu\text{m}$$

2.2.2 Elliptical and Circular Polarization

For general values of δ the wave given by Eq. (2.10) is elliptically polarized. The resultant field vector \mathbf{E} will both rotate and change its magnitude as a function of the angular frequency ω . Elimination of the $(\omega t - kz)$ dependence between Eqs. (2.8) and (2.9) for a general value of δ yields,

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta = \sin^2\delta \quad (2.13)$$

which is the general equation of an ellipse. Thus as Fig. 2.6 shows, the endpoint of \mathbf{E} will trace out an ellipse at a given point in space. The axis of the ellipse makes an angle α relative to the x axis given by

$$\tan 2\alpha = \frac{2E_{0x}E_{0y}\cos\delta}{E_{0x}^2 - E_{0y}^2} \quad (2.14)$$

Aligning the principal axis of the ellipse with the x axis gives a better picture of Eq. (2.13). In that case, $\alpha = 0$, or, equivalently, $\delta = \pm\pi/2, \pm3\pi/2, \dots$, so that Eq. (2.13) becomes

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 = 1 \quad (2.15)$$

This is the equation of an ellipse with the origin at the center and with amplitudes of the semi-axes equal to E_{0x} and E_{0y} .

When $E_{0x} = E_{0y} = E_0$ and the relative phase difference $\delta = \pm\pi/2 + 2m\pi$, where $m = 0, \pm 1, \pm 2, \dots$, then the light is *circularly polarized*. In this case, Eq. (2.15) reduces to

$$E_x^2 + E_y^2 = E_0^2 \quad (2.16)$$

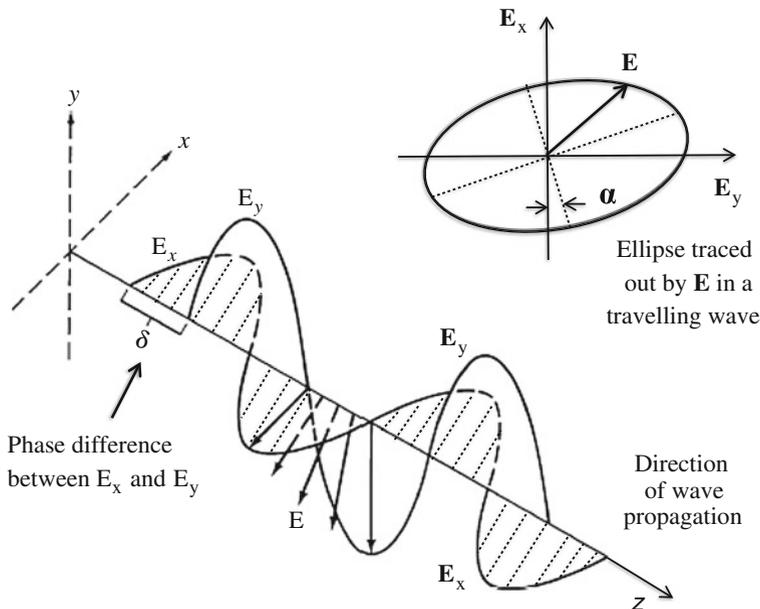


Fig. 2.6 Elliptically polarized light results from the addition of two linearly polarized waves of unequal amplitude having a nonzero phase difference δ between them

which defines a circle. Choosing the phase difference to be $\delta = +\pi/2$ and using the relationship $\cos(a+b) = (\cos a)(\cos b) + (\sin a)(\sin b)$ to expand Eq. (2.9), then Eqs. (2.8) and (2.9) become

$$\mathbf{E}_x(z, t) = \mathbf{e}_x E_0 \cos(\omega t - kz) \tag{2.17a}$$

$$\mathbf{E}_y(z, t) = -\mathbf{e}_y E_0 \sin(\omega t - kz) \tag{2.17b}$$

In this case, the endpoint of \mathbf{E} will trace out a circle at a given point in space, as Fig. 2.7 illustrates. To see this, consider an observer located at some arbitrary point z_{ref} toward whom the wave is moving. For convenience, pick the reference point at $z_{\text{ref}} = \pi/k$ at $t = 0$. Then from Eq. (2.17a) it follows that

$$\mathbf{E}_x(z, t) = -\mathbf{e}_x E_0 \text{ and } \mathbf{E}_y(z, t) = 0 \tag{2.18}$$

so that \mathbf{E} lies along the negative x axis as Fig. 2.7 shows. At a later time, say at $t = \pi/2\omega$, the electric field vector has rotated through 90° and now lies along the positive y axis. Thus as the wave moves toward the observer with increasing time, the resultant electric field vector \mathbf{E} rotates clockwise at an angular frequency ω . It makes one complete rotation as the wave advances through one wavelength. Such a light wave is *right circularly polarized*.

