Chapter 2
Hydrogeological Parameters Calculation

Hydrogeological parameters of aquifer are the essential and crucial basic data in the designing and construction progress of geotechnical engineering and groundwater dewatering, which are directly related to the reliability of these parameters.

There are three types of hydrogeological parameters that reflect the hydraulic properties of aquifer, as follows:

The first type is the parameters that represent the properties of aquifer. Hydraulic conductivity ($K$) and transmissibility ($T$) represent the aquifer’s permeability. The water reserving capacity is represented by the specific yield ($\mu$) in unconfined aquifer and storage coefficient ($\mu'$) in confined aquifer. The rate of water head conduction is represented by groundwater table conductivity in unconfined aquifer and pressure transitivity in confined aquifer, which are both $a$.

The second type parameters show the interaction of aquifers after dewatering, including leakage coefficient ($\sigma$) and leakage factor ($B$).

The third type parameters refer to the capacity of water exchange between aquifers and the external environment. It includes parameters that refer to the receiving capacity of external recharge and the degree of water loss. The former includes infiltration coefficients ($\alpha$) of precipitation, river and irrigation, and the latter mainly for coefficient of phreatic evaporation.

There are many methods in hydrogeological parameter calculation. Laboratory tests and pumping and injection tests are the most common methods in geotechnical engineering design and construction. With the data of long-term groundwater observation, hydrogeological parameters can also be back calculated by analytical and numerical solutions and optimization method.

2.1 Hydrogeological Tests

In geotechnical engineering, hydrogeological in situ tests include pumping test, recharge test, infiltration test, injection test, water pressure test, connection test, groundwater flow direction and velocity test et al. These tests are used to calculate
hydrogeological parameters and find out the hydraulic connection between different aquifers and between groundwater and surface water. Hydrogeological and geotechnical engineering design and construction conditions should be considered when selecting test method.

### 2.1.1 Pumping Test

Pumping test is one of the most common geotechnical engineering investigation methods in finding out the permeability and calculating the parameters of aquifers. Different types of pumping tests are applied in different engineering programs according to their objectives and hydrogeological conditions.

Pumping tests can be divided into three types according to the operation and the number of wells, shown in Table 2.1.

#### 2.1.1.1 Objective, Task, and Types of Pumping Test

1. **Objective and task of pumping test**

   Pumping test is on the basis of well flow theory. During this test, groundwater is pumped out through the main well and the change of flow rate in observation wells is measured. Meanwhile, the variation of state and distribution of seepage field in the time and space is also measured. Pumping test is aimed at finding out the hydrogeological condition of engineering construction field, quantifying the water amount of pumping wells and aquifers, calculating the hydrogeological parameters and finally providing a basis for groundwater solution program.

   The main tasks of pumping test are as follows:

   1. Measure the variation of drawdown with the change of discharge of wells or drilling holes, then calculate the unit inflow and estimate the maximum yielding water of the aquifer.
   2. Determine the hydrogeological parameters of aquifer, including hydraulic conductivity, transmissibility, specific yield, storage coefficient, pressure transitivity, leakage factor, and influence radius et al.
   3. Measure the shape of cone of depression, and its expanding progress.
   4. Find out the hydraulic connection between different aquifers and between groundwater and surface water.

<table>
<thead>
<tr>
<th>Table 2.1 Pumping test classification and applied range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
</tr>
<tr>
<td>Simple pumping test in drillings or exploration wells</td>
</tr>
<tr>
<td>Pumping test without observation well</td>
</tr>
<tr>
<td>Pumping test with observation wells</td>
</tr>
</tbody>
</table>
(5) Determine the aquifer boundary condition, including its location and properties.

(6) Conduct pumping simulation to provide necessary data for well-group design, which includes determining reasonable distance and diameter of wells, drawdown and the flux of water.

2. Types of pumping test

According to different classification principles, pumping tests can be classified as follows:

(1) Steady flow pumping test and unsteady flow pumping test, according to groundwater flow state on the basis of well flow theory.

   (a) Steady flow pumping test is an early common method, which requires the test must last for a long time after meeting the stable flow and drawdown. Steady flow theory is used in calculation of aquifer’s parameters, such as hydraulic conductivity, influence radius, etc. However, groundwater flows are mostly unsteady in nature; only the areas which have abundant and stable water supply can form a relatively steady seepage field. Therefore, its application is limited.

   (b) Unsteady flow pumping test has been used universally since 1970s in our country. It requires the water discharge or water table to remain constant. Generally, it is the water discharge flux that remains constant or staged constant and the water table changes with time. The duration of the unsteady flow pumping test is determined by s-lgf curve. If the aquifer has an infinite recharge boundary, then pumping can be terminated after an inflection point appears on the curve. While if the aquifer has a constant head boundary, impermeable boundary, or leakage recharge, there are generally two inflection points.

   The results of unsteady flow theories and formulas can be more accurate than steady flow theories, and so the former has a wider application. It can calculate more parameters, such as transmissibility, specific yield, storage coefficient, pressure transmission coefficient, leakage factor and so on. Also it can determine the simple boundary conditions and take full advantages of all the information provided throughout the whole pumping process. However, the calculation is much more complex that needs higher technical standards for observation. Generally, for the early unsteady stage and later steady stage, relevant formulas are applied, respectively, to calculate the parameters in different stages.

(2) Single well pumping test and multiwells pumping test, depending on whether there is observation well(s).

   (a) Single well pumping test is the pumping test that only has one pumping well, which also known as main well, and has no observation well. It is simple and less expensive, but not very accurate, which makes it suit for preliminary investigation stage. The main well is usually set at the place
where is rich in groundwater. Aquifer’s water abundance, permeability, and the relationship between pumping discharge and drawdown can be found through single well pumping test.

(b) Multiwells pumping test is the pumping test that has a pumping well and one or more observation well(s). It has a wider application. It can determine not only the hydraulic conductivity and pumping discharge, but also the anisotropy of hydraulic conductivity, the radius and shape of the depression cone, the width of supply area, the reasonable well spacing, interference coefficient, and the hydraulic connection between groundwater and surface water. Besides, seepage velocity test also can be taken during the pumping test. This kind of pumping test costs a lot, but the results of which are more accurate. Therefore, it is more used in detailed investigation stage than preliminary investigation stage. In the area which has the value of water supply, at least one group of multiwells pumping test should be taken.

(3) Fully penetrating well pumping test and partially penetrating well pumping test according to the type of pumping well.

Generally, fully penetrating well pumping test is the primary choice, for its comprehensive well flow theory. Only in the condition that the aquifer is thick and homogeneous, or in the specialized study of filter’s effective length, the partially penetrating well pumping test is adopted.

(4) Layering pumping test and combination pumping test according to aquifer’s condition involved in test.

(a) Layering pumping test is the pumping test that conducted the test for separate aquifers to determine each aquifer’s hydrogeological characters and parameters.

(b) Combination pumping test is the pumping test that tests several layers of aquifers in one pumping well. The results reflect the average value of those aquifers’ hydrogeological parameters. In the condition that the layers are not numerous, the approximate value of each aquifer’s parameters can be determined by recharging the well layer by layer and conducting combination pumping test accordingly.

3. Arrangement of main well and observation wells

Main well should be considered arranging in the following locations: the main water source aquifer, aquifer with large thickness and abundant water, the possible connection part between surface water and groundwater, fault or karst-concentrated zone, the representative control region, such as boundaries of different sections and aquifers.

The design of observation wells in the plane and profile layout depends on the test tasks, accuracy, feature size, aquifer’s character, as well as data processing and calculation methods and other factors. If only to eliminate “well loss” or “water jump” effects, just one observation well near the pumping well need to be arranged. If to obtain reliable hydrogeological parameters, one to four rows of observation wells can be arranged according to aquifer’s character and groundwater flow condition, shown in Tables 2.2, 2.3 and Fig. 2.1.
Table 2.2  Distance between main well and observation wells

<table>
<thead>
<tr>
<th>Aquifer’s characters</th>
<th>Hydraulic conductivity (m/day)</th>
<th>Groundwater type</th>
<th>Distance (m)</th>
<th>Influence radius (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>First well</td>
<td>Second well</td>
</tr>
<tr>
<td>Hard with developing fissures</td>
<td>&gt;60</td>
<td>Confined</td>
<td>15–20</td>
<td>30–40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unconfined</td>
<td>10–15</td>
<td>20–30</td>
</tr>
<tr>
<td>Hard with slight developing fissures</td>
<td>60–20</td>
<td>Confined</td>
<td>6–8</td>
<td>10–15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unconfined</td>
<td>5–7</td>
<td>8–12</td>
</tr>
<tr>
<td>Pure cobble, gravel and coarse-medium sand</td>
<td>&gt;60</td>
<td>Confined</td>
<td>8–10</td>
<td>15–20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unconfined</td>
<td>4–6</td>
<td>10–15</td>
</tr>
<tr>
<td>Cobble and gravel with fine particles</td>
<td>60–20</td>
<td>Confined</td>
<td>5–7</td>
<td>8–12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unconfined</td>
<td>3–5</td>
<td>6–8</td>
</tr>
<tr>
<td>Anisotropic sand</td>
<td>20–5</td>
<td>Confined</td>
<td>3–5</td>
<td>6–8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unconfined</td>
<td>2–3</td>
<td>4–6</td>
</tr>
</tbody>
</table>

Table 2.3  Arrangement of observation lines

<table>
<thead>
<tr>
<th>Aquifer’s characters</th>
<th>Arrangement of observation lines</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homogeneous and isotropic</td>
<td>Small water gradient One line that is perpendicular to groundwater flow direction</td>
<td>Figure 2.1 (1)</td>
</tr>
<tr>
<td>Large water gradient Two lines that are perpendicular and parallel to groundwater flow direction</td>
<td>Figure 2.1 (2)</td>
<td></td>
</tr>
<tr>
<td>Heterogeneous and anisotropic</td>
<td>Small water gradient Two lines that are perpendicular to groundwater flow direction and one line that is parallel to groundwater flow direction</td>
<td>Figure 2.1 (3)</td>
</tr>
<tr>
<td>Large water gradient Two lines that are perpendicular to groundwater flow direction and two lines that are parallel to groundwater flow direction</td>
<td>Figure 2.1 (4)</td>
<td></td>
</tr>
</tbody>
</table>

The number, distance, and depth of observation wells depend on the test task, accuracy, and pumping type. There should be no less than three observation wells arranged in one line to figure out the shape of the depression cone. For parameter
calculations, only two observation wells in one line are needed for a steady pumping test, and usually three wells for an unsteady pumping test to take full use of all observation data. If the test task is to find out the hydraulic connection or boundary characters, the observation wells should not be less than two.

The distance between observation wells should be small near the main well and became larger far from the main well. The distance between the main well and the closest observation well depends on the permeability of aquifer and the drawdown, which can be several meters to 20 m on the principle of in favor of controlling the shape of depression cone and avoiding the turbulence and 3D flow around the observation well. For unsteady flow pumping tests, observation wells should be evenly distributed on a logarithmic axis and ensure the observation of the initial water table changes. The empirical distance data of observation wells can be found in the relevant handbooks.

The depth of observation wells generally is required to be 5–10 m deep in tested aquifers, except for thin aquifers. If the aquifer is heterogeneous, the depth and the filter’s position of the observation wells should be in accord with the main well.

2.1.1.2 Technical Requirements for Pumping Tests

1. Steady flow pumping test
   (1) Drawdown
   Generally, at least three drawdowns should be made to determine the relation between water discharge and drawdown (Q-s curve), which can judge the correctness of tests and indicate the water discharge. While, only one drawdown is enough if the maximum drawdown is <1 m in the following conditions: the requirement to test accuracy is not very high, the test is taken in a secondary aquifer, the water discharge is too small (<0.1 L/s m), and the pumping equipment has limitations. If the Q-s relation has been determined and the correctness of pumping tests can be ensured, only two time tests need to be taken out. This is because that there are no more than two unknown coefficients in Q-s relation and the type of Q-s curve can be determined by two times pumping tests using the coefficient \( n \) in \( Q_2/Q_1 = \sqrt{S_2/S_1} \), where \( n < 1 \) is unmoral, \( n = 1 \) is linear type, \( 1 < n < 2 \) is exponential type, \( n = 2 \) is parabolic type, \( n > 2 \) is logarithmic type. Although this method can save one test workload process, it has poor reliability.

   The maximum drawdown is mainly determined by the test purpose. When calculating the parameters, the drawdown should be smaller to avoid turbulence and 3D flow. When calculating for groundwater resource evaluation and dewatering, the drawdown should be able to extrapolate to the design requirements. When determining the boundary properties and hydraulic connection, the drawdown should be large enough to fully expose the problems, for the impermeability of some layers is related to the waterhead difference on both sides of boundary. The maximum
drawdown \((s_{\text{max}})\) can be \(1/3-1/2\) of aquifer’s thickness in unconfined aquifer, and can be the distance between static water table and the aquifer’s roof. The rest drawdowns can be evenly distributed \((s_1 = s_{\text{max}}/3, s_2 = s_{\text{max}}/2)\), which is convenient for drawing \(Q-s\) curve. The minimum drawdown and the difference of each two drawdowns usually is no <1 m. In geotechnical engineering, construction design, or groundwater dewatering design, formal pumping test conducted three times, and the difference between each drawdown is more appropriate than 1 m.

(2) Stable duration time

Stable duration time refers to the time that the pumping test lasts after the seepage field reaches approximate stabilization. The time from the beginning of pumping to steady seepage field depends on the groundwater type, aquifer’s parameters, boundary and recharge conditions, and drawdown value. This time is longer when it is in unconfined or leakage aquifers, or in the condition of poor water recharge or large drawdown. The duration time is different in different investigation stage, test purpose, and aquifer condition. Generally, it should meet the requirement of test reliability. It is easier to find out the slight and trending change and the false stability that is caused by temporary recharge.

The stable duration time does not need to be long when calculating parameters, usually <24 h. In other conditions usually it will be 48–72 h. No matter what the test purpose is, it should not <2–4 h of the farthest observation well.

Generally, the stable stage is reaching when the variation of water table in pumping well is <1 % of drawdown. If the drawdown is small, the limitation is 3–5 cm. When pumped with air compressor, the variation of water table in main well allows up to 20–30, and 2–3 cm in observation well, but no trending change is allowed. The variation of water discharge should not exceed 5 %.

