This book is intended for graduate students and researchers who have interest in functional analysis, in general and summability theory, in particular. It describes several useful topics in summability theory along with applications. The book consists of nine chapters and is organized as follows:

Chapter “An Introduction to Summability Methods” is introductory in nature. This chapter focuses on the historical development of summability theory right from Cauchy’s concept to till date. Summability methods developed from the two basic processes—$T$-process and $\phi$-process—have also been discussed in this chapter.

Chapter “Some Topics in Summability Theory” deals with the study of some classical and modern summability methods, and the connections among them. In fact, results concerning summability by weighted mean method, the $(M, \lambda_n)$ method, the Abel method, and the Euler method are presented. Then the sequence space $K_r$, $r \geq 1$ being a fixed integer, is defined and a Steinhaus type theorem is proved. The space $A_r$ is then studied in the context of sequences of 0’s and 1’s. Further, the core of a sequence is studied, an improvement of a result of Sherbakhoff is proved and a very simple proof of Knopp’s core theorem is then deduced. Finally, a study of the matrix class $(\ell, \ell)$ is presented.

Chapter “Summability and Convergence Using Ideals” is concentrated on different concepts of summability and convergence using the notions of ideals and essentially presents the basic developments of these notions. This chapter starts with the first notion of ideal convergence and goes on to discuss in detail how the notion has been extended over the years from single sequences to double sequences and nets. This chapter also discusses some of the most recent advances made in this area, in particular applications of ideal convergence to the theory of convergence of sequences of functions. Some problems are also listed which still remain open.

In chapter “Convergence Acceleration and Improvement by Regular Matrices”, a new, non-classical convergence acceleration concept is studied and compared with the well-known classical convergence acceleration concept. It is shown that the new concept allows to compare the speeds of convergence for a larger set of
sequences than the classical convergence acceleration concept. Also, regular matrix methods that improve and accelerate the convergence of sequences and series are studied. The results described in this chapter are further applied to increase the order of approximation of Fourier expansions and Zygmund means of Fourier expansions in certain Banach spaces.

Chapter “On Summability, Multipliability and Integrability” deals with the study of summability and multipliability of vector families indexed by well-ordered sets of real numbers. These concepts generalize the classical notions of convergence of infinite series and products. The studies are also motivated by problems in integration theory of functions of one variable. In particular, the chapter describes the relation between integrability and product integrability on the one side, and summability and multipliability on the other side. Applications in the theory of differential equations with impulses and distributional differential equations are presented, and concrete examples are introduced to illustrate the derived theoretical results.

In chapter “Multi-dimensional Summability Theory and Continuous Wavelet Transform”, the connection between multi-dimensional summability theory and continuous wavelet transform is investigated. Two types of \( \theta \)-summability of Fourier transforms are considered, the circular and rectangular summability. Norm and almost everywhere convergence of the \( \theta \)-means are shown for both types. The inverse wavelet transform is traced back to summability means of Fourier transforms. Using the results concerning the summability of Fourier transforms, norm and almost everywhere convergence of the inversion formula are obtained for functions from the \( L_p \) and Wiener amalgam spaces.

In chapter “Absolute Riesz and Related Summability Methods”, several theorems dealing with the absolute Riesz summability of infinite series have been given. Additionally, some theorems which are generalization of these theorems to absolute matrix summability have been given by using several different sequences.

Chapter “Some Applications of Summability Theory” discusses some applications of summability theory in sequence spaces defined by certain functions and summability methods, which are related to statistical convergence and their applications. Several topological and geometric properties of the sequence spaces, such as the \( (\beta) \)-property, Banach–Saks property, Kadec–Klee property, Opial property, etc., are also discussed. Then some applications of summability theory to Tauberian theorems, both in ordinary sense and in statistical sense are discussed. Finally, some results related to the Tauberian theory characterized by weighted summability methods such as, the generalized de la Vallée–Poussin method, generalized Nörlund-Cesàro, etc., are presented.

Chapter “Degree of Approximation of Functions Through Summability Methods” first discusses a result on the degree of approximation of functions belonging to the \( \text{Lip}(\alpha, r) \) class, using almost Riesz summability method of its infinite Fourier series. Then a result concerning the degree of approximation of the conjugate of a function \( f \) belonging to \( \text{Lip}(\xi(t), r) \) class by Euler \( (E, q) \) summability of conjugate series of its Fourier series has been established. The results discussed in this chapter generalize several existing results.
The editors are very much thankful to all learned referees for their valuable and helpful suggestions, and friends for encouragement and moral support.

Guwahati, India
Bloomington, USA
February 2016
Current Topics in Summability Theory and Applications
Dutta, H.; E. Rhoades, B. (Eds.)
2016, XIII, 431 p., Hardcover
ISBN: 978-981-10-0912-9