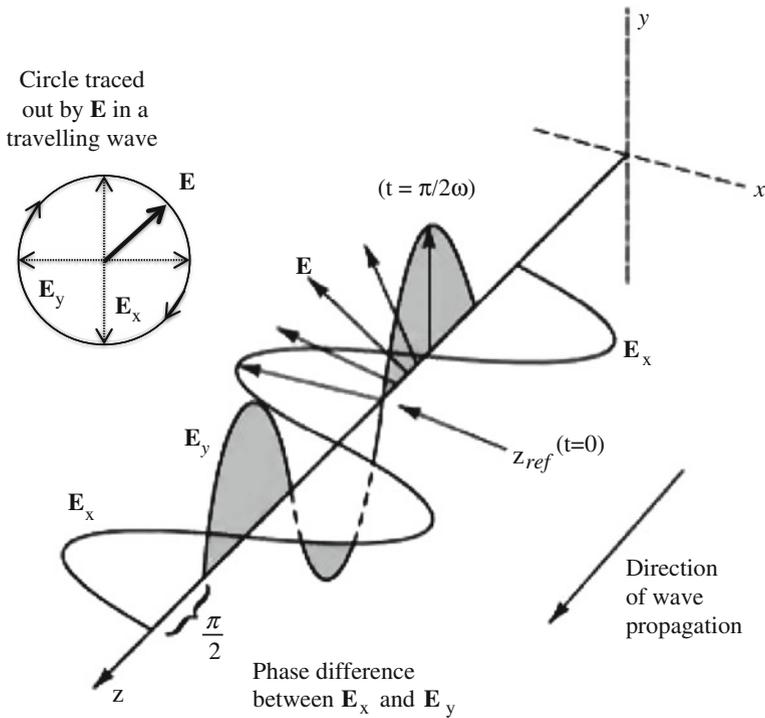


Fig. 2.7 Addition of two equal-amplitude linearly polarized waves with a relative phase difference $\delta = \pi/2 + 2m\pi$ results in a right circularly polarized wave

If the negative sign is selected for δ , then the electric field vector is given by

$$\mathbf{E} = E_0[\mathbf{e}_x \cos(\omega t - kz) + \mathbf{e}_y \sin(\omega t - kz)] \tag{2.19}$$

Now \mathbf{E} rotates counterclockwise and the wave is *left circularly polarized*.

2.3 Quantized Photon Energy and Momentum

The wave theory of light adequately accounts for all phenomena involving the transmission of light. However, in dealing with the interaction of light and matter, such as occurs in the emission and absorption of light, one needs to invoke the quantum theory of light, which indicates that optical radiation has particle as well as wave properties. The particle nature arises from the observation that light energy is always emitted or absorbed in discrete (quantized) units called *photons*. The photon energy depends only on the frequency ν . This frequency, in turn, must be measured by observing a wave property of light.

The relationship between the wave theory and the particle theory is given by *Planck's Law*, which states that the energy E of a photon and its associated wave frequency ν is given by the equation

$$E = h\nu = hc/\lambda \quad (2.20)$$

where $h = 6.625 \times 10^{-34}$ J·s is Planck's constant and λ is the wavelength. The most common measure of photon energy is the *electron volt* (eV), which is the energy an electron gains when moving through a 1-volt electric field. Note that $1 \text{ eV} = 1.60218 \times 10^{-19}$ J. As is noted in Eq. (1.3), for calculation simplicity, if λ is expressed in μm then the energy E is given in eV by using $E(\text{eV}) = 1.2405/\lambda$ (μm). The linear momentum p associated with a photon of energy E in a plane wave is given by

$$p = E/c = h/\lambda \quad (2.21)$$

The momentum is of particular importance when examining photon scattering by molecules (see Chap. 6).

When light is incident on an atom or molecule, a photon can transfer its energy to an electron within this atom or molecule, thereby exciting the electron to a higher electronic or vibrational energy levels or quantum states, as shown in Fig. 2.8a. In this process either all or none of the photon energy is imparted to the electron. For example, consider an incoming photon that has an energy $h\nu_{12}$. If an electron sits at an energy level E_1 in the ground state, then it can be boosted to a higher energy level E_2 if the incoming photon has an energy $h\nu_{12} = E_2 - E_1$. Note that the energy absorbed by the electron from the photon must be exactly equal to the energy required to excite the electron to a higher quantum state. Conversely, an electron in an excited state E_3 can drop to a lower energy level E_1 by emitting a photon of energy exactly equal to $h\nu_{13} = E_3 - E_1$, as shown in Fig. 2.8b.

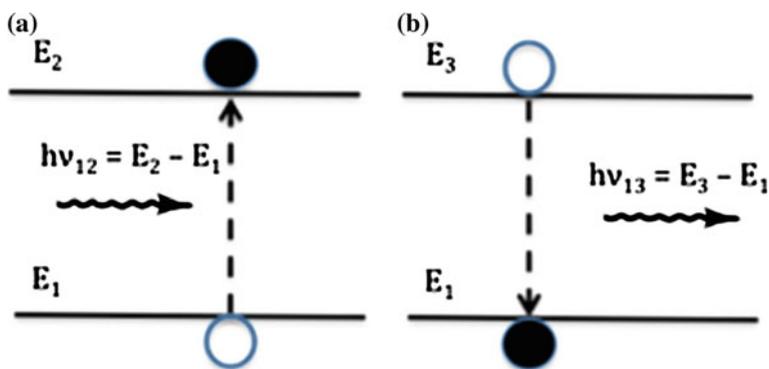


Fig. 2.8 Electron transitions between the ground state and higher electronic or vibrational energy levels

Example 2.3 Two commonly used sources in biomedical photonics are Er:YAG and CO₂ lasers, which have peak emission wavelengths of 2.94 and 10.6 μm, respectively. Compare the photon energies of these two sources.