(3) Water table and water discharge

Natural stable water table should be observed before pumping. Water table should be observed hourly. The water table that does not change in 2 h or only changes 2 cm in 4 h is the stable water table. If the natural water table fluctuates, the average value is desirable as the natural stable water table, or eliminating the interference effects.

During pumping, the water table and water discharge should be measured at the same time. The interval time for observation should be close first and loose afterward, for example 5–10 min first and 15–30 min afterward, which should e according to the specific requirements.

When pumping is stopped or broken off, recovery water table should be measured with the same interval time. The standards for stable water table judgment is the same with above. If there is difference between natural and recovery stable water table, the drawdowns should be amended by the weighted arithmetic average of the difference regarding the time.
2. Unsteady flow pumping test

Unsteady flow pumping test can be divided into constant-flow test and constant-drawdown test. The former is much used in practice. The latter is used when in artesian well or modeling dewatering or groundwater mining.

(1) Water discharge and water table

The requirements for water discharge and water table measuring is the same with steady flow pumping test. It should be especially noticed that the flow or water table should be constant from the beginning to the end of pumping.

During pumping, the water table and water discharge should be measured at the same time. When pumping is stopped or broken off, recovery water table should be measured. The interval time in unsteady flow pumping test should be smaller than it in steady flow pumping test, especially in first 10–30 min. For example, it could be observed at 1, 2, 3, 4, 6, 8, 10, 15, 20, 25, 30 min, then at every 30 min.

(2) Stable duration time

The stable duration time for unsteady flow pumping test also depends on test tasks and purposes, hydrogeological conditions, test type, water discharge, and calculation method. It has big differences in different pumping test, which also has no uniform regulations. For the parameter calculation alone, the duration time usually does not exceed 48–96 h in our country. However, the variation is 6–600 h according to global data, and 48–96 h is the most choice.

If the aquifer is a borderless confined aquifer, curve-matching and linear graphic methods are in common use. The former only requires the early pumping data, while the latter needs pumping data for two pairs of log-periods. These mean that the total pumping time needs to be three pairs of log-period, which is 1000 min, about 17 h. So the pumping usually lasts for 1–2 days. If there are more than one observation wells, all of them should meet the above requirements. If the water discharge is ladder-like distributed, the last ladder should also continue to meet the above requirements.

In leakage flow, if inflected point method is used in parameter calculation, the duration time should be long enough to judge the maximum drawdown. If linear graphic method is used, the duration time can be shorter. If the data of steady stage is used, it also should meet the requirements for steady flow.

If the test purpose is to determine the boundary location and character, the duration time should be long enough to finish the job. For example, if there is constant head boundary, steady stage should be reached; for linear impermeable boundary, the second line segment in s-lg$t$ curve should occur and the pumping generally lasts more than 100 min. Some impermeable boundary can be permeable when waterhead difference is high enough, so the duration time should ensure that the water table drawdown near boundary value reaches a predetermined value.

The test duration could be long in following circumstances: using large group wells pumping test to determine the boundary property, using the hydrogeological numerical method to calculate the parameters of heterogeneous area, and modeling water supply and unwatering.
3. Measurement of water temperature and weather temperature
Water temperature and weather temperature should be measured every 2–4 h. Other
groundwater physical properties should be recorded if necessary.

4. Water sampling
At the end of pumping test, water samples should be taken for full chemical
analysis, bacteria analysis, or other special analysis. The sample for chemical
analysis should be no <2000 mL and analyzed in one week after sampling. As for
bacteria analysis, 500 mL sample is needed, which should be sealed with wax and
analysed within 6 h after sampling. Special analysis should be taken according to
requirements.

2.1.1.3 Test Equipment and Appliances

Test equipment mainly refers to pumping equipment, such as water pump. Test
appliances include flowmeter, water table indicator, water thermometer, and timer.
Besides, drainage should be constructed and communication tools should be set.

1. Pumping equipment

There are many types of pumping equipment, in which the horizontal centrifugal
pump, deep-well pump, and air compressor.

(1) Horizontal centrifugal pump

Centrifugal pump has a simple structure and small size, which is easy to handle
and adjust the flux. It can pump large quantities of water that are even mixed with a
mass of sand, but the pumping head is small, only 5–9 m. It is commonly used in
shallow well pipes and volume water or group wells.

(2) Deep-well pump

The main advantage of deep-well pump is that it can pump deep water evenly.
However it is hard to adjust the flux, and not suit for water with high sand content.
It can be used in wells that the water table is more than 10 m and with less sand.

(3) Air compressor

Air compressor has simple structure and can be easily handling and can pump
water with a mass of sand. It is not affected by the slight curve of pipe well.
However, the efficiency of the air compressor is only 15–25 %, which leads too
much wasted power. It is not able to pump evenly and stably, and cannot run a long
time. Sometime it cannot meet the engineering needs, so it is not suitable for
large-scale pumping test work.

However, it is usually used in drilling washing. In order to save cost and time, it
is also used in pumping test after washing work.

(4) Other pump types

There are many other pump types, which can be chosen according to specific
conditions. For example, axial flow pump is suitable for volume and shallow water
while jet pump and rob pump are suitable for the opposite condition; water hammer
pump is suitable for the condition that small flux and less energy. Submersible pump is suitable for deep water and low sand content.

In short, the choice of pump depends on static groundwater table, designed outlet water, dynamic water table, well diameter, sand content, and other requirements. Generally, the pumping water should be more than the designed outlet water.

2. Utensils for flow measurement

(1) Weir box

Weir box is the most common flowmeter. A triangle weir box is suitable for small flow as shown in Fig. 2.2, and a trapezoid weir box is suitable for mass flow. Usually, weir box is made of steel, but in the group well pumping test it can be made of brick or wood for the amount of temporary weir box is too much.

Flow calculation formulas for triangle weir box are as follows:

when

\[ H = 0.021 - 0.200 \text{ m} : \quad Q = 1.4H^{\frac{5}{2}} \]  

(2.1)

when

\[ H = 0.301 - 0.350 \text{ m} : \quad Q = 1.343H^{\frac{5}{2}} \]  

(2.2)

when

\[ H = 0.201 - 0.300 \text{ m} : \quad Q = \frac{1}{2}(1.4 + 1.343)H^{\frac{5}{2}} \]

where \( H \) is the water head, in m, which is measured by steel ruler; the ruler is 0.8–1.0 m far from overflow plate, and its zero point and the crest of weir are in the same horizontal line; \( Q \) is the water flow, \( \text{m}^3/\text{s} \).

Fig. 2.2  Triangle weir box
(2) Orifice flowmeter

The principle of orifice flowmeter is to set a thin-walled hole with a certain diameter near the end of the outlet pipe and measure the waterhead of the two sides of orifice or of the position at a certain distance from the orifice if the flowmeter is at the end of water pipe. The waterhead is only dependent on flow velocity, if the diameters of water pipe and orifice are determined. So the quantity of flow can be calculated from that waterhead. There are two types of orifice flowmeters as shown in Figs. 2.3 and 2.4. The orifice flowmeter is portable and accurate, but not suitable for air compressors.

The following formula can be used to calculate flow in unit time:

\[
Q = 0.0125Ed^2\sqrt{\frac{H}{1000}} \quad \text{(water temperature: } 1-20^\circ\text{C)} \quad (2.3)
\]

where \(Q\) is the quantity of flow, \(m^3/h\); \(d\) is the diameter of orifice, \(mm\); \(H\) is the waterhead difference, \(mm\); \(E\) is the coefficient determined by the diameters of

Fig. 2.3 Installation of orifice flowmeter

Fig. 2.4 Segment orifice flowmeter
orifice and water pipe, as well as the connection method of orifice. If the flange plate is used, then:

$$E = \frac{k}{\sqrt{1 - B^4}} = 0.606 + 1.25(B - 0.41)^2$$  \hspace{1cm} (2.4)

$$B = \frac{d}{D}$$  \hspace{1cm} (2.5)

where $k$ is the drainage coefficient.

(3) YKS-1 impeller orifice instantaneous flowmeter

The flow velocity, which is used to calculate the flow, can be measured by the impeller speed. The impeller speed is measured by electronic device. This type of flowmeter is small, light, and easy to use, which however is also not suitable for air compressors.

(4) Water meter

It is used together with centrifugal pump or deep-well pump. The water should be clear and there should be no sand or mud in it to keep the water meter work normally. The measurement error is ±2–3 %.

3. Water table indicator

The common water table indicators include electronic ones and float-type ones. The former ones indicate the water table by an ammeter, a bulb, or a loudspeaker. Recently, the pressure indicators and capacitor-based indicators are getting recognized. All the above-mentioned types belong to contact measurement. The no-contact ultrasonic water gauge is a new type with bright prospects.

2.1.1.4 Comprehensive Analysis of Pumping Test Data

1. Site data analysis

During the pumping test, the water table and flow should be observed and recorded carefully. Besides, following diagrams should be drawn to know the test progress, find out anomaly, and lay a foundation for indoor data statistic.

(1) Steady flow pumping test

(a) Draw water discharge versus time and drawdown versus time curves for main well.

The normal curve is drawn in Fig. 2.5. At the beginning of pumping, the values of drawdown and water discharge are all big and unstable. Over time, they become stable. According to the changing trend of these curves, the start and end of the stable phase can be determined reasonably.

(b) Draw drawdown versus time curves for observation wells if there is, such as $s_1$ curves for OW1, OW2 et al. in Fig. 2.5.

(c) Draw flow versus drawdown curves ($Q = f(s)$ curves).

Draw the point that represents a certain flow under certain second stable drawdown. Connect all the points to get the flow versus drawdown curve, as shown in Fig. 2.6. The meanings of these curves are:
2.1 Hydrogeological Tests

Fig. 2.5 Water discharge versus time and drawdown versus time curves

Fig. 2.6 $Q = f(s)$ curves

Curve 1—the curve for confined groundwater.
Curve 2—the curve for unconfined groundwater, confined–unconfined groundwater, or confined groundwater that is influenced by a 3D flow or turbulent flow, or by the resistance of well wall and filter.
Curve 3—the curve for groundwater with deficient water supply or the flow cross-section is blocked during pumping.
Curve 4—if the pump faucet is at the same position with filter, this curve indicates that pumping is affected by a 3D flow or turbulent flow, which makes it correct; if the pump faucet is above filter, this curve indicates that the results of pump test are wrong and the test should be redone.
Curve 5—this curve refers that under a certain drawdown $s$, the pump flow $Q$ will be constant; this curve occurs when the drawdown is too large.
The $Q = f(s)$ curves can be used to understand the hydraulic characteristics of aquifer and yield capacity of dilled hole, to predict the maximum yield quantity, and to verify whether the results of pumping test are correct or not.

(d) Draw unit pumping-flow versus drawdown curves ($q = f(s)$ curves)

Connect the points that refer to the drawdown with a certain unit pumping-flow of the same drill hole will get the unit pumping-flow versus drawdown curve, as shown in Fig. 2.7. The meanings of these curves are the same with Fig. 2.6.

(e) Draw water table recovery curves

The method is same with the draw drawdown—time curves.

If the pumping test is normal, the curve should be rising linearly at first, and then the rise becomes slow, and finally turns horizontal. The wavy curves indicate that the observation results are wrong.

The water table recovery curves can be used to estimate the groundwater type and permeability performance of the stratum. If the water table recovers quickly, it may be the confined aquifer or strong permeable stratum. Conversely, if the water table recovers slowly, it is usually an unconfined aquifer or aquitard.

(2) Unsteady flow pumping test

(a) Draw drawdown versus time curves ($s$-$t$ curves) with the same method referred above. If the unsteady flow pumping test time is short, then magnify the time scale of abscissa. If the data include main well and observation wells, the $s$-$t$ curves of them can be drawn in the same figure.

(b) Draw drawdown versus logarithmic time curves ($s$-$\log t$ curves).

(c) Draw double logarithmic curves of drawdown versus time ($\log s$-$\log t$ curves).

(d) Draw double logarithmic curves of observation well drawdown versus distance to main well ($\log s$-$\log r$ curves).

(e) Draw water table recovery curves with logarithmic time ($s'$-$\log(1 + t_p/t')$ curves), where $s'$ is the remaining drawdown, m; $t_p$ is the time from the start of pumping to the end of pumping, min; $t'$ is the time of water table recovery, starting from the end of pumping, min.

Fig. 2.7 $q = f(s)$ curves
2. Indoor data analysis

(1) Draw a comprehensive result figure of pumping test, which includes: geological drilling histogram, technical structure graph of drill hole construction, \(Q-t\), \(s-t\) curves, \(Q-s\) curves, \(q-s\) curves, table of pumping test results, table of water quality analysis, and drill hole layout plan.

(2) Calculate the hydrogeology parameters of aquifer: based on the data of steady flow pumping test and/or unsteady flow pumping test, calculate the hydrogeology parameters with multiple method and fill the summary sheet.

(3) Estimate the maximum flow of the drill hole.

(4) Write the work summary of pumping test which includes: the purposes and principles of pumping test, test method, test process, major achievements, abnormal phenomena during test and their solutions, quality analysis, and conclusions and so on.

2.1.2 Water Pressure Test

2.1.2.1 Test Purposes

The purposes for water pressure test are: exploring the fissured properties and permeability of rock and soil layers; calculating the parameters, such as unit water sucking amount \((\omega)\); providing bases for relevant design.

2.1.2.2 Test Types

Water pressure tests can be divided into following types:

1. Multistage water pressure test, synthesized water pressure test, and one-stage water pressure test according to test stages.
2. One-point water pressure test, three-point water pressure test, and multipoint water pressure test according to the number of flux-pressure relationship point.
3. Low-pressure test and high-pressure test according to pressure degree.
4. Water column pressure test, gravity flow water pressure test, and mechanical water pressure test according to pressure source, which are shown in Figs. 2.8, 2.9 and 2.10.

2.1.2.3 Main Parameters

1. Steady flux
   It refers to the steady flux that is pressed into the field under certain hydrogeological conditions and pressure.
Keep the pressure constant and measure the flux every 10 min. When it meets one of following criterions, the water flow can be considered as stable, according to the Code of Water Pressure Test in Borehole for Water Resources and Hydropower Engineering (SL31-2003):

(1) The difference between maximum and minimum value of four consecutive readings is <10% of final reading $Q_L$, which is $Q_{\text{max}} - Q_{\text{min}} < Q_L/10$.
(2) The flow is reducing gradually till four consecutive readings are all <0.5 L/min, that is $0.5 \text{ L/min} > Q_1 > Q_2 > Q_3 > Q_4$.
(3) The flow is increasing gradually till four consecutive readings are no longer increase.

In simple water pressure test, it can be lower than above standards.
2. Pressure stage and pressure value

(1) Total test pressure

Total pressure for water pressure test refers to the average pressure that acts on the test section. It is measured with the height of water, that is "1 m water height" = 0.98 N/cm$^2$ = 9.8 kPa ≈ 1 N/cm$^2$. The total pressure can be calculated by the following formula:

$$P = P_b + P_z + P_s$$

where $P$ is the total test pressure, N/cm$^2$; $P_b$ is the reading of pressure gauge, N/cm$^2$; $P_z$ is the pressure of water column, N/cm$^2$; $P_s$ is the pressure loss that in single-pipe column plunger from pressure gauge to the bottom of plunger, N/cm$^2$.

(2) Zero line (0–0 line) and pressure loss

Water column pressure refers to the pressure of water from zero line to the middle of pressure gauge. Therefore, the zero line (0–0) for pressure calculation should be determined first. There are three conditions, as follows:

(a) When the water table is below the test section, the 0–0 line is the horizontal line through 1/2 of the test section, shown in Fig. 2.11.

(b) When the water table is in the test section, the 0–0 line is the horizontal line through 1/2 of the test section that is above the water table, shown in Fig. 2.12.

(c) When the water table is above the test section, the 0–0 line is the water table, as shown in Fig. 2.13.

The pressure is measured from water table, which should be determined before test.

Standards for stable groundwater table are as follows:

If the natural groundwater table is not affected by outer factors, or changes little, it can be determined by the average value of 2–3 times observation.
If the groundwater table changes, the stable water table is observed by following steps. At the initial observation stage, the interval time should be short, and then observed every 10 min. When the water table is no longer changed, or the rate of change of the three consecutive readings of water table is <1 cm/min (that is 10 cm/10 min), the last measured water table could be considered as stable water table.
If the initial water table is higher than the stable water table in drilling, it will gradually decrease to stable, shown in Fig. 2.14. The stable standard is: \( H_3 - H_2 \leq 10 \) cm, \( H_2 - H_1 \leq 10 \) cm and the decreasing rate is <1 cm/min.

If the initial water table is lower than the stable water table in drilling, it will gradually increase to stable, shown in Fig. 2.15. The stable standard is: \( H_1 - H_2 \leq 10 \) cm, \( H_2 - H_3 \leq 10 \) cm and the decreasing rate is <1 cm/min.

The pressure loss \( P_s \) can occur in following the conditions: uniform diameter, sudden change of diameter (to bigger or smaller).

(a) Pressure loss in uniform diameter

The water pressure loss when flowing in uniform diameter can be calculated as:

\[
\Delta P_{s1} = 0.49 \lambda \cdot \frac{l}{d} \cdot \frac{v^2}{g}
\]

where \( \Delta P_{s1} \) is the pressure loss in the uniform diameter pipe, N/cm²; \( l \) is the length of pipe, m; \( d \) is the inner diameter of pipe, m; \( v \) is the velocity of water, m/s; \( g \) is the

![Fig. 2.14 Duration curve of water table decreasing](image)

![Fig. 2.15 Duration curve of water table increasing](image)
acceleration of gravity, 9.81 m/s$^2$; $\lambda$ is the friction coefficient, 0.02–0.03 for steel pipe.

(b) Pressure loss in sudden expansion diameter pipe

$$\Delta P_{s2} = 0.49 \cdot \frac{(v_1 - v_2)^2}{g}$$

where $\Delta P_{s2}$ is the pressure loss in the sudden expansion diameter pipe, N/cm$^2$; $v_1$ is the velocity of water in small diameter segment, m/s; $v_2$ is the velocity of water in large diameter segment, m/s.

(c) Pressure loss in sudden reduction diameter pipe

$$\Delta P_{s3} = 0.49\alpha \cdot \frac{v^2}{g}$$

where $\Delta P_{s3}$ is the pressure loss in the sudden reduction diameter pipe, N/cm$^2$; $v$ is the velocity of water in small diameter segment, m/s; $g$ is the acceleration of gravity, 9.81 m/s$^2$; $\alpha$ is the resistance coefficient, see Table 2.4.

In engineering investigation, usually choose one-point water pressure test, and the water pressure is 30 N/cm$^2$.

(3) Length of test segment
The length of test segment usually is 5 m.
If the rock core is intact ($\omega = 0.01$ L min$^{-2}$ m$^{-2}$), it can be lengthen, but not longer than 10 m. For tectonic fracture zones, karst segments, sand and gravel layers with strong permeability, the length should be determined by specific condition. If the length of rock core is <20 cm, it can be included into the test segment. For tilt test drilling, the test length is the actual length of the drilling.

### Table 2.4 Resistance coefficients

<table>
<thead>
<tr>
<th>$d_2/d_1$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.5</td>
<td>0.42</td>
<td>0.33</td>
<td>0.25</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Notes $d_1$ is the larger diameter and $d_2$ is the smaller diameter.

2.1.2.4 Test Data Compilation

1. Test data reliability judgment
The reliability of one-point water pressure test data depends on the quality of drilling and pressure process. Ensure the data reliability by following test programs: drill with clean water → wash the test drilling → set plunger → observe stable water table → press water, keep the pressure constant and read $Q$ → error check → loosen and pull out the plunger.
2. Test outcome and application

(1) Unit water sucking amount $\omega$

The major outcome of water pressure test is unit water sucking mount ($\omega$), which can be calculated as:

$$\omega = \frac{Q}{LP}$$  \hspace{1cm} (2.10)

where $\omega$ is the unit water sucking amount, L/min m$^2$; $Q$ is the steady packing flow in drill hole, L/min; $L$ is the length of test section, m; $P$ is the total applied pressure, N/cm$^2$.

The decimal of $\omega$ is limited to 0.01.

The $\omega$ got from water pressure test usually less than the real value, so it unsafe to use it in engineering design.

(2) Estimate the hydraulic conductivity $K$ according to $\omega$

If the distance from the lower end of test section to the aquifer’s bottom is larger than the length of test section, the aquifer’s hydraulic conductivity $K$ can be estimated as:

$$K = 0.527\omega \lg \frac{0.66L}{r}$$  \hspace{1cm} (2.11)

where $K$ is the hydraulic conductivity, m/day; $L$ is the length of test section, m; $r$ is the radius of drill hole or filter, m; $\omega$ is the unit water sucking amount, L/min m$^2$.

If the distance from the lower end of test section to the aquifer’s bottom is less than the length of test section, the aquifer’s hydraulic conductivity $K$ can be estimated as:

$$K = 0.527\omega \lg \frac{1.32L}{r}$$  \hspace{1cm} (2.12)

The meanings of symbols are the same with above.

(3) Relations between unit water sucking amount and rock fracture

The relations between unit water sucking amount and rock fracture coefficient are shown in Table 2.5.

<table>
<thead>
<tr>
<th>Unit water sucking amount (L/min m$^2$)</th>
<th>Fracture coefficient</th>
<th>Rock evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.001</td>
<td>&lt;0.2</td>
<td>Complete</td>
</tr>
<tr>
<td>0.001–0.01</td>
<td>0.2–0.4</td>
<td>Relatively complete</td>
</tr>
<tr>
<td>0.01–0.1</td>
<td>0.4–0.6</td>
<td>Some fracture</td>
</tr>
<tr>
<td>0.1–0.5</td>
<td>0.6–0.8</td>
<td>More fracture</td>
</tr>
<tr>
<td>&gt;0.5</td>
<td>&gt;0.8</td>
<td>Cracked</td>
</tr>
</tbody>
</table>
2.1.2.5 Test Equipment and Demands

1. Pipeline: inner pipe use steel and outer pipe use rubber.
2. Water supply equipment: if the water pressure test is for geological investigation, it’s better to adopt gravity flow type water pressure test. The flow of the pump should be no <100 L/min under 150 N/cm² pressure, and the flow pressure should be stable. The pump should be with an agile and reliable valve.
3. Pressure gauges: the pressure gauge should be qualified with an accuracy no <2.5%; the working pressure is usually in the 1/3–1/4 measuring range; when lightly knock the pressure gauge during working, the pointer change should be no more than 2 % of the measuring range; the pointer can return to zero when stop loading.
5. Water table measurement: measurement bell and plumb; electronic water table indicator.

2.1.3 Water Injection Test

Drilling water injection test is a simple measurement method for aquifer permeability in field, the principles of which is similar with pumping test.

Drilling water injection test is usually used in following conditions: (1) the water table too deep to be pumped; (2) the rock and soil layer is dry.

The testing apparatus are shown in Fig. 2.16.

Inject water in drilling continuously and constantly to form constant water table. The duration of stable time depends on test purposes and requirements, which is

![Fig. 2.16 Schematic of drilling water injection test](image-url)
usually 2–8 h. This kind of test can be used in calculating the hydraulic conductivity \( K \) and unit water sucking amount \( \omega \).

According to engineering experience, in horizontal aquifer with huge thickness, \( K \) can be calculated by following formulas:

When \( l/r \leq 4 \):

\[
K = \frac{0.08Q}{rs\sqrt{\frac{l}{2r} + \frac{1}{4}}}
\]

(2.13)

when \( l/r > 4 \):

\[
K = \frac{0.366Q}{ls\lg\frac{2l}{r}}
\]

(2.14)

where \( l \) is the length of filter, m; \( Q \) is the constant injection water amount, \( m^3/\text{day} \); \( s \) is the waterhead in drilling, m; \( r \) is the radius of drilling or filter, m.

The hydraulic conductivity that calculated above is 15–20 % less than it calculated by pumping test formulas.

The \( K_1 \) for single layer and \( K \) for double layers can be calculated in two tests. Since \( KL = K_1l_1 + K_2l_2 \), so that \( K_2 = (KL - K_1l_1)/l_2 \).

If the water table is deep and the medium is uniform, and in the condition that \( 50 < h/r < 200 \) and the water height in drilling is higher than 1 m, \( K \) can be calculated by Eq. (2.15):

\[
K = 0.423\frac{Q}{h^2\lg\frac{2h}{r}}
\]

(2.15)

where \( h \) is the water height in drilling, m.

The error of \( K \) that calculated by Eq. (2.15) is <10 %.

### 2.1.4 Infiltration Test

Infiltration test is a simple method that is taken in trial pit to measure the hydraulic conductivity of vadose zone in field. The most common methods are trial pit method, single-loop method, and double-loop method as shown in Table 2.6.

### 2.1.4.1 Test Method

1. Trial pit method

Trial pit method is the test that conducted in trial pit, which is 30–50 m deep. The shape can be square, whose length of side is 30 cm, or can be round with the
37.75 cm diameter. The water table is 3–5 m beneath the pit bottom. Lay a layer of gravel sand with a thickness of 2 cm. Control the flow continuous balancing and the water a constant thickness (10 cm) since test starting. When the injection water amount is stable and then lasts for 2–4 h, the test can be completed.

If it is the coarse sand, gravel or cobble layer that tested, the water thickness \(Z\) should be kept in 2–5 cm. When \((H_k + Z + l)/l \approx 1\), hydraulic conductivity can be calculated by following formula: \(K = \frac{Q}{F} = \nu\), where \(H_k\) is the capillary pressure head (in m), can be found in Table 2.7; \(l\) is the water penetration depth when test completes, which can be determined after excavation or by water content analysis.

The sketch map of the trial pit method is shown in Fig. 2.17. This method is usually used in sand, which is not much influenced by capillary pressure. As for clay, the result is usually on high side.

2. Single-loop method
Embed an iron loop that has a height of 20 cm, a diameter of 37.70 cm and an area of 1000 cm² on the pit bottom. At the start of the test, control the water column more than 10 cm high in the loop by Mariotte bottle. When the infiltration amount

<table>
<thead>
<tr>
<th>Test method</th>
<th>Test sketch map</th>
<th>Advantages and disadvantages</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial pit method</td>
<td><img src="image1" alt="Trial pit method sketch map" /></td>
<td>1. Simple device; 2. Effected by lateral penetration, leading to low accuracy</td>
<td>When there is anti-seepage measurement on the wall of round pit, (F = \pi r^2). When there is no anti-seepage measurement, (F = \pi r (r + 2Z)), where (r) is the radius of pit bottom, (Z) is the thickness of water in pit</td>
</tr>
<tr>
<td>Single-loop method</td>
<td><img src="image2" alt="Single-loop method sketch map" /></td>
<td>1. Simple device; 2. Lateral penetration is not considered, leading to low accuracy</td>
<td></td>
</tr>
<tr>
<td>Double-loop method</td>
<td><img src="image3" alt="Double-loop method sketch map" /></td>
<td>1. Simple device; 2. Effect of lateral penetration is excluded, so that the results are accurate</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6 Infiltration test method (from Handbook of Engineering Geology 1992)
Q is constant, the test is complete and the infiltration rate can be calculated by Eq. (2.16), which is equal to the hydraulic conductivity of soil layer.

\[ v = \frac{Q}{F} = K \]  

(2.16)

In addition, infiltration rate can be calculated by following steps: measured the infiltration amount in a certain period of time (e.g. 30 min); compute the average infiltration rate value; draw the infiltration rate duration curve, shown as Fig. 2.18.

---

**Table 2.7** Capillary pressure head \( (H_k) \) of different soil type (from Handbook of Engineering Geology 1992)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>( H_k ) (m)</th>
<th>Soil type</th>
<th>( H_k ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty clay (SC)</td>
<td>1.0</td>
<td>Fine clayed sand (SM)</td>
<td>0.3</td>
</tr>
<tr>
<td>Clay (CLS)</td>
<td>0.8</td>
<td>Silty sand</td>
<td>0.2</td>
</tr>
<tr>
<td>Clayed silt (CL)</td>
<td>0.6</td>
<td>Fine sand</td>
<td>0.1</td>
</tr>
<tr>
<td>Sandy silt (MLS)</td>
<td>0.4</td>
<td>Medium sand</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Notes** The \( H_k \) values in above table are always lower

---

**Fig. 2.17** Sketch map for trial pit method applied in clay

**Fig. 2.18** Seepage velocity duration curve in infiltration test
It can be found that the infiltration rate decreases with time and tend to be a constant, which can be considered as hydraulic conductivity.