Solution: Using the relationship $E = hc/\lambda$ from Eq. (2.20) yields

$$\begin{aligned} E(2.94 \mu\text{m}) &= (6.625 \times 10^{-34} \text{ J} \cdot \text{s}) (3 \times 10^8 \text{ m/s}) / (2.94 \times 10^{-6} \text{ m}) \\ &= 6.76 \times 10^{-20} \text{ J} = 0.423 \text{ eV} \end{aligned}$$

Similarly, $E(10.6 \mu\text{m}) = 0.117 \text{ eV}$

Example 2.4 Compare the photon energies for an ultraviolet wavelength of 300 nm and an infrared wavelength of 1550 nm.

Solution: From Eq. (2.20), $E(300 \text{ nm}) = 4.14 \text{ eV}$ and $E(1550 \text{ nm}) = 0.80 \text{ eV}$.

Example 2.5 (a) Consider an incoming photon that boosts an electron from a ground state level E_1 to an excited level E_2 . If $E_2 - E_1 = 1.512 \text{ eV}$, what is the wavelength of the incoming photon? (b) Now suppose this excited electron loses some of its energy and moves to a slightly lower energy level E_3 . If the electron then drops back to level E_1 and if $E_3 - E_1 = 1.450 \text{ eV}$, what is the wavelength of the emitted photon?

Solution: (a) From Eq. (2.20), $\lambda_{\text{incident}} = 1.2405/1.512 \text{ eV} = 0.820 \mu\text{m} = 820 \text{ nm}$. (b) From Eq. (2.20), $\lambda_{\text{emitted}} = 1.2405/1.450 \text{ eV} = 0.855 \mu\text{m} = 855 \text{ nm}$.

2.4 Reflection and Refraction

The concepts of reflection and refraction can be described by examining the behavior of the light rays that are associated with plane waves, as is shown in Fig. 2.1. When a light ray encounters a smooth interface that separates two different dielectric media, part of the light is reflected back into the first medium. The remainder of the light is bent (or refracted) as it enters the second material. This is shown in Fig. 2.9 for the interface between a glass material and air that have refractive indices n_1 and n_2 , respectively, where $n_2 < n_1$.

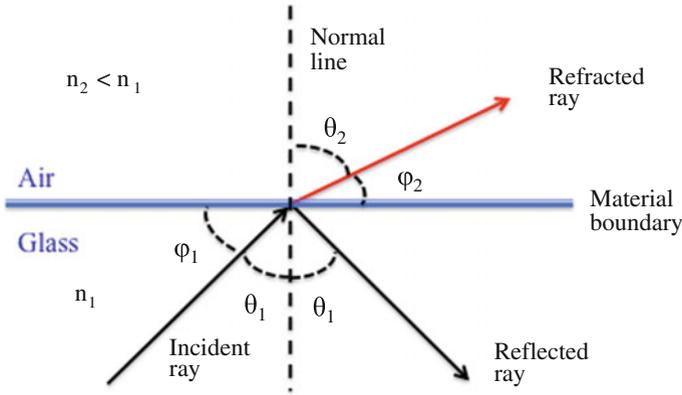


Fig. 2.9 Reflection and refraction of a light ray at a material boundary

2.4.1 Snells' Law

The bending or refraction of light rays at a material interface is a consequence of the difference in the speed of light in two materials with different refractive indices. The relationship at the interface is known as *Snell's law* and is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (2.22)$$

or, equivalently, as

$$n_1 \cos\varphi_1 = n_2 \cos\varphi_2 \quad (2.23)$$

where the angles are defined in Fig. 2.9. The angle θ_1 between the incident ray and the normal to the surface is known as the *angle of incidence*.

In accordance with the law of reflection, the angle θ_1 at which the incident ray strikes the interface is the same as the angle that the reflected ray makes with the interface. Furthermore, the incident ray, the reflected ray, and the normal to the interface all lie in a common plane, which is perpendicular to the interface plane between the two materials. This common plane is called the *plane of incidence*. When light traveling in a certain medium reflects off of a material that has a higher refractive index (called an optically denser material), the process is called *external reflection*. Conversely, the reflection of light off of a material that has a lower refractive index and thus is less optically dense (such as light traveling in glass being reflected at a glass–air interface) is called *internal reflection*.

As the angle of incidence θ_1 in an optically denser material increases, the refracted angle θ_2 approaches $\pi/2$. Beyond this point no refraction is possible as the incident angle increases and the light rays become totally internally reflected. The application of Snell's law yields the conditions required for *total internal reflection*. Consider Fig. 2.10, which shows the interface between a glass surface and air. As a

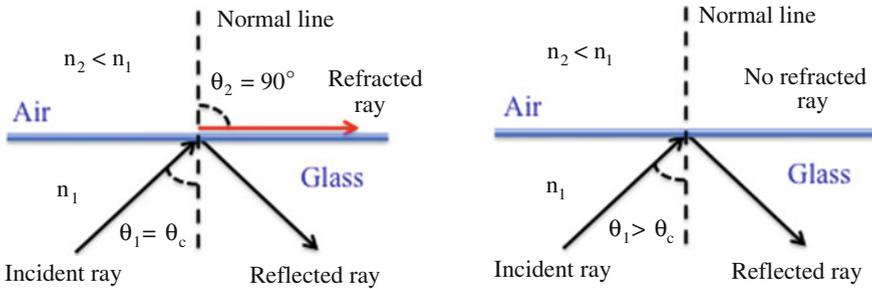


Fig. 2.10 Representation of the critical angle and total internal reflection at a glass-air interface, where n_1 is the refractive index of glass

light ray leaves the glass and enters the air medium, the ray gets bent toward the glass surface in accordance with Snell's law. If the angle of incidence θ_1 is increased, a point will eventually be reached where the light ray in air is parallel to the glass surface. This situation defines the *critical angle of incidence* θ_c . The condition for total internal reflection is satisfied when the angle of incidence θ_1 is greater than the critical angle, that is, all the light is reflected back into the glass with no light penetrating into the air.