3. Double-loop method

Embed two iron loops on the bottom of the trial pit. The outer one has a diameter of 0.5 m and inner one is 0.25 m. Keep the water table the same in both loops by Mariotte bottle. Calculate hydraulic conductivity according to the data that is obtained from the inner loop. The water in inner loop only infiltrates vertically, which can exclude the effect of lateral infiltration and makes the results more accurate.

2.1.4.2 Parameters Calculation

When the infiltration water amount is tend to be constant, the hydraulic conductivity can be calculated by following equation, in which the capillary pressure has been considered.

\[
K = \frac{Ql}{F(H_k + Z + l)}
\] (2.17)

where \(Q\) is the constant infiltration amount, cm\(^3\)/min; \(F\) is the infiltration area of inner loop, cm\(^2\); \(Z\) is the thickness of water in inner loop, cm; \(H_k\) is the capillary pressure head, cm; \(l\) is the infiltration depth when test complete, cm.

If the infiltration can be steady for a very long time, \(K\) can be calculated by Eq. (2.18):

\[
K = \frac{V_1}{Ft_1a_1} \left[ a_1 + \ln(1 + a_1) \right]
\] (2.18)

\[
a_1 = \frac{\ln(1 + a_1) - \frac{t_2}{t_1} \ln \left( 1 - \frac{a_1V_2}{V_1} \right)}{1 - \frac{t_1}{t_2} V_1}
\] (2.19)

where \(V_1, V_2\) are the total infiltration amount during \(t_1\) and \(t_2\), m\(^3\); \(t_1\) and \(t_2\) are the cumulative time; \(F\) is the infiltration area of inner loop, cm\(^2\); \(a_1\) is the alternative factor, calculated by trial method.

2.1.4.3 Test Data Compilation

1. Draw the layout of pit plane position.
2. Draw the hydrogeological cross-sectional view and the test device.
3. Draw the penetration rate duration curve.
4. Calculate the hydraulic conductivity.
5. Organize the original recording sheets.
2.2 Measurement of Groundwater Table, Flow Direction and Seepage Velocity

2.2.1 Measurement of Groundwater Table

Groundwater table is the naturally relative stable water table, which means it has no obvious up or down trend during a period. Water table can be measured by water table indicator, which should be chosen according to engineering properties, construction conditions and measurement accuracy.

2.2.1.1 Measurement Bell

Measurement bell is a common tool used in borehole and observation hole. It is a metal cylinder with a diameter of 25–40 mm and length of 50–80 mm. The top is closed, connecting with a measuring line, and its accuracy is 1–2 cm. It can make a sound after contacting with water, which may hardly to identify when water table is too low.

2.2.1.2 Battery Water Table Indicator

Battery water table indicator consists of electrodes, wires, μA ampere meter and dry battery. The accuracy is about 1 cm. It is convenient to use, making it available for all boreholes with any diameter or depth.

2.2.1.3 Auto Water Table Recorder

It adopts the clock spring principle so that it can automatically record water table. It can be used in wells with a diameter larger than 89 mm. The accuracy is about ±1.5 cm.

2.2.2 Measurement of Groundwater Flow Direction

Groundwater flow direction can be measured by three-point method, shown in Fig. 2.19. Set three boreholes to create a near equilateral triangle. Measure the water table in these boreholes, and then draw the water table contour map. The direction that is perpendicular to contours and point to the descent side is the groundwater flow direction.
Besides, groundwater flow direction also can be measured by artificial radioisotopes method. Trickle the radioactive tracer into a single well, and then measure the concentration of the tracer around the well. The direction with the highest concentration of tracer is the flow direction.

### 2.2.3 Measurement of Seepage Velocity

#### 2.2.3.1 Hydraulic Gradient Method

Measure the hydraulic gradient between adjacent water table contours on water table contour map. Groundwater flow velocity can be calculated by Darcy’s law:

\[ v = KI \]  

where \( v \) is the seepage velocity, m/day; \( K \) is the hydraulic conductivity of aquifer, m/day; and \( I \) is the hydraulic gradient.

#### 2.2.3.2 Indicator or Tracer Method

Indicators and radioactive tracer can be used in situ seepage velocity measurement. Here are some requirements: borehole should be set in a typical position of the tested aquifer; groundwater around borehole is steady laminar flow.

Set tracer injection hole and observation hole along flow direction line. Two assistant observation holes can be set to prevent tracer or indicator bypassing the main observation hole, shown as Fig. 2.20. The distance between Delivery hole (DH) and Observation hole (OH) depends on the permeability of soil or rock, shown in Table 2.8.

Draw indicator concentration versus time curve and use the time corresponding to the peak or average concentration to calculate the actual flow velocity:
where \( u \) is the average actual groundwater flow velocity, m/h; \( l \) is the distance between injection hole and observation hole, m; \( t \) is the time mentioned above, s.

Seepage velocity can be calculated by following formula:

\[
\nu = nu
\]

where \( n \) is the porosity.

The seepage velocity also can be tested in situ by single well artificial radioisotopes method. The common radioactive tracers include \(^3\text{H}, \, ^{51}\text{Cr}, \, ^{60}\text{Co},\) \(^{82}\text{Br}, \, ^{131}\text{I}, \, ^{137}\text{Cs}\) et al. According to the tracer’s concentration versus time curve, the average actual flow velocity \( u \) can be calculated by following formula, shown in Fig. 2.21.

\[
U = \frac{V}{st} \ln \left( \frac{C_0}{C} \right)
\]

### Table 2.8 Distances between injection hole and observation hole

<table>
<thead>
<tr>
<th>Rock or soil type</th>
<th>Distance (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silt</td>
<td>1–2</td>
</tr>
<tr>
<td>Fine sand</td>
<td>2–5</td>
</tr>
<tr>
<td>Coarse sand with gravel</td>
<td>5–15</td>
</tr>
<tr>
<td>Fractured rock</td>
<td>10–15</td>
</tr>
<tr>
<td>Limestone high karst degree</td>
<td>&gt;50</td>
</tr>
</tbody>
</table>
where $C_0$ and $C$ are tracer’s concentration of time $T = 0$ and $T = t$ respectively, μg/L; $t$ is the time duration of observation, h; $s$ is the vertical cross section area of isolated part that water pass through, m$^2$; $V$ is water volume in isolated part, m$^3$.

Single well method is shown in Fig. 2.21. Other measurement methods are shown in Table 2.9.

### 2.3 Capillary Rise Height Determination

Methods for capillary rise height determination are as follows:

#### 2.3.1 Direct Observation Method

This method could apply to silty soil and clayed soil, which has a large capillary rise height. A boundary of wet and dry soils can be easily observed in trial pit, the distance from which to water table is the capillary rise height.

#### 2.3.2 Water Content Distribution Curve Method

##### 2.3.2.1 Plastic Limit Test Method

This method could apply to silty soil and clayed soil. Soil samples are taken every 15–20 cm above the water table, and their water content and plastic limit are tested. Then two curves that water content and plastic limit against depth are obtained, whose intersection depth to water table is the capillary rise height.
### Table 2.9 The measurement methods for actual groundwater flow velocity (from Handbook of Engineering Geology 1992)

<table>
<thead>
<tr>
<th>Method</th>
<th>Principle</th>
<th>Indicator</th>
<th>Distance between IH and OH</th>
<th>Amount</th>
<th>Operation</th>
<th>Identification</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Chemical method</strong></td>
<td>Determine the time that salt appear and its concentration change through chemical analysis</td>
<td>NaCl</td>
<td>&gt;5 m</td>
<td>10–15 kg</td>
<td>There are two ways for tracer delivery: (1) Put the tracer cylinder, which has a conic valve at bottom, to predetermined depth and open the valve to make the solution spill into the delivery hole; (2) Bury the cylinder, which has a taphole that connects a rubber hose to ground, into predetermined depth, then pull the solution in through rubber hose. Sample the water from observation holes by sample collector that no more than 50 cm² cross-sectional area.</td>
<td>Determine the concentration of chlorine by titration. Titrating the solution until the color of water sample turns into brownish red and not fade. Method for NO₂⁻ concentration test: first, compound testing powder; then dissolve 0.5 g testing powder into 50 mL water sample; compare its color with standard solution. The method for NO₃⁻ concentration test is the same with above. But the result is the total concentration of NO₂⁻ and NO₃⁻. This result subtract the concentration of NO₂⁻ will get the concentration of NO₃⁻. Use fluorescent colorimeter to confirm the existence of dyestuff and determine its concentration. Or pull self-confected solution with different concentration into colorimeter, and compare its color with water sample at regular time intervals.</td>
<td>(1) Of all these methods, the best one is taking the nitrate as indicator. Its advantages include: high sensitivity, less disturbance, simple and easy operation, good repeatability, low cost, and easy to get. However, the NaNO₂ is unstable and has certain toxicity. By comparison, NaNO₃ has low toxicity and low sensitivity, and needs for rectify of NO₃⁻ concentration. (2) The preparation method of testing powder for NO₂⁻ and NO₃⁻ concentration test: porphyrize 100 g BaSO₄, 75 g citric acid, 4 g sulfanilic acid, and 2 g α-naphthylamine; then mix them uniformly, store the mixture in brown bottle and keep dry. For NO₃⁻ concentration test, mix another 10 g MnSO₄ and 4 g zinc powder. The NO₂⁻ and NO₃⁻ concentration of standard colorimetric solution is 0.001 mg/L. (3) When taking the nitrate as indicator, it is must to pre-sample one bottle of testing water, for comparison in case there is an exception during test.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CaCl₂</td>
<td>3–5 m</td>
<td>5–10 kg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NH₄Cl</td>
<td>&lt;3 m</td>
<td>3–5 kg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NaNO₂</td>
<td>&gt;5 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>NaNO₃</td>
<td>&gt;5 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Colorimetric method</strong></td>
<td>Use the concentration change of reagent color to determine the time that is taken by the reagent pass through the two holes</td>
<td>Alkaline water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fluorescent yellow, fluorescent red, eosin</td>
<td>Every 5 cm</td>
<td>Incompact stratum 1–5 g</td>
<td>Use fluorescent colorimeter to confirm the existence of dyestuff and determine its concentration. Or pull self-confected solution with different concentration into colorimeter, and compare its color with water sample at regular time intervals.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Weak acid water</td>
<td></td>
<td>Karst fracture stratum 1–10 g</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Congo red, methylene blue, aniline blue</td>
<td>10–30 g</td>
<td>10–40 g</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Method</td>
<td>Principle</td>
<td>Indicator</td>
<td>Operation</td>
<td>Identification</td>
<td>Notes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>---------------</td>
<td>---------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrolytic method</td>
<td>Determine the electrolyte movement and distribution condition in observation hole by special electrical test equipment</td>
<td>Ammonium chloride</td>
<td>The delivery method is the same with above. A special electrode, which is isolated from the casing pipe, is put into the observation hole. The electric circuit is from the electrode and through battery, ammeter, regulating rheostat, finally to the casing pipe of delivery hole</td>
<td>Measure the current intensity at regular time intervals to confirm the existence of tracer and determine its concentration</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electricize method</td>
<td>Dissolve the salt into groundwater. The flow of saline water makes the electric field change near the delivery hole</td>
<td>Salt</td>
<td>Put the salt into the delivery hole at the position where correspond to the aquifer. Put the electrode A into drill hole and insert the electrode B in the ground where is 20H away from drill hole (H is the bury depth of aquifer), and make MN = (2–4) H (electrode N is set on upstream)</td>
<td>The equipotential line that observed on ground changes from circle to ellipse. The flow direction of groundwater is parallel to the ellipse’s long axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atomic method of radioactive tracer</td>
<td>Determine the time that the tracer takes to pass through the observation hole by special equipment</td>
<td>Tritium (H$^3$), iodine (I$^{131}$), bromine (Br$^{82}$), sodium (Na$^{23}$), sulfur (S$^{35}$) et al.</td>
<td>If the velocity of groundwater is $10^{-2}$–$10^{-3}$ cm/s, then the delivery distance is 50–1 m</td>
<td>The intensity of radioactive source usually is 10–15 mci</td>
<td>Deliver the tracer into center hole, then record the radioactivity degree at regular intervals in observation holes by detect device, which takes Cr-M counter tube as probe and scaler</td>
<td>Take the projection value of maximum radioactive intensity on time axis as the time that the tracer passing through the observation hole</td>
<td></td>
</tr>
</tbody>
</table>
2.3.2.2 Maximum Molecular Water Absorption Method

This method could apply to sand, that is high column method for medium-coarse sand and water absorption medium method for fine sand. Soil samples are taken every 15–20 cm above the water table, and their maximum water absorption and natural water content are tested. Then two curves that natural water content and maximum water absorption against depth are obtained, whose intersection depth to water table is the capillary rise height.

2.4 Pore Water Pressure Determination

Pore water pressure of saturated soil foundation changes during foundation treatment and base construction. It's crucial to measure pore water pressure due to its big effect on soil deformation and stability.

Engineering projects that pore water pressure measurement should be taken and its aims are shown in Table 2.10.

2.4.1 Pore Water Pressure Gauge and Measurement Methods

Pore water pressure gauges should be chosen in accordance with measurement aims, period, and soil permeability. Their accuracy, sensitivity, and range should meet the needs. Table 2.11 shows the instrument types and their application conditions.

2.4.2 Calculation Formulas

Calculation formulas for different types of pore water pressure gauges are shown in Table 2.12.

<table>
<thead>
<tr>
<th>Table 2.10 Engineering projects and measurement aims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engineering projects</td>
</tr>
<tr>
<td>Preloading foundation</td>
</tr>
<tr>
<td>Dynamic consolidation</td>
</tr>
<tr>
<td>Prefabricated pile construction</td>
</tr>
<tr>
<td>Engineering dewatering</td>
</tr>
<tr>
<td>Landslide</td>
</tr>
</tbody>
</table>
### Table 2.11 Pore water pressure gauges and their application conditions

<table>
<thead>
<tr>
<th>Pore water pressure gauge</th>
<th>Application conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riser pipe pressure gauge (open type)</td>
<td>Hydraulic conductivity $&gt;10^{-4}$ cm</td>
</tr>
<tr>
<td>Water pressed pressure gauge (hydraulic type)</td>
<td>Low hydraulic conductivity</td>
</tr>
<tr>
<td></td>
<td>Accuracy $&gt;2$ kPa</td>
</tr>
<tr>
<td></td>
<td>Period of measurement $&lt;1$ month</td>
</tr>
<tr>
<td>Electric pressure gauge</td>
<td>All kinds of soil</td>
</tr>
<tr>
<td>Vibration string type</td>
<td>Accuracy $&lt;2$ kPa</td>
</tr>
<tr>
<td></td>
<td>Period of measurement $&gt;1$ month</td>
</tr>
<tr>
<td>Differential transformer type</td>
<td>All kinds of soil</td>
</tr>
<tr>
<td></td>
<td>Accuracy $&lt;2$ kPa</td>
</tr>
<tr>
<td></td>
<td>Period of measurement $&gt;1$ month</td>
</tr>
<tr>
<td>Resistance type</td>
<td>All kinds of soil</td>
</tr>
<tr>
<td></td>
<td>Accuracy $&lt;2$ kPa</td>
</tr>
<tr>
<td></td>
<td>Period of measurement $&lt;1$ month</td>
</tr>
<tr>
<td>Pneumatic pressure gauge (air pressure type)</td>
<td>All kinds of soil</td>
</tr>
<tr>
<td></td>
<td>Accuracy $&gt;10$ kPa</td>
</tr>
<tr>
<td></td>
<td>Period of measurement $&lt;1$ month</td>
</tr>
<tr>
<td>Piezo-cone static penetration apparatus</td>
<td>All kinds of soil</td>
</tr>
<tr>
<td></td>
<td>Short period of measurement</td>
</tr>
</tbody>
</table>

### Table 2.12 Calculation formulas for different types of pore water pressure gauges

<table>
<thead>
<tr>
<th>Type</th>
<th>Calculation equation</th>
<th>Symbols</th>
</tr>
</thead>
</table>
| Hydraulic type        | $u = P_a + \rho_w h$ | $u$—Pore water pressure, kPa  
                        |                      | $P_a$—Gauge reading, kPa  
                        |                      | $h$—Distance between pore water pressure gauge and the base table of piezometer, cm  
                        |                      | $\rho_w$—Water density, g/cm³ |
| Air pressure type     | $u = c + aP_a$       | $c, a$—Calibration constants of pressure gauge  
                        |                      | $K$—Sensitivity coefficient of pore water pressure gauge, measured in kPa/Hz² for vibration string type and kPa/με for resistor type  
| Vibration string type | $u = K(f_0^2 - f^2)$ | $f_0$—Frequency of pore water pressure gauge at zero, Hz  
                        |                      | $f$—Frequency of pore water pressure gauge after pressed, Hz  
| Resistor type         | $u = K(\varepsilon_1 - \varepsilon_0)$ | $\varepsilon_1$—Reading of pore water pressure gauge after pressed, με  
                        |                      | $\varepsilon_0$—Reading of pore water pressure gauge before pressed, με  
| Differential resistor type | $u = (A - A_0)K$ | $A$—Initial reading, V  
                        |                      | $A_0$—Measured value, V  
                        |                      | $K$—Calibration coefficient, kPa/V  

2.5 Hydrogeological Parameters Calculation in Steady Flow Pumping Test

Hydraulic conductivity $K$ and conductivity coefficient $T$ can be calculated by steady flow formulas with test data. Hydraulic conductivity represents the aquifer’s permeability, which equal to seepage velocity when hydraulic gradient $I = 1$. Hydraulic conductivity relates to properties of both aquifer and liquid. Conductivity coefficient $T = KM$ ($M$ represents aquifer’s thickness).

2.5.1 Calculation of Hydraulic Conductivity

When single well pumping test reaching to a steady state, the drawdown $s$ and water discharge $Q$ can be measured. Generally, three group data, which are $s_1$ and $Q_1$, $s_2$ and $Q_2$, $s_3$ and $Q_3$ should be got.

2.5.1.1 Dupuit Formula

Dupuit formula can be applied to hydraulic conductivity calculation for homogeneous, isopachous, and infinite horizontal extent aquifer with a fully penetrating pumping well.

Confined aquifer:

$$K = \frac{Q}{2\pi M s_w} \ln \frac{R}{r_w}$$  \hspace{1cm} (2.23)

Unconfined aquifer:

$$K = \frac{Q}{\pi (H_0^2 - h_w^2)} \ln \frac{R}{r_w}$$  \hspace{1cm} (2.24)

where $K$ is the aquifer’s hydraulic conductivity, LT$^{-1}$; $Q$ (in L$^3$T$^{-1}$) and $s_w$ (in L) are water discharge and drawdown, respectively, when pumping test reaching to a steady state; $R$ is the radius of influence of the pumping well, L; $r_w$ is the radius of the pumping well, L; $M$ is the aquifer’s thickness, L; $H_0$ is the natural water table of unconfined aquifer, L; $h_w$ is the water table in pumping well when pumping test in unconfined aquifer reaching to a steady state, L.

Formulas should be chosen correctly according to hydrogeological conditions, boundary conditions and well structure.
2.5.1.2 Three-Dimensional Single Well Formula

Dupuit formula does not consider 3D flow near pumping well, so the drawdown it used is lager, which lead to the calculated $K$ is usually less than actual value. Three-dimensional formula is an amendment of Dupuit formula, as Eq. (2.25).

$$s = \frac{Q}{2\pi KM} \ln \frac{R}{r_w} \pm \frac{Q^2}{g\pi^2 r_w^4} \left( \frac{f}{6M^2 D} Z^3 + \frac{Z^2}{M^2} - \frac{fM}{24D} - \frac{1}{3} \right)$$

(2.25)

where $D$ is the diameter of the pumping well, L; $f$ is the friction coefficient of the filter tube, which is equal to $64/Re$ in laminar flow; $g$ is the acceleration of gravity, m/s$^2$; $Z$ is the distance between the bottom of the filter tube to the bottom of the aquifer, L. Other symbols are the same with that of Dupuit formula.

For ease of use, two parts of the former formula can be represented by $A$ and $C$, respectively, that are:

$$A = \frac{Q}{2\pi KM} \ln \frac{R}{r_w}$$

$$C = \frac{1}{g\pi^2 r_w^4} \left( \frac{f}{6M^2 D} Z^3 + \frac{Z^2}{M^2} - \frac{fM}{24D} - \frac{1}{3} \right)$$

So the 3D formula can be expressed as:

$$s = AQ \pm CQ^2$$

(2.26)

or

$$s = s_w \pm CQ^2$$

(2.27)

where $s_w$ is the drawdown in pumping well, L; $s$ is the amendment drawdown considered 3D flow, L.

$A$ and $C$ are constants when size of pumping well is determined. That means $s$ and $Q$ has a parabolic relation and $A$, $C$ can be obtained by graphic method.

Making $\varepsilon = \frac{s}{Q}$ as unit flow drawdown, then $s$-$Q$ parabola will simplify to $\varepsilon$-$Q$ line as follow, shown in Fig. 2.22.

$$\varepsilon = A + CQ$$

(2.28)

where $A$ is the intercept; $C$ is the slope, which can be rewritten as:

$$C = \frac{s_i+1/Q_i+1 - s_i/Q_i}{Q_i+1 - Q_i}$$

(2.29)
where $s_i$ and $s_{i+1}$ are the drawdowns for the $i$th pumping and $(i + 1)$th pumping, respectively; $Q_i$ and $Q_{i+1}$ are the water discharge for the $i$th pumping and $(i + 1)$th pumping, respectively.

Steps for hydraulic conductivity calculation using single well pumping data are as follows:

1. Draw $s_w$ (or $\Delta h_w^2$)-$Q$ curve

Draw $s_w$-$Q$ curve for confined aquifer or $\Delta h_w^2$-$Q$ curve for unconfined aquifer according to steady flow pumping test data. Here, $\Delta h_w^2 = H_0^2 - h_w^2$.

There are three types of $s_w$ (or $\Delta h_w^2$)-$Q$ curves in engineering practice, shown in Fig. 2.23.

2. Calculate $K$

   (1) If $s_w$ (or $\Delta h_w^2$)-$Q$ curve is a straight line, as line (1) in Fig. 2.23, that means the groundwater flow is two dimensional and $C = 0$ in Eq. (2.26). So hydraulic conductivity $K$ can be calculated by following formulas:

   Fully penetrating well in confined aquifer:
   
   $$ K = \frac{Q}{2\pi s_w M} \ln \frac{R}{r_w} $$

   Fully penetrating well in unconfined aquifer:
   
   $$ K = \frac{Q}{\pi (H_0^2 - h_w^2)} \ln \frac{R}{r_w} = \frac{Q}{\pi \Delta h_w^2} \ln \frac{R}{r_w} \quad (2.30) $$

Fig. 2.22 $s$-$Q$ curve

Fig. 2.23 $s_w$ (or $\Delta h_w^2$)-$Q$ curve
Dupuit formula that be used in unconfined aquifer should meet the need that the water gradient of cone of depression <1/4. If the drawdown in unconfined aquifer is less than one tenth of the aquifer’s thickness, formulas for confined aquifer can be used and \( M \) will be replaced with \( H_0 \), which represents the aquifer’s thickness.

For partially penetrating pumping well, corresponding formulas should be chosen.

(2) Curves (2), (3) in Fig. 2.23 represent three-dimensional groundwater flow, so that \( \varepsilon - Q \) curve should be drawn as shown in Fig. 2.22, where \( \varepsilon = \frac{s}{Q} \) in confined aquifer and \( \varepsilon = \frac{\Delta h^2}{Q} \) in unconfined aquifer. Intercept \( A \) can be obtained from it and then hydraulic conductivity \( K \) can be calculated as follows:

Fully penetrating well in confined aquifer:

\[
K = \frac{Q}{2\pi AM}\ln\frac{R}{r_w}
\]  

(2.31)

Fully penetrating well in unconfined aquifer:

\[
K = \frac{1}{\pi A}\ln\frac{R}{r_w}
\]  

(2.32)

2.5.1.3 Steady Pumping Test with Observation Wells

Hydraulic conductivity can be calculated in steady pumping test with observation wells, which is usually more than two. Specific steps are as follows:

1. Draw \( s_w \) (or \( \Delta h^2 \))-\( \lg r \) curve curves according to pumping test data, shown in Fig. 2.24, where \( \Delta h^2 = H_0^2 - h^2 \).

2. Calculate \( K \) by following formula:

Fully penetrating well in confined aquifer:

\[
K = \frac{Q}{2\pi M(s_1 - s_2)}\ln\frac{r_2}{r_1} = \frac{2.3Q}{2\pi M m_r}
\]  

(2.33)

Fig. 2.24 \( s_w \) (or \( \Delta h^2 \))-\( \lg r \) curve
where \((r_1, s_1)\) and \((r_2, s_2)\) are the coordinate values of two random points of the straight-line portion in \(s-\lg r\) curve; \(m_r\) is the intercept of the straight-line portion, 
\[
m_r = \frac{s_1 - s_2}{\lg r_1 - \lg r_2}.
\]

Fully penetrating well in unconfined aquifer:

\[
K = \frac{Q}{\pi (\Delta h_1^2 - \Delta h_2^2) \ln \frac{r_2}{r_1}} = \frac{2.3Q}{\pi m_r}
\]

(2.34)

where \((r_1, \Delta h_1^2)\) and \((r_1, \Delta h_2^2)\) are the coordinate values of two random points of the straight-line portion in \(\Delta h^2 - \lg r\) curve;

\[
\Delta h^2 = H_0^2 - h^2
\]

where \(H\) is the thickness of unconfined aquifer before pumping, \(L\); \(h\) is the height of water column that from aquifer’s bottom to water surface in observation well, \(L\); \(m_r\) is the intercept of the straight-line portion, 
\[
m_r = \frac{\Delta h_1^2 - \Delta h_2^2}{\lg r_2 - \lg r_1}.
\]

2.5.1.4 Choice of Formulas and Empirical Values

Formulas for hydraulic conductivity calculation can be chosen from Tables 2.13, 2.14, 2.15, 2.16 and 2.17, according to different engineering condition. Empirical values can be chosen from Tables 2.18, 2.19, and 2.20.

2.5.2 Calculation of Radius of Influence

Radius of influence of pumping well is one of the aquifer’s original data in hydraulic conductivity calculation. It can be determined by steady pumping test with observation wells. There are two kinds of methods, as follows:

2.5.2.1 Graphing Method

Draw \(s\) (or \(\Delta h^2\))-\(\lg r\) curves according to pumping test data, shown in Fig. 2.24. Lengthen the straight-liner segment to \(\lg r\) axial, and the intersection \(R\) is the radius of influence of pumping well in hypothetical cylindrical aquifer. According to the conception of “reference influence radius”, \(R\) is a constant that does not change with water discharge. So all \(s\) (or \(\Delta h^2\))-\(\lg r\) curves should intersect in one point, which is \(R\). If they did not, reasons should be found, for example the change of recharge condition (Fig. 2.25).
2.5.2.2 Formula Method

Steps of influence radius calculation with data of steady pumping test are as follows:

1. Draw $s$ (or $\Delta h^2$)-$\lg r$ curves according to pumping test data.
2. Choose two points A ($\lg r_1$, $s_1$) and B ($\lg r_2$, $s_2$) or A ($\lg r_1$, $\Delta h^2_1$) and B ($\lg r_2$, $\Delta h^2_2$) in straight-liner segment, then calculate $R$ by following formulas:
### Table 2.14 Partially penetrating well in unconfined aquifer (filter unsubmerged) (from Handbook of hydrogeological investigation of water supply 1977)

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Formulas</th>
<th>Application condition</th>
</tr>
</thead>
</table>
| ![Graph](image1) | $K = \frac{0.73Q}{l + s_w} \left[ \frac{l}{\lg r_w} + \frac{l}{\lg \frac{4r_w}{s_w}} \right]$ | 1. Filter installed near the top of aquifer  
2. $l < 0.3H$  
3. Aquifer has a large thickness |
| ![Graph](image2) | $K = \frac{0.16Q}{l(s - s_l)} \left( 2.3 \lg \frac{l}{r_w} - \text{arsh} \frac{l}{r_1} \right)$ where $l = l_0 - 0.5(s + s_l)$ | 1. Filter installed near the top of aquifer  
2. $l < 0.3H$  
3. $s < 0.3l$  
4. Have one observation well; radius $r_1 < 0.3$ |
| ![Graph](image3) | $K = \frac{0.73Q}{s_w} \left[ \frac{l + s_w}{\lg r_w} + \frac{2m}{l} \left( 2 \lg \frac{2A}{m} - \lg \frac{l}{r_w} \right) \right]$ where $m$ is the distance between the middle of filter and the aquifer bottom; $A$ depends on $l/m$ | 1. Filter installed near the top of aquifer  
2. $l > 0.3H$  
3. No observation well |
| ![Graph](image4) | $K = \frac{0.366Q(lg R - lg r_w)}{H_1 s_w}$ where $H_1$ is the distance between bottoms of filter and aquifer | No observation well |
| ![Graph](image5) | $K = \frac{0.366Q}{ls_w} \left[ \frac{0.66l}{r_w} \right]$ | 1. Pumping under river  
2. Filter installed in the middle or near the top of aquifer  
3. $c > \frac{l}{lnw}$ (Usually $c < (2-3)$ cm) |