To find the critical angle, consider Snell's law as given by Eq. (2.22). The critical angle is reached when $\theta_2 = 90^\circ$ so that $\sin \theta_2 = 1$. Substituting this value of θ_2 into Eq. (2.22) shows that the critical angle is determined from the condition

$$\sin \theta_c = \frac{n_2}{n_1} \quad (2.24)$$

Example 2.6 Consider the interface between a smooth biological tissue with $n_1 = 1.45$ and air for which $n_2 = 1.00$. What is the critical angle for light traveling in the tissue?

Solution: From Eq. (2.24), for light traveling in the tissue the critical angle is

$$\theta_c = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} 0.690 = 43.6^\circ$$

Thus any light ray traveling in the tissue that is incident on the tissue-air interface at an angle θ_1 with respect to the normal (as shown in Fig. 2.9) greater than 43.6° is totally reflected back into the tissue.

Example 2.7 A light ray traveling in air ($n_1 = 1.00$) is incident on a smooth, flat slab of crown glass, which has a refractive index $n_2 = 1.52$. If the incoming ray makes an angle of $\theta_1 = 30.0^\circ$ with respect to the normal, what is the angle of refraction θ_2 in the glass?

Solution: From Snell's law given by Eq. (2.24),

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{1.00}{1.52} \sin 30^\circ = 0.658 \times 0.50 = 0.329$$

Solving for θ_2 then yields $\theta_2 = \sin^{-1}(0.329) = 19.2^\circ$.

2.4.2 The Fresnel Equations

As noted in Sect. 2.2, one can consider unpolarized light as consisting of two orthogonal plane polarization components. For analyzing reflected and refracted light, one component can be chosen to lie in the plane of incidence (the plane containing the incident and reflected rays, which here is taken to be the yz -plane) and the other of which lies in a plane perpendicular to the plane of incidence (the xz -plane). For example, these can be the E_y and E_x components, respectively, of the electric field vector shown in Fig. 2.5. These then are designated as the *perpendicular polarization* (E_x) and the *parallel polarization* (E_y) components with maximum amplitudes E_{0x} and E_{0y} , respectively.

When an unpolarized light beam traveling in air impinges on a nonmetallic surface such as biological tissue, part of the beam (designated by E_{0r}) is reflected and part of the beam (designated by E_{0t}) is refracted and transmitted into the target material. The reflected beam is partially polarized and at a specific angle (known as *Brewster's angle*) the reflected light is completely perpendicularly polarized, so that $(E_{0r})_y = 0$. This condition holds when the angle of incidence is such that $\theta_1 + \theta_2 = 90^\circ$. The parallel component of the refracted beam is transmitted entirely into the target material, whereas the perpendicular component is only partially refracted. How much of the refracted light is polarized depends on the angle at which the light approaches the surface and on the material composition.

The amount of light of each polarization type that is reflected and refracted at a material interface can be calculated using a set of equations known as the *Fresnel equations*. These field-amplitude ratio equations are given in terms of the perpendicular and parallel *reflection coefficients* r_x and r_y , respectively, and the perpendicular and parallel *transmission coefficients* t_x and t_y , respectively. Given that E_{0i} , E_{0r} , and E_{0t} are the amplitudes of the incident, reflected, and transmitted waves, respectively, then

$$r_{\perp} = r_x = \left(\frac{E_{0r}}{E_{0i}} \right)_x = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad (2.25)$$

$$r_{\parallel} = r_y = \left(\frac{E_{0r}}{E_{0i}} \right)_y = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \quad (2.26)$$

$$t_{\perp} = t_x = \left(\frac{E_{0t}}{E_{0i}} \right)_x = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} \quad (2.27)$$

$$t_{\parallel} = t_y = \left(\frac{E_{0t}}{E_{0i}} \right)_y = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \quad (2.28)$$

If light is incident perpendicularly on the material interface, then the angles are $\theta_1 = \theta_2 = 0$. From Eqs. (2.25) and (2.26) it follows that the reflection coefficients are

$$r_x(\theta_1 = 0) = -r_y(\theta_2 = 0) = \frac{n_1 - n_2}{n_1 + n_2} \quad (2.29)$$

Similarly, for $\theta_1 = \theta_2 = 0$, the transmission coefficients are

$$t_x(\theta_1 = 0) = t_y(\theta_2 = 0) = \frac{2n_1}{n_1 + n_2} \quad (2.30)$$

Example 2.8 Consider the case when light traveling in air ($n_{\text{air}} = 1.00$) is incident perpendicularly on a smooth tissue sample that has a refractive index $n_{\text{tissue}} = 1.35$. What are the reflection and transmission coefficients?

Solution: From Eq. (2.29) with $n_1 = n_{\text{air}}$ and $n_2 = n_{\text{tissue}}$ it follows that the reflection coefficient is

$$r_y = -r_x = (1.35 - 1.00)/(1.35 + 1.00) = 0.149$$

and from Eq. (2.30) the transmission coefficient is

$$t_x = t_y = 2(1.00)/(1.35 + 1.00) = 0.851$$

The change in sign of the reflection coefficient r_x means that the field of the perpendicular component shifts by 180° upon reflection.