Fully penetrating well in confined aquifer:

$$\lg R = \frac{s_1 \lg r_2 - s_2 \lg r_1}{s_1 - s_2}$$  \hspace{1cm} (2.35)
### Table 2.15 Fully penetrating well in confined aquifer (from Handbook of hydrogeological investigation of water supply 1977)

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Formulas</th>
<th>Application condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pumping well</td>
<td>Observation well 1</td>
<td>Observation well 2</td>
</tr>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.732Q}{2H} \sqrt{r}$</td>
<td>No observation well</td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.732Q}{2H-2r} \sqrt{r}$</td>
<td>Have one observation well</td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.732Q}{2H-2r-2l} \sqrt{r}$</td>
<td>Have two observation well</td>
</tr>
</tbody>
</table>

### Table 2.16 Fully/partially penetrating well in confined aquifer (from Handbook of hydrogeological investigation of water supply 1977)

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Formulas</th>
<th>Application condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pumping well</td>
<td>Observation well 1</td>
<td>Observation well 2</td>
</tr>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.366Q}{M} \sqrt{r}$</td>
<td>1. In confined or unconfined aquifer 2. Filter is next to the top or bottom of aquifer 3. $l/r &gt; 5$ and $l &lt; 0.3M$</td>
</tr>
<tr>
<td></td>
<td>$a = 1.6$ Гиринский</td>
<td>$a = 1.32$ В.Д.Бабушкин</td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.366Q}{M} \sqrt{r}$</td>
<td>1. In confined aquifer 2. Filter is next to the top of aquifer 3. $l/r &gt; 5$ and $l &gt; 0.3M$ (by Muskat formula)</td>
</tr>
<tr>
<td></td>
<td>$a = l/M$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.366Q}{M} \sqrt{r}$</td>
<td>Dupuit formula No observation well</td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.366Q}{M} \sqrt{r}$</td>
<td>Dupuit formula Have one observation well</td>
</tr>
<tr>
<td></td>
<td>$K = \frac{0.366Q}{M} \sqrt{r}$</td>
<td>Dupuit formula Have one observation well</td>
</tr>
</tbody>
</table>
### Table 2.17  Water table recovery method (from Handbook of hydrogeological investigation of water supply 1977)

<table>
<thead>
<tr>
<th>Graphs</th>
<th>Formulas</th>
<th>Application condition</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td>$K = \frac{1.57r_w(h_2 - h_1)}{t(h_1 + h_2)}$</td>
<td>Flat bottom well and trial pit with large diameter in confined aquifer</td>
<td>Obtain a series of $k$ that related to water table recovery time, and draw $k$-$t$ curve, in which the constant value of hydraulic conductivity can be determined, as shown in following figure</td>
</tr>
<tr>
<td><img src="image2" alt="Graph" /></td>
<td>$K = \frac{r_w(h_2 - h_1)}{t(h_1 + h_2)}$</td>
<td>Spherical well and trial pit with large diameter in confined aquifer</td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Graph" /></td>
<td>$K = \frac{3.5r_w^2}{(H + 2r)^2} \ln \frac{h_2}{h_1}$</td>
<td>Fully penetrating well in unconfined aquifer</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Graph" /></td>
<td>$K = \frac{2r_w}{H - h_1} \ln \frac{H - h_1}{H - h_2}$</td>
<td>1. Fully penetrating well in unconfined aquifer 2. Seepage just occurs on the bottom of well with large diameter</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2.18  Empirical values of hydraulic conductivity $K$ (from Handbook of Engineering Geology 1992)

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$K$ (m/day)</th>
<th>Soil type</th>
<th>$K$ (m/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy silt</td>
<td>&lt;0.05</td>
<td>Fine sand</td>
<td>1–5</td>
</tr>
<tr>
<td>Clayed silt</td>
<td>0.05–0.1</td>
<td>Medium sand</td>
<td>5–20</td>
</tr>
<tr>
<td>Silty clay</td>
<td>0.1–0.5</td>
<td>Coarse sand</td>
<td>20–50</td>
</tr>
<tr>
<td>Loess</td>
<td>0.25–0.05</td>
<td>Gravel</td>
<td>100–200</td>
</tr>
<tr>
<td>Silty sand</td>
<td>0.5–1.0</td>
<td>Gravel-boulder</td>
<td>20–150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Boulder</td>
<td>500–1000</td>
</tr>
</tbody>
</table>
Fully penetrating well in unconfined aquifer:

\[
\lg R = \frac{\Delta h_1^2 \lg r_2 - \Delta h_2^2 \lg r_1}{\Delta h_1^2 - \Delta h_2^2}
\]

(2.36)

### 2.5.2.3 Choice of Formulas and Empirical Values

Table 2.21 shows some formulas for influence radius calculation. Most results are approximate values. Empirical values can be chosen from Table 2.22.
Table 2.21 Formulas for influence radius calculation

<table>
<thead>
<tr>
<th>Calculation formulas</th>
<th>Application conditions</th>
<th>Notes</th>
</tr>
</thead>
</table>
| \( \lg R = \frac{s_1 \lg r_2 - s_2 \lg r_1}{s_1 - s_2} \) | 1. Confined aquifer  
2. With two observation wells | Accurate (By Dupuit formula) |
| \( \lg R = \frac{s_1 (2H - s_1) \lg r_2 - s_2 (2H - s_2) \lg r_1}{(s_1 - s_2)(2H - s_1 - s_2)} \) | 1. Unconfined aquifer  
2. With two observation wells | Accurate (By Dupuit formula) |
| \( \lg R = \frac{s_1 \lg r_2 - s_2 \lg r_1}{s_1 - s_2} \) | 1. Confined aquifer  
2. With one observation well | Calculated value is larger than actual value |
| \( \lg R = \frac{s_1 (2H - s_1) \lg r_2 - s_2 (2H - s_2) \lg r_1}{(s_1 - s_2)(2H - s_1 - s_2)} \) | 1. Unconfined aquifer  
2. With one observation well | Calculated value is larger than actual value |
| \( \lg R = 2.73K\sqrt{\frac{Q}{r}} + \lg r \) | 1. Confined aquifer  
2. No observation well | Calculated value is larger than actual value |
| \( \lg R = 1.366K(2H - s) \frac{s}{Q} \) | 1. Unconfined aquifer  
2. No observation well | Calculated value is larger than actual value |
| \( R = 10s\sqrt{K} \) | 1. Confined aquifer  
2. No observation well | Rough calculation (By W. Sihardt formula) |
| \( R = 2s\sqrt{HK} \) | 1. Unconfined aquifer  
2. No observation well | Rough calculation (By И.П. Кусаин formual) |
| \( R = \sqrt{\frac{12e}{\mu}} \sqrt{\frac{QK}{\pi}} \) | 1. Unconfined aquifer  
2. Fully penetrating well | (By Kozeny formula) |
| \( R = 3\sqrt{\frac{KnH}{\mu}} \) | Unconfined aquifer | (By Weber formula) |
| \( R = \frac{Q}{2KH} \) | Confined aquifer | Rough calculation (By E.E. Керкис formula) |

Symbols in this table:

- \( s_1, s_2 \): Drawdowns for observation wells, m
- \( r_1, r_2 \): Distance between pumping well and observation well, m
- \( r \): Radius of pumping well, m
- \( H/M \): Thickness of unconfined/confined aquifer, m
- \( K \): Hydraulic conductivity, m/day
- \( t \): Time, d
- \( \mu \): Specific yield
- \( i \): Hydraulic gradient
2.5.3 Case Study

Layout of pumping test wells in Tongji University is shown in Fig. 2.26. The filters of both main well and observation wells are all drilled in the same aquifer, which contains mainly fine to medium sand with gravel and coarse sand. The buried depth

![Fig. 2.26 Layout of pumping test wells](image)

Table 2.22 Empirical values of influence radius $R$

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Primary particle diameter (mm)</th>
<th>Weight ratio (%)</th>
<th>Influence radius $R$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty sand</td>
<td>0.05–0.1</td>
<td>&lt;70</td>
<td>25–50</td>
</tr>
<tr>
<td>Fine sand</td>
<td>0.1–0.25</td>
<td>&gt;70</td>
<td>50–100</td>
</tr>
<tr>
<td>Medium sand</td>
<td>0.25–0.5</td>
<td>&gt;50</td>
<td>100–200</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>0.5–1.0</td>
<td>&gt;50</td>
<td>300–400</td>
</tr>
<tr>
<td>Very coarse sand</td>
<td>1.0–2.0</td>
<td>&gt;50</td>
<td>400–500</td>
</tr>
<tr>
<td>Small gravel</td>
<td>2.0–3.0</td>
<td></td>
<td>500–100</td>
</tr>
<tr>
<td>Medium gravel</td>
<td>3.0–5.0</td>
<td></td>
<td>600–1500</td>
</tr>
<tr>
<td>Large gravel</td>
<td>5.0–10.0</td>
<td></td>
<td>1500–3000</td>
</tr>
</tbody>
</table>

$2 m \quad 20 m \quad 30 m$

![Fig. 2.27 $Q$-$s$ curve](image)
of the aquifer is 67.7 m, and the thickness is 23.1 m. Diameters of main well and observation wells are 305 mm and 152 mm, respectively.

Three steady pumping tests were taken and the matching line that water discharge $Q$ versus drawdown $s$ is shown in Fig. 2.27. Other tests data are shown in Table 2.23.

### 2.5.3.1 Calculation of Influence Radius

1. Influence radius of pumping well is 930 m, which determined by graphing method, shown in Fig. 2.28.
2. $R$ can also be calculated by formulas as follows:

\[
\begin{align*}
\lg R_1 &= \frac{s_1 \lg r_2 - s_2 \lg r_1}{s_1 - s_2} = \frac{0.52 \lg 30 - 0.31 \lg 3}{0.52 - 0.31} = 2.9532, \quad \text{and} \quad R_1 = 898 \text{ m} \\
\lg R_2 &= \frac{0.70 \lg 30 - 0.43 \lg 3}{0.70 - 0.43} = 3.0697, \quad \text{and} \quad R_2 = 1174 \text{ m} \\
\lg R_3 &= \frac{0.81 \lg 30 - 0.47 \lg 3}{0.81 - 0.47} = 2.8595, \quad \text{and} \quad R_3 = 724 \text{ m} \\
R &= \frac{R_1 + R_2 + R_3}{3} = 932 \text{ m}
\end{align*}
\]

### Table 2.23 Data of steady pumping tests

<table>
<thead>
<tr>
<th>Test number</th>
<th>Water discharge $Q$ (m$^3$ day$^{-1}$)</th>
<th>Drawdown (m) $s_0$ for MW</th>
<th>$s_1$ for OW1</th>
<th>$s_2$ for OW2</th>
<th>$s_3$ for OW3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1570</td>
<td>1.385</td>
<td>0.52</td>
<td>0.36</td>
<td>0.31</td>
</tr>
<tr>
<td>2</td>
<td>1954</td>
<td>1.635</td>
<td>0.70</td>
<td>0.47</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>2384</td>
<td>2.22</td>
<td>0.81</td>
<td>0.61</td>
<td>0.47</td>
</tr>
</tbody>
</table>

### Fig. 2.28 $s$-lg$r$ curve
2. 

2.5.3.2 Calculation of Hydraulic Conductivity $K$

1. Hydraulic conductivity $K$ can be calculated by single well’s test data, as follows:

$$K = \frac{Q}{2\pi Ms} \ln \frac{R}{r_0}$$

$$K_1 = \frac{1570}{2\pi \times 23.1 \times 1.385} \ln \frac{932}{0.152} = 68.11 \text{ m/day}$$

$$K_2 = \frac{1954}{2\pi \times 23.1 \times 1.635} \ln \frac{932}{0.152} = 71.81 \text{ m/day}$$

$$K_3 = \frac{2384}{2\pi \times 23.1 \times 2.22} \ln \frac{932}{0.152} = 64.53 \text{ m/day}$$

$$K = \frac{K_1 + K_2 + K_3}{3} = 68.15 \text{ m/day}$$

2. Hydraulic conductivity $K$ can be calculated by two more wells’ test data, as follows:

$$K = \frac{2.30Q}{2\pi M} \times \frac{1}{m_r} = \frac{2.30Q}{2\pi M} \times \frac{\lg r_3 - \lg r_1}{s_1 - s_3}$$

$$K_1 = \frac{2.30 \times 1570}{2\pi \times 23.1} \times \frac{\lg 30 - \lg 3}{0.52 - 0.31} = 118.5 \text{ m/day}$$

$$K_2 = \frac{2.30 \times 1954}{2\pi \times 23.1} \times \frac{\lg 30 - \lg 3}{0.70 - 0.43} = 114.7 \text{ m/day}$$

$$K_3 = \frac{2.30 \times 2384}{2\pi \times 23.1} \times \frac{\lg 30 - \lg 3}{0.81 - 0.47} = 111.1 \text{ m/day}$$

$$K \approx 115 \text{ m/day}.$$
2.6 Hydrogeological Parameters Calculation in Unsteady Flow Pumping Test

2.6.1 Transmissibility, Storage Coefficient, and Pressure Transitivity Coefficient Calculation for Confined Aquifer

Transmissibility $T$ represents the ability of aquifer’s water conductivity. It is the numeric equivalent of the product of hydraulic conductivity times aquifer’s thickness ($T = KM$), which means it is the seepage flow under the condition of unit hydraulic gradient, unit time, and unit width.

Storage coefficient $\mu^*$ (or elastic specific yield) represents the water storage capacity of aquifer in pressurization or water yield capacity in pressure reduction. It refers to water volume that is stored or released from unit area aquifer when waterhead having a unit change. It is determined by the elastic property of water and aquifer skeleton.

Pressure transitivity coefficient $(a)$ represents the waterhead conduction velocity of confined aquifer. It is defined as $a = T/\mu^*$.