The field amplitude ratios can be used to calculate the *reflectance* R (the ratio of the reflected to the incident flux or power) and the *transmittance* T (the ratio of the transmitted to the incident flux or power). For linearly polarized light in which the vibrational plane of the incident light is perpendicular to the interface plane, the total reflectance and transmittance are

$$R_{\perp} = \left(\frac{E_{0r}}{E_{0i}} \right)_x^2 = R_x = r_x^2 \quad (2.31)$$

$$R_{\parallel} = \left(\frac{E_{0r}}{E_{0i}} \right)_y^2 = R_y = r_y^2 \quad (2.32)$$

$$T_{\perp} = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} \left(\frac{E_{0t}}{E_{0i}} \right)_x^2 = T_x = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} t_x^2 \quad (2.33)$$

$$T_{\parallel} = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} \left(\frac{E_{0t}}{E_{0i}} \right)_y^2 = T_y = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} t_y^2 \quad (2.34)$$

The expression for T is a bit more complex compared to R because the shape of the incident light beam changes upon entering the second material and the speeds at which energy is transported into and out of the interface are different.

If light is incident perpendicularly on the material interface, then substituting Eq. (2.29) into Eqs. (2.31) and (2.32) yields the following expression for the reflectance R

$$R = R_{\perp}(\theta_1 = 0) = R_{\parallel}(\theta_1 = 0) = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \quad (2.35)$$

and substituting Eq. (2.30) into Eqs. (2.33) and (2.34) yields the following expression for the transmittance T

$$T = T_{\perp}(\theta_2 = 0) = T_{\parallel}(\theta_2 = 0) = \frac{4n_1 n_2}{(n_1 + n_2)^2} \quad (2.36)$$

Example 2.9 Consider the case described in Example 2.8 in which light traveling in air ($n_{\text{air}} = 1.00$) is incident perpendicularly on a smooth tissue sample that has a refractive index $n_{\text{tissue}} = 1.35$. What are the reflectance and transmittance values?

Solution: From Eq. (2.35) and Example 2.8 the reflectance is

$$R = [(1.35 - 1.00)/(1.35 + 1.00)]^2 = (0.149)^2 = 0.022 \text{ or } 2.2 \%$$

From Eq. (2.36) the transmittance is

$$T = 4(1.00)(1.35)/(1.00 + 1.35)^2 = 0.978 \text{ or } 97.8 \%$$

Note that $R + T = 1.00$.

Example 2.10 Consider a plane wave that lies in the plane of incidence of an air-glass interface. What are the values of the reflection coefficients if this lightwave is incident at 30° on the interface? Let $n_{\text{air}} = 1.00$ and $n_{\text{glass}} = 1.50$.

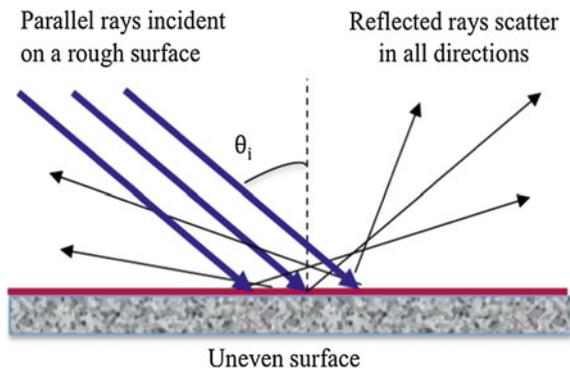
Solution: First from Snell's law as given by Eq. (2.22), it follows that $\theta_2 = 19.2^\circ$ (see Example 2.7). Substituting the values of the refractive indices and the angles into Eqs. (2.25) and (2.26) then yield $r_x = -0.241$ and $r_y = 0.158$. As noted in Example 2.8, the change in sign of the reflection coefficient r_x means that the field of the perpendicular component shifts by 180° upon reflection.

2.4.3 Diffuse Reflection

The amount of light that is reflected by an object into a certain direction depends greatly on the texture of the reflecting surface. Reflection off of smooth surfaces such as mirrors, polished glass, or crystalline materials is known as *specular reflection*. In specular reflection the surface imperfections are smaller than the wavelength of the incident light, and basically all of the incident light is reflected at a definite angle following Snell's law. *Diffuse reflection* results from the reflection of light off of surfaces that are microscopically rough, uneven, granular, powdered, or porous. This type of reflection tends to send light in all directions, as Fig. 2.11 shows. Because most surfaces in the real world are not smooth, most often incident light undergoes diffuse reflection. Note that Snell's law still holds at each incremental point of incidence of a light ray on an uneven surface.

Diffuse reflection also is the main mechanism that results in scattering of light within biological tissue, which is a *turbid medium* (or random medium) with many different types of intermingled materials that reflect light in all directions. Such diffusely scattered light can be used to probe and image spatial variations in

Fig. 2.11 Illustration of diffuse reflection from an uneven surface



macroscopic optical properties of biological tissues. This is the basis of elastic scattering spectroscopy, also known as diffuse reflectance spectroscopy, which is a non-invasive imaging technique for detecting changes in the physical properties of cells in biological tissues. Chapter 9 covers this topic in more detail.

2.5 Interference

All types of waves including lightwaves can interfere with each other if they have the same or nearly the same frequency. When two or more such lightwaves are present at the same time in some region, then the total wavefunction is the sum of the wavefunctions of each lightwave. Thus consider two monochromatic lightwaves of the same frequency with complex amplitudes $U_1(\mathbf{r}) = \sqrt{I_1} \exp(i\varphi_1)$ and $U_2(\mathbf{r}) = \sqrt{I_2} \exp(i\varphi_2)$, as defined in Eq. (2.2), where φ_1 and φ_2 are the phases of the two waves. Superimposing these two lightwaves yields another monochromatic lightwave of the same frequency

$$U(\mathbf{r}) = U_1(\mathbf{r}) + U_2(\mathbf{r}) = \sqrt{I_1} \exp(i\varphi_1) + \sqrt{I_2} \exp(i\varphi_2) \quad (2.37)$$

where for simplicity the explicit dependence on the position vector \mathbf{r} was omitted on the right-hand side. Then from Eq. (2.5) the intensities of the individual lightwaves are $I_1 = |U_1|^2$ and $I_2 = |U_2|^2$ and the intensity I of the composite lightwave is

$$\begin{aligned} I &= |U|^2 = |U_1 + U_2|^2 = |U_1|^2 + |U_2|^2 + U_1^* U_2 + U_1 U_2^* \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \varphi \end{aligned} \quad (2.38)$$

where the phase difference $\varphi = \varphi_1 - \varphi_2$.

The relationship in Eq. (2.38) is known as the *interference equation*. It shows that the intensity of the composite lightwave depends not only on the individual intensities of the constituent lightwaves, but also on the phase difference between the waves. If the constituent lightwaves have the same intensities, $I_1 = I_2 = I_0$, then

$$I = 2I_0(1 + \cos \varphi) \quad (2.39)$$

If the two lightwaves are in phase so that $\varphi = 0$, then $\cos \varphi = 1$ and $I = 4I_0$, which corresponds to *constructive interference*. If the two lightwaves are 180° out of phase, then $\cos \pi = -1$ and $I = 0$, which corresponds to *destructive interference*.

Example 2.11 Consider the case of two monochromatic interfering lightwaves. Suppose the intensities are such that $I_2 = I_1/4$. (a) If the phase difference $\varphi = 2\pi$, what is the intensity I of the composite lightwave? (b) What is the intensity of the composite lightwave if $\varphi = \pi$?

Solution: (a) The condition $\varphi = 2\pi$ means that $\cos \varphi = 1$, so that the two waves are in phase and interfere constructively. From Eq. (2.32) it follows that

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} = I_1 + I_1/4 + 2(I_1 I_1/4)^{1/2} = (9/4)I_1$$

(b) The condition $\varphi = \pi$ means that $\cos \varphi = -1$, so that the two waves are out of phase and interfere destructively. From Eq. (2.38) it follows that

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2} = I_1 + I_1/4 - 2(I_1 I_1/4)^{1/2} = I_1/4$$

2.6 Optical Coherence

As noted in Sect. 2.2, the complex envelope of a polychromatic waveform typically exists for a finite duration and thus has an associated finite optical frequency bandwidth. Thus, all light sources emit over a finite range of frequencies $\Delta\nu$ or wavelengths $\Delta\lambda$, which is referred to as a *spectral width* or *linewidth*. The spectral width most commonly is defined as the full width at half maximum (FWHM) of the spectral distribution from a light source about a central frequency ν_0 . Equivalently, the emission can be viewed as consisting of a set of finite wave trains. This leads to the concept of *optical coherence*.

The time interval over which the phase of a particular wave train is constant is known as the *coherence time* t_c . Thus the coherence time is the temporal interval over which the phase of a lightwave can be predicted accurately at a given point in space. If t_c is large, the wave has a high degree of temporal coherence. The corresponding spatial interval $l_c = ct_c$ is referred to as the *coherence length*. The importance of the coherence length is that it is the extent in space over which the wave is reasonably sinusoidal so that its phase can be determined precisely. An illustration of the coherence time is shown in Fig. 2.12 for a waveform consisting of random finite sinusoids.

The coherence length of a wave also can be expressed in terms of the linewidth $\Delta\lambda$ through the expression

$$l_c = \frac{4 \ln 2}{\pi} \frac{\lambda_0^2}{\Delta\lambda} \quad (2.40)$$

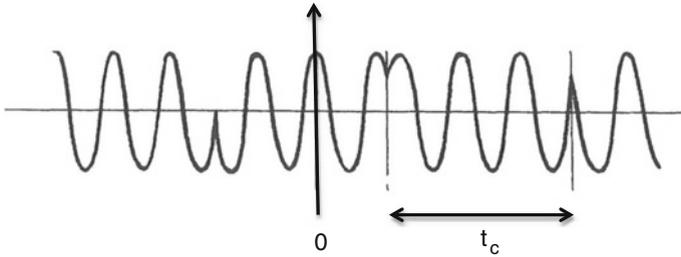


Fig. 2.12 Illustration of the coherence time in random finite sinusoids

Example 2.12 What are the coherence time and coherence length of a white light laser, which emits in the range 380 nm to 700 nm?