$T$, $\mu^*$, and $a$ can be calculated according to unsteady pumping test data. If the thickness of aquifer has already been measured, the hydraulic conductivity $K$ can also be calculated.

In horizontal, homogeneous, isotropic, and infinite extending confined aquifer without vertical recharge and thickness change, Theis formula can be written as:

$$s(r, t) = \frac{Q}{4\pi T} \int_u^\infty e^{-u} \frac{d\omega}{u} = \frac{Q}{4\pi T} W(u) \quad (2.37)$$

$$u = \frac{r^2 \mu^*}{4Tt}, \quad \text{or} \quad \frac{1}{u} = \frac{4Tt}{r^2 \mu^*} \quad (2.38)$$

where $s(r, t)$ is the drawdown of certain point within influence area, m; $t$ is the time form the beginning of pumping, h; $r$ is the distance between calculating point and pumping well, m; $W(u)$ is the well formula which can be found from $W(u)$ table. The meaning of other symbols are the same with above.

2.6.1.1 Calculation Method

$T$, $\mu^*$, and $a$ can be calculated by curve-matching method, linear graphic method, and water table recovery method, as shown in Table 2.24. Data for these methods are got from unsteady pumping test with constant water discharge.
<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curve-matching method</td>
<td>1. Plot the t-s data on log-log graph paper</td>
<td>( T = \frac{Q}{4\pi} W(u) ) (2.39)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Superimpose this plot on the ( W(u) - u ) type curve sheet of the same size and scale as the t-s plot, so that the plotted points match the type curve. The axes of both graphs must be kept parallel.</td>
<td>( \mu^* = \frac{4Q}{\pi \rho C^2} ) (2.40)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Select a match point, which can be any point in the overlap area of the curve sheets. Write down the ( W(u) ), ( \mu^* ), ( s ) and ( t ) values of match point</td>
<td>( a = \frac{T}{\pi} ) (2.41)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Determine ( T ), ( \mu^* ) and ( a )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drawdown (s)-time (t) curve</td>
<td>1. Plot the t-s data on log-log graph paper</td>
<td>( T = \frac{Q}{4\pi} W(u) ) (2.39)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Superimpose this plot on the ( W(u) - u ) type curve sheet of the same size and scale as the t-s plot, so that the plotted points match the type curve. The axes of both graphs must be kept parallel.</td>
<td>( \mu^* = \frac{4Q}{\pi \rho C^2} ) (2.40)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Select a match point, which can be any point in the overlap area of the curve sheets. Write down the ( W(u) ), ( \mu^* ), ( s ) and ( r^2 ) values of match point</td>
<td>( a = \frac{T}{\pi} ) (2.41)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Determine ( T ), ( \mu^* ) and ( a )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2.24 (continued)

<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear graphic method</strong></td>
<td>1. Plot the ( s-t ) data on semi-log graph paper</td>
<td>( T = \frac{2.303}{4.63} ) (2.43)</td>
<td><img src="image" alt="Fig.2.31 s-t curve in semi-log coordinate" /></td>
</tr>
<tr>
<td></td>
<td>2. Extent the line to ( \text{lg} ) axial, then get the intercept ( t_0 )</td>
<td>( \mu^* = \frac{2.303}{\rho} ) (2.44)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Determine the straight slope ( i ), which numerically equal to ( \Delta s ) that corresponding to one time period (( \Delta \text{lg} t = 1 ))</td>
<td>( a = \frac{T}{\rho} ) (2.41)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Determine ( T, \mu^* ) and ( a )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Drawdown (s)-time (t) line</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Plot the ( s-r ) data on semi-log graph paper</td>
<td>( T = \frac{2.303}{4.63} ) (2.45)</td>
<td><img src="image" alt="Fig.2.32 s-r curve in semi-log coordinate" /></td>
</tr>
<tr>
<td></td>
<td>2. Extent the line to ( \text{lg} r ) axial, then get the intercept ( r_0 )</td>
<td>( \mu^* = \frac{2.303}{r_0} ) (2.40)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Determine the straight slope ( i ), which numerically equal to ( \Delta s ) that corresponding to one time period (( \Delta \text{lg} r = 1 ))</td>
<td>( a = \frac{T}{r_0} ) (2.41)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4. Determine ( T, \mu^* ) and ( a )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Water table recovery method</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1. Plot the ( s'-t'/t' ) data on semi-log graph paper, where ( s' ) is the rest drawdown, ( t' ) is the time of water table recovery, ( t_0 ) is the time of pumping, and ( t = t' + t_0 )</td>
<td>( T = \frac{2.303}{4.63} ) (2.43)</td>
<td><img src="image" alt="Fig.2.33 s-t'/t' curve in semi-log coordinate" /></td>
</tr>
<tr>
<td></td>
<td>2. Determine the straight slope ( i ), which numerically equal to ( \Delta s ) that corresponding to one time period (( \Delta \text{lg} t' = 1 ))</td>
<td>( a = 0.44 \left( \frac{s'}{r_0} \right) \cdot 10^2 ) (2.46)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3. Determine ( T, \mu^* ) and ( a )</td>
<td>( \mu^* = \frac{T}{s'} ) (2.47)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( s \) (m), \( r \) (m), \( t \) (min), \( \Delta s \), \( \Delta \text{lg} t \)
The biggest advantage of curve-matching method is that all test data can be used to improve the calculation accuracy. This method also has disadvantages. That is when the $r$ is small and $T$ is large, the steep part of observation curve appears in the first 2 min, which is difficult to measure. While the part that is easy to measure is too gentle to fit accurately. These all lead to low accuracy of parameters. The solutions are: On the one hand, using the data from initial pumping stage as much as possible. On the other hand, setting the observation wells far from pumping well when $T$ is large, so that the drawdown can be measured accurately.

There are many advantages of linear graphic method, for example all test data can be used and the randomness of fitting curve method can be avoided. The disadvantage of this method is that the pumping time needs to be long to meet the requirement of $u \leq 0.01$, especially when $T$ is small and $r$ is large. Besides, the long pumping time may leads to the deviation of intercept and gradient, which will result in larger $T$ and smaller $\mu^e$.

The water table recovery method can avoid the interference from pumping equipment and water discharge variation during pumping period. The drawdown versus time curve of water table recovery period usually much more regular, which can improve the accuracy of parameters calculation.

Apply as many methods as possible to calculate the parameters when using unsteady pumping test data, and compare the results to choose the best one.

### 2.6.1.2 Case Study

The hydrogeological condition is the same as it in Sect. 2.5.3.

1. $\lg s - \lg t$ curve-matching method

$\lg s - \lg t$ curve of Observation well 2 and Observation well 3 are shown in Fig. 2.34.

**Fig. 2.34** $\lg s - \lg t$ curves

![Fig. 2.34 $\lg s - \lg t$ curves](image)
Results:
Observation well 2:

\[ W(u) = 1, \quad 1/u = 10^3, \quad s = 0.061 \text{ m}, \quad t = 14.1 \text{ min}; \]

\[ T = \frac{Q}{4\pi s} W(u) = \frac{2384}{4\pi \times 0.061} \times 1 = 3.11 \times 10^3 \text{ m}^2/\text{day}; \]

\[ \mu^s = \frac{4Tt}{r^2(1/u)} = \frac{4 \times 3.14 \times 10^3 \times 14.1}{20^2 \times 102 \times 1440} = 3.05 \times 10^{-4}; \]

\[ a = \frac{T}{\mu^s} = \frac{3.11 \times 10^3}{3.05 \times 10^{-4}} = 1.02 \times 10^7 \text{ m}^2/\text{day}. \]

Observation well 3:

\[ W(u) = 1, \quad 1/u = 10^2, \quad s = 0.062 \text{ m}, \quad t = 10 \text{ min}; \]

\[ T = \frac{2384}{4\pi \times 0.062} \times 1 = 3.06 \times 10^3 \text{ m}^2/\text{day}; \]

\[ \mu^s = \frac{4 \times 3.06 \times 10^3 \times 10}{30^2 \times 10^2 \times 1440} = 9.44 \times 10^{-4}; \]

\[ a = \frac{3.06 \times 10^3}{9.44 \times 10^{-4}} = 3.24 \times 10^6 \text{ m}^2/\text{day}. \]

2. Linear graphic method

s-lgt curve of Observation well 2 and Observation well 3 are shown in Fig. 2.35.

Results:
Observation well 2:

\[ T = \frac{2.30Q}{4\pi \Delta s} = \frac{2.30 \times 2384}{4\pi \times (0.50 - 0.36)} = 3.12 \times 10^3 \text{ m}^2/\text{day}; \]

\[ \mu^t = \frac{2.25T_0}{r^2} = \frac{2.25 \times 3.12 \times 10^3 \times 0.22}{20^2 \times 1440} = 2.68 \times 10^{-3}; \]

\[ a = \frac{T}{\mu^t} = 1.2 \times 10^6 \text{ m}^2/\text{day}. \]
Observation well 3:

\[
T = \frac{2.30 \times 2384}{4\pi \times (0.40 - 0.26)} = 3.12 \times 10^3 \text{ m}^2/\text{day};
\]
\[
\mu^* = \frac{2.25 T t_0}{r^2} = \frac{2.25 \times 3.12 \times 10^3 \times 0.12}{30^2 \times 1440} = 6.5 \times 10^{-4};
\]
\[
a = \frac{T}{\mu^*} = 4.8 \times 10^6 \text{ m}^2/\text{day}.
\]

3. Water table recovery method

$s-\lg \frac{t_p + t'}{t}$ curve of Observation well 2 and Observation well 3 are shown in Fig. 2.36.

Results:

Observation well 2:

\[
T = \frac{2.30Q}{4\pi \Delta s} = \frac{2.30 \times 2384}{4\pi \times (0.276 - 0.14)} = 3.21 \times 10^3 \text{ m}^2/\text{day};
\]
\[
a = \frac{0.44r^2}{t_0} = \frac{0.44 \times 20^2}{0.5} \times 10^{0.64} = 1.08 \times 10^7 \text{ m}^2/\text{day};
\]
\[
\mu^* = \frac{T}{a} = \frac{3.21 \times 10^3}{1.08 \times 10^7} = 2.97 \times 10^{-4}.
\]
Table 2.25 Summary sheet for aquifer parameters

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Data</th>
<th>Aquifer parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K$ (m/day)</td>
</tr>
<tr>
<td>Steady flow</td>
<td>Single well</td>
<td>68.15</td>
</tr>
<tr>
<td></td>
<td>Multiple wells</td>
<td>115.0</td>
</tr>
<tr>
<td>Unsteady flow</td>
<td>Fitting curve method</td>
<td>Observation well 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Observation well 3</td>
</tr>
<tr>
<td></td>
<td>Linear graphic method</td>
<td>Observation well 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Observation well 3</td>
</tr>
<tr>
<td></td>
<td>Water table recovery method</td>
<td>Observation well 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Observation well 3</td>
</tr>
<tr>
<td></td>
<td>Average value</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td></td>
</tr>
</tbody>
</table>

Observation well 3:

\[
T = \frac{2.30 \times 2384}{4\pi \times (0.242 - 0.11)} = 3.31 \times 10^3 \text{ m}^2/\text{day};
\]

\[
a = 0.44 \times \frac{30^2}{0.5} \times 10^{0.47} = 2.88 \times 10^6 \text{ m}^2/\text{day};
\]

\[
\mu^* = \frac{T}{a} = \frac{3.31 \times 10^3}{2.88 \times 10^6} = 1.15 \times 10^{-3}.
\]

Table 2.25 is the summary for hydraulic parameters.

2.6.2 Transmissibility, Storage Coefficient, Leakage Coefficient, and Leakage Factor Calculation for Leaky Aquifers

Leakage coefficient $\sigma (=K'/M')$ and leakage factor $B$ represent the leakage characteristic of aquitard. Leakage recharge amount is related to hydraulic conductivity $K'$ and thickness $M'$ of aquitard.

Leakage coefficient $\sigma$ represents the recharging amount in a unit area from aquitard to pumped aquifer under a unit waterhead difference.
Leakage factor \( B = \sqrt{TM'/K'} \), where \( T \) is the transmissibility of main aquifer, \( K' \) and \( M' \) are the hydraulic conductivity and thickness of aquitard. The smaller the \( K' \) is and the bigger the \( M' \) is, the larger the \( B \) is.

Hydraulic parameters of leaky aquifers can be calculated according to unsteady pumping test data by Hantush-Jacob formula, as follows:

\[
\begin{align*}
 s &= \frac{Q}{4\pi T} \int_0^\infty e^{-y-y D^2} dy = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right) \\
 u &= \frac{r^2 \mu^*}{4Tt} \quad \text{or} \quad \frac{1}{u} = \frac{4Tt}{r^2 \mu^*} 
\end{align*}
\]  

where \( T, \mu^*, a, \sigma \) and \( B \) can be calculated by curve-matching method, yielding point method and tangent method, as shown in Table 2.26.

### 2.6.3 Specific Yield, Storage Coefficient, Hydraulic Conductivity and Transmissibility Calculation of Unconfined Aquifer

Specific yield \( \mu \) refers to the volume of gravitational water that drained from unit area of unconfined aquifer when water table decreases a unit meter. Free saturation rate, also called saturation deficit, refers to the volume of water that recharge to unit area of unconfined aquifer when water table rise a unit meter. Usually, specific yield and free saturation rate are numerically same.

#### 2.6.3.1 Bolton Formulas

Early pumping stage:

\[
 s = \frac{Q}{4\pi T} W(u_d, \frac{r}{D})
\]  

Middle pumping stage:

\[
 s = \frac{Q}{4\pi T} K_0\left(\frac{r}{D}\right)
\]  

Late pumping stage:

\[
 s = \frac{Q}{4\pi T} W(u_y, \frac{r}{D})
\]

\[
 u_d = \frac{r^2 \mu^*}{4Tt}, \quad u_y = \frac{r^2 u}{4Tt}
\]
Table 2.26 Methods for parameters calculation of confined aquifer (revised from Handbook of hydrogeological investigation of water supply 1977)

<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
</table>
| Drawdown (s−t) time curve   | 1. Plot the s−t data on double logarithmic graphic paper  
2. Superimpose this plot on the W(u, r/B)(1/ut) type curve sheet of the same size and scale as the s−t plots, and find the best overlapping curve that the plotted points match the type curve. The axes of both graphs must be kept parallel  
3. Select a match point, which can be any point in the overlap area of the curve sheets. Write down the W(u, r/B), 1/ut, s and t values of match point  
4. Calculate T, μ*, a, B and σ                                                                 | $T = \frac{Q}{4\pi p}W(u, r/B)$  
$\mu^* = \frac{1}{2\pi T}$  
$a = \frac{T}{p}$  
$B = r/[r/B]$  
$K_i/M_i = T/B^2$                                                                 | ![Fig.2.37](image)                                                                                   |-------|
| Yield point method          | 1. Plot the s−t data on semi-log graphic paper. Determine the maximum drawdown $s_{max}$ with extrapolation method, and calculate the drawdown of yield point $s_p = s_{max}/2$  
2. Basing on $s_p$, figure out the position of yield point p and the corresponding $t_p$  
3. Draw the tangent line of the s−lg t curve at the point p, and determine its slope $i_p$  
4. According to formula $d/(r/B)$, $K_0(r/B) = 2.30 (s_p/i_p)$, determine the value of $r/B$ and $d/(r/B)$ from Hantusi function table  
5. Calculate $T$, $\mu^*$, $B$ and $K_i/M_i$  
6. Verification                                                                 | $B = r/[r/B]$  
$T = \frac{2.30Q}{4\pi p} e^{-(r/B)}$  
$\mu^* = \frac{2B}{2\pi}$  
$a = \frac{T}{p}$  
$K_i/M_i = T/B^2$                                                                 | ![Fig.2.38](image)                                                                                   |-------|
| Tangent method              | 1. Plot the s−t data on semi-log graph paper and determine the maximum drawdown $s_{max}$ with extrapolation method  
2. Pick one point p at the s−lg t curve and its coordinates are $t_p$ and $s_p$  
3. Draw the tangent line of the s−lg t curve at the point p, and determine its slope $i_p$  
4. According to formula $f(\delta) = 2.30 (s_{max}−s_p)/i_p$, determine the value of $\delta$, $e^\delta$ and $\delta(\delta)$ from Hantusi function table  
5. Calculate $T$, $B$, $K_i/M_i$ and $\mu^*$; according to formula $K_i \left( \frac{r}{B} = \frac{2\pi T}{Q} s_{max} \right)$, determine the value of $r/B$ from $\mu^*$ function table                                                                 | $T = \frac{2.30Q}{4\pi p} e^{−d}$  
$B = r/[r/B]$  
$K_i/M_i = T/B^2$  
$\mu^* = \frac{2B}{2\pi}$  
$a = \frac{T}{p}$                                                                 | ![Fig.2.39](image)                                                                                   |-------|
Table 2.27 Bolton formulas for parameters calculation of unconfined aquifer

<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
</table>
| Drawdown (s)-time (t) curve-matching method | 1. Plot the t-s data on double logarithmic graph paper  
2. Superimpose these plots on the standard curves of group A to find out one curve that can best match these plots. Pick one match point and record its coordinates W(ud, r/D), 1/ud, s and t, and also the r/D value of the match curve. Calculate T and μ*  
3. Superimpose the rest plots on the standard curves of group B to find out one curve that can best match these plots. Keep the r/D value constant and pick one match point, whose coordinates are W(uy, r/D), 1/uy, s and t. Calculate T and μ  
4. When the value of s/H₀ is large, the value of T will change. Thus the drawdown should be amended by formula s' = s - s²/2H₀  
5. If using Theis curves replace standard group B curves, then the time tₜₑₓₜ can be calculated from ⁴ₜₜₑₓₜ r/D curve because ⁴ₜₜₑₓₜ = 1/2 (s')² = s²/2H₀ | \[ T = \frac{Q}{4\pi} \left[ W(ud, \frac{r}{\sqrt{D}}) \right] \]  
\[ \mu^* = \frac{4T}{r^2[1/ud]} \]  
\[ T = \frac{Q}{4\pi} \left[ W(uy, \frac{r}{\sqrt{D}}) \right] \]  
\[ \mu = \frac{4T(\alpha)}{r^2[1/uy]} \]  
\[ \eta = \frac{\mu^* + \mu}{\mu} \]  
\[ \alpha = \frac{4T}{r^2[1/uy]} \]  
\[ l = \frac{4T}{r^2[1/uy]} \]  
\[ g = l + l/2 \]  
\[ a = \frac{T}{\mu} \] | Fig.2.40 (from Prickett, 1965) |
| Linear graphic method | 1. Plot the s-t data on semi-log graph paper  
2. Determine T, S and a by Theis formulas, as shown in Table 2.19 | \[ T = \frac{2^{330Q}}{4\pi r \alpha} \]  
\[ \mu = \frac{2^{330D}}{r} \]  
\[ a = \frac{1}{\mu} \] | Fig.2.41 |
Table 2.28 Newman formulas for parameters calculation of unconfined aquifer

<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
</table>
| s-t curve-matching   | 1. Plot the t-s data on double logarithmic graph paper  
2. Superimpose these plots on the standard group B curves to find out one curve that can best match these plots and record its \( \beta \) value. Pick one match point and record its coordinates \( s_b, t_b, s, \) and \( t \), then calculate \( T \) and \( \mu \).  
3. Superimpose the front part of plots on the standard curves of group A to find out one curve that can best match these plots and its \( \beta \) value should be the same with step 2. Pick one match point, whose coordinates are \( s_b, t_b, s, \) and \( t \). Calculate \( T \) and \( \mu^* \). The values of \( T \) that are calculated in step 2 and step 3 should be close.  
4. Calculate radial permeability \( K_r \), ratio of anisotropy permeability \( K_d \), vertical permeability \( K_z \) and \( \sigma \). | \( T = \frac{Q}{4 \pi T_{t_b}} \) (2.71)  
\( \mu = \frac{T}{r^2 |t|} \) (2.72)  
\( T = \frac{Q}{4 \pi |t|} \) (2.73)  
\( \mu^* = \frac{T}{r^2 |t|} \) (2.74)  
\( K_r = \frac{T}{H_0} \) (2.75)  
\( K_d = \beta \frac{T}{r^2 |t|} \) (2.76)  
\( K_z = K_d K_r \) (2.77)  
\( \sigma = \frac{\mu^*}{\mu} \) (2.78) | ![Fig.2.42](From Newman, 1972) |
| Linear graphic method | 1. Plot the t-s data on semi-log graph paper  
2. Determine the slope \( i_1 \) of the linear part of s-lgt curve. Extend this linear part and get the intercept \( t_1 \) of the abscissa with \( s_b = 0 \). Calculate \( T \) and \( \mu \).  
3. Extend the horizontal part of the measured curve toward the right to intersect with the later linear part in one point, whose coordinate is \( t_b \). Calculate \( t_b \) and \( \beta \).  
4. If the front part of the measured curve is linear and parallel to the later linear part, then determine the slope \( i_E \) of this line and extend it to get the intercept \( t_E \) of the abscissa with \( s_b = 0 \). Calculate \( T \) and \( \mu^* \). The values of \( T \) that calculated in step 2 and this step should be close.  
5. Calculate \( K_r, K_d, K_z \) and \( \sigma \). | \( T = \frac{Q_0}{4 \pi b} \) (2.43)  
\( \mu = \frac{2.25T_b}{b^2} \) (2.44)  
\( t_b = \frac{Q_b}{b^2} \) (2.79)  
\( \beta = \frac{0.195}{b} \) (2.80)  
\( T = \frac{2.30Q}{4 \pi b} \) (2.43)  
\( \mu^* = \frac{2.25T_b}{b^2} \) (2.40)  
\( K_r = \frac{T}{H_0} \) (2.75)  
\( K_d = \beta \frac{H_b^2}{b^2} \) (2.76)  
\( K_z = K_d K_r \) (2.77)  
\( \sigma = t_E/t_1 \) (2.81) | ![Fig.2.43](From Newman, 1973) |
<table>
<thead>
<tr>
<th>Method</th>
<th>Calculation steps</th>
<th>Formulas</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water table recovery</td>
<td>1. Plot the $s'(t')$ data on the semi-log graph paper</td>
<td>$T = \frac{2\times10^4}{4\times10^3}$ (2.43)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2. Determine the slope $i_L$ of the line</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\Delta \log \left( \frac{t}{t'} \right) = 1$$

![Graph](image)
Parameters of unconfined aquifer can be calculated by curve-matching method and linear graphic method according to Bolton Formula, shown in Table 2.27.

### 2.6.3.2 Newman Formula

Newman formula is:

\[
s(r, t) = \frac{Q}{4\pi T} \int_0^\infty 4yJ_0(y\beta^{1/2}) \left[ \omega_0(y) + \sum_{n=1}^{\infty} \omega_n(y) \right] dy \quad (2.68)
\]

where:

\[
\omega_0 = \frac{\{1 - \exp[-t_n\beta(y^2 - \gamma_0^2)]\} \theta(y_0)}{\{y_0^2 + (1 + \sigma)\gamma_0^2 - [(y^2 - \gamma_0^2)^2/\sigma]\} \gamma_0} \quad (2.69)
\]

\[
\omega_n = \frac{\{1 - \exp[-t_n\beta(y^2 + \gamma_n^2)]\} \theta(y_n)}{\{y_n^2 - (1 + \sigma)\gamma_n^2 - [(y^2 + \gamma_n^2)^2/\sigma]\} \gamma_n} \quad (2.70)
\]

Parameters of unconfined aquifer can be calculated by curve-matching method and linear graphic method according to Bolton Formula, shown in Table 2.28.

### 2.7 Other Methods for Hydrogeological Parameters Calculation

#### 2.7.1 Transmissibility and Well Loss Calculation

Typically, well loss is ignored when using pumping test data to calculate aquifer parameters, which can lead to inaccuracy of those parameters.

This section will introduce a method that can consider the loss for transmissibility and well loss constant calculation.

#### 2.7.1.1 Basic Theory

In confined aquifer, the Jacob approximation formula for Theis formula is:

\[
s = \frac{2.30Q}{4\pi T} \lg \frac{2.25Tr}{r^2 \mu^s} \quad (u < 0.01) \quad (2.82)
\]
It can be found from Eq. (2.82) that the value of drawdown is only related to the properties of aquifer, which is not suitable for calculation. The drawdown of pumping well mainly consists of two parts: The first part is the drawdown that related to aquifers’ properties; the second part is that caused by turbulence flow around pumping well, which also know as well loss. The later is hard to be estimated and is various for different pumping wells. It can be approximately calculated by following formulas:

$$s'_w = CQ^2$$  \hspace{1cm} (2.83)

where $s'_w$ is the drawdown that caused by well loss, L; $C$ is the well loss constant, T$^2$K$^{-5}$; $Q$ is the water discharge, L$^3$T$^{-5}$.

So the total drawdown at pumping time $t$ is:

$$s_g = s + s'_w = \frac{2.30Q}{4\pi T} \lg \frac{2.25Ti}{r^2} + CQ^2$$  \hspace{1cm} (2.84)

where $a = \frac{2.30Q}{4\pi T}$; $b = \lg \frac{2.25T}{r^2}$.  

Change the water discharge for $n$ times:

$$s_{gj} = a(b + \lg t_j)Q + CQ^2$$  \hspace{1cm} (2.85)

where $A = a(b + \lg t_j)$; $s_{gj}$ is the drawdown at time $j$, L; $t_j$ is the time, $j = 1, 2, \ldots, m$.

Formula (2.85) is a linear equation, shown in Fig. 2.45.
The intercepts that correspond to time \( t_1, t_2, \ldots, t_m \) are \( A_1, A_2, \ldots, A_m \), so:

\[
A_m - A_j = a(b + \log t_m) - a(b + \log t_j)
\]

\[
= a \log \left( \frac{t_m}{t_j} \right) = \frac{2.30}{4\pi T} \log \left( \frac{t_m}{t_j} \right)
\]

where \( j = 1, 2, \ldots, m \).

So:

\[
T = \frac{2.30 \log \left( \frac{t_m}{t_j} \right)}{4\pi (A_m - A_j)} \tag{2.86}
\]

Well loss constant \( C \) and transmissibility \( T \) can be calculated by Eqs. (2.85) and (2.86), respectively. Storage coefficient \( \mu^s \) can be calculated by Eq. (2.84), then (2.87) can be got:

\[
\log \mu^s = \log \frac{2.25 T t}{r_w^2} + \frac{4\pi T}{2.30} \left( C Q - \frac{s_g}{Q} \right) \tag{2.87}
\]

If \( r \) is replaced by \( r_w \), the effective radius of well \( r_c \) is no less than the actual well radius \( r_w \), and the above formula can be rewritten as:

\[
\log \mu^s \leq \log \frac{2.25 T t}{r_w^2} + \frac{4\pi T}{2.30} \left( C Q - \frac{s_g}{Q} \right) \tag{2.88}
\]

This formula is used to estimate the range of storage coefficient.

### 2.7.1.2 Application Steps

#### Case Study

The hydrogeological condition is the same as Sect. 2.5.3. The pumping test data are shown in Tables 2.29 and 2.30.

#### Table 2.29 Drawdown \( s \) (m)

<table>
<thead>
<tr>
<th>Flow ( Q ) (m(^3)/day)</th>
<th>Observation time ( t ) (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>1570</td>
<td>1.24</td>
</tr>
<tr>
<td>2335</td>
<td>1.92</td>
</tr>
<tr>
<td>2540</td>
<td>2.10</td>
</tr>
<tr>
<td>2757</td>
<td>2.36</td>
</tr>
</tbody>
</table>
1. Draw $s_g/Q - Q$ lines, shown in Fig. 2.46.
2. Transmissibility calculation

$$T_1 = \frac{2.3\log(40/10)}{4\pi(0.735 - 0.70) \times 10^{-3}} = 3.15 \times 10^3 \text{ m}^2/\text{day}$$

$$T_2 = \frac{2.3\log(150/10)}{4\pi(0.77 - 0.70) \times 10^{-3}} = 3.08 \times 10^3 \text{ m}^2/\text{day}$$

3. Well loss constant calculation

$$C = \frac{(0.79 - 0.70) \times 10^{-3}}{1600} = 5.6 \times 10^{-8}$$

### 2.7.2 Calculation of Transmissibility Coefficient and Water Storage Coefficient by Sensitivity Analysis Method Based on Pumping Test Data

#### 2.7.2.1 Sensitivity Analysis

When simulating an aquifer system, researchers have to determine the admissible deviation. If the admissible deviation does not affect the simulation results
significantly, the parameters of