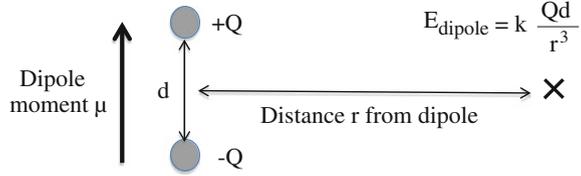
Solution: The wavelength bandwidth is $\Delta\lambda = 320$ nm with a center wavelength of 540 nm. Using the relationship in Eq. (2.40) then gives a coherence length $l_c = 804$ nm. The coherence time thus is $t_c = l_c/c = 2.68 \times 10^{-15}$ s = 2.68 fs.

2.7 Lightwave-Molecular Dipole Interaction

The interaction effects between light and biological material can be understood by considering the electromagnetic properties of light and of molecules. First consider the structure and electronic properties of molecules. The most common atoms found in biological molecules are carbon, hydrogen, nitrogen, and oxygen. The most abundant atom is oxygen because it is contained in proteins, nucleic acids, carbohydrates, fats, and water. A molecule can be viewed as consisting of positively charged atomic nuclei that are surrounded by negatively charged electrons. Chemical bonding in a molecule occurs because the different constituent atoms share common electron pairs. If the atoms are identical, for example, two oxygen atoms in an O_2 molecule, the common electron pair is usually located between the two atoms. In such a case the molecule is symmetrically charged and is called a *nonpolar molecule*.

If the atoms in a molecule are not identical, then the shared electron pair is displaced toward the atom that has a greater attraction for common electrons. This condition then creates what is known as a *dipole*, which is a pair of equal and opposite charges $+Q$ and $-Q$ separated by a distance d , as shown in Fig. 2.13. The dipole is described in terms of a vector parameter called the *dipole moment*, which has a magnitude μ given by

Fig. 2.13 A dipole is defined as two charges $+Q$ and $-Q$ separated by a distance d



$$\mu = Qd \quad (2.41)$$

A molecule that has a permanent dipole moment is called a *polar molecule*. Examples of polar molecules are water (H_2O), ammonia (NH_3), hydrogen chloride (HCl), and the amino acids arginine, lysine, and tyrosine. Symmetric nonpolar molecules such as oxygen, nitrogen, carbon dioxide (CO_2), methane (CH_4), ammonium (NH_4), carbon tetrachloride (CCl_4), and the amino acids glycine and tryptophan have no permanent dipole moments.

The interaction of the oscillating electric field of a lightwave and the dipole moment of a molecule is a key effect that can help describe light-tissue interactions. In addition, an energy exchange between two oscillating dipoles is used in fluorescence microscopy. When an external electric field interacts with either polar or nonpolar molecules, the field can distort the electron distribution around the molecule. In both types of molecules this action generates a temporary induced dipole moment μ_{ind} that is proportional to the electric field E . This induced dipole moment is given by

$$\mu_{\text{ind}} = \alpha E \quad (2.42)$$

where the parameter α is called the *polarizability* of the molecule. Thus when a molecule is subjected to the oscillating electric field of a lightwave, the total dipole moment μ_{T} is given by

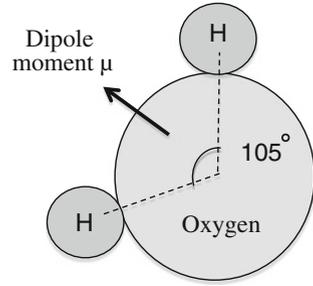
$$\mu_{\text{T}} = \mu + \mu_{\text{ind}} = Qd + \alpha E \quad (2.43)$$

As is shown in Fig. 2.13, the resultant electric field E_{dipole} at a distance r from the dipole in any direction has a magnitude

$$E_{\text{dipole}} = k \frac{Qd}{r^3} \quad (2.44)$$

where $k = 1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$.

Fig. 2.14 Representation of the dipole moment for a water molecule



Example 2.13 What is the dipole moment of a water molecule?

Solution: As Fig. 2.14 shows, water (H₂O) is an asymmetric molecule in which the hydrogen atoms are situated at a 105° angle relative to the center of the oxygen atom. This structure leads to a dipole moment in the symmetry plane with the dipole pointing toward the more positive hydrogen atoms. The magnitude of this dipole moment has been measured to be

$$\mu = 6.2 \times 10^{-30} \text{ C} \cdot \text{m}$$

2.8 Summary

Some optical phenomena can be explained using a wave theory whereas in other cases light behaves as though it is composed of miniature particles called photons. The wave nature explains how light travels through an optical fiber and how it can be coupled between two adjacent fibers, but the particle theory is needed to explain how optical sources generate light and how photodetectors change an optical signal into an electrical signal.

In free space a lightwave travels at a speed $c = 3 \times 10^8$ m/s, but it slows down by a factor $n > 1$ when entering a material, where the parameter n is the index of refraction (or refractive index) of the material. Example values of the refractive index for materials related to biophotonics are 1.00 for air, about 1.36 for many tissue materials, and between 1.45 and 1.50 for various glass compounds. Thus light travels at about 2.2×10^8 m/s in a biological tissue.

When a light ray encounters a boundary separating two media that have different refractive indices, part of the ray is reflected back into the first medium and the remainder is bent (or refracted) as it enters the second material. As will be discussed

in later chapters, these concepts play a major role in describing the amount of optical power that can be injected into a fiber and how lightwaves travel along a fiber.

Other important characteristics of lightwaves for a wide variety of biophotonics microscopy methods, spectroscopy techniques, and imaging modalities are the polarization of light, interference effects, and the properties of coherence.

2.9 Problems

- 2.1 Consider an electric field represented by the expression

$$\mathbf{E} = \left[100e^{i30^\circ} \mathbf{e}_x + 20e^{-i50^\circ} \mathbf{e}_y + 140e^{i210^\circ} \mathbf{e}_z \right]$$

Express this as a measurable electric field as described by Eq. (2.8) at a frequency of 100 MHz.

- 2.2 A particular plane wave is specified by $y = 8 \cos 2\pi(2t - 0.8z)$, where y is expressed in micrometers and the propagation constant is given in μm^{-1} . Find (a) the amplitude, (b) the wavelength, (c) the angular frequency, and (d) the displacement at time $t = 0$ and $z = 4 \mu\text{m}$. Answers: (a) $8 \mu\text{m}$; (b) $1.25 \mu\text{m}$; (c) 4π ; (d) $2.472 \mu\text{m}$.
- 2.3 Light traveling in air strikes a glass plate at an angle $\theta_1 = 57^\circ$, where θ_1 is measured between the incoming ray and the normal to the glass surface. Upon striking the glass, part of the beam is reflected and part is refracted. (a) If the refracted and reflected beams make an angle of 90° with each other, show that the refractive index of the glass is 1.540. (b) Show that the critical angle for this glass is 40.5° .
- 2.4 A point source of light is 12 cm below the surface of a large body of water ($n_{\text{water}} = 1.33$). What is the radius of the largest circle on the water surface through which the light can emerge from the water into air ($n_{\text{air}} = 1.00$)? Answer: 13.7 cm.
- 2.5 A right-angle prism (internal angles are 45, 45, and 90°) is immersed in alcohol ($n = 1.45$). What is the refractive index the prism must have if a ray that is incident normally on one of the short faces is to be totally reflected at the long face of the prism? Answer: 2.05.
- 2.6 Show that the critical angle at an interface between doped silica with $n_1 = 1.460$ and pure silica with $n_2 = 1.450$ is 83.3° .
- 2.7 As noted in Sect. 2.4.2, at a certain angle of incidence there is no reflected parallel beam, which is known as Brewster's law. This condition holds when the reflection coefficient r_y given by Eq. (2.26) is zero. (a) Using Snell's law from Eq. (2.22), the condition $n_1 \cos \theta_2 = n_2 \cos \theta_1$ when $r_y = 0$, and the relationship $\sin^2 \alpha + \cos^2 \alpha = 1$ for any angle α , show there is no parallel

reflection when $\tan \theta_1 = n_2/n_1$. (b) Show that $\theta_1 + \theta_2 = 90^\circ$ at the Brewster angle.

- 2.8 Consider the perpendicular and parallel *reflection coefficients* r_x and r_y , given by Eqs. (2.25) and (2.26), respectively. By using Snell's law from Eq. (2.22) and the identity $\sin(\alpha \pm \beta) = (\sin \alpha)(\cos \beta) \pm (\cos \alpha)(\sin \beta)$, eliminate the dependence on n_1 and n_2 to write r_x and r_y as functions of θ_1 and θ_2 only. That is, show that this yields

$$r_x = r_\perp = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}$$

$$r_y = r_\parallel = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}$$

- 2.9 Consider the case when light traveling in air ($n_{\text{air}} = 1.00$) is incident perpendicularly on a smooth tissue sample that has a refractive index $n_{\text{tissue}} = 1.50$. (a) Show that the reflection and transmission coefficients are 0.20 and 0.80, respectively. (b) Show that the reflectance and transmittance values are $R = 0.04$ and $T = 0.96$.
- 2.10 Show that the reflection coefficients r_x and r_y , given by Eqs. (2.25) and (2.26) can be written in terms of only the incident angle θ_1 and the refractive index ratio $n_{21} = n_2/n_1$ as

$$r_\perp = r_x = \frac{\cos \theta_1 - (n_{21}^2 - \sin^2 \theta_1)^{1/2}}{\cos \theta_1 + (n_{21}^2 - \sin^2 \theta_1)^{1/2}}$$

$$r_\parallel = r_y = \frac{n_{21}^2 \cos \theta_1 - (n_{21}^2 - \sin^2 \theta_1)^{1/2}}{n_{21}^2 \cos \theta_1 + (n_{21}^2 - \sin^2 \theta_1)^{1/2}}$$

- 2.11 Consider a plane wave that lies in the plane of incidence of an air-glass interface. (a) Show that the values of the reflection coefficients are $r_x = -0.303$ and $r_y = 0.092$ if this lightwave is incident at 45° on the interface. (b) Show that the values of the transmission coefficients are $t_x = 0.697$ and $t_y = 0.728$. Let $n_{\text{air}} = 1.00$ and $n_{\text{glass}} = 1.50$.
- 2.12 Consider the case of two monochromatic interfering lightwaves. Suppose the intensities are such that $I_2 = I_1$. (a) If the phase difference $\phi = 2\pi$, show that the intensity I of the composite lightwave is $4I_1$. (b) Show that the intensity of the composite lightwave is zero when $\phi = \pi$.
- 2.13 The frequency stability given by $\Delta\nu/\nu$ can be used to indicate the spectral purity of a light source. Consider a Hg^{198} low-pressure lamp that emits at a wavelength 546.078 nm and has a spectral bandwidth $\Delta\nu = 1000$ MHz. (a) Show that the coherence time is 1 ns. (b) Show that the coherence length is 29.9 cm. (c) Show that the frequency stability is 1.82×10^{-6} . Thus the frequency stability is about two parts per million.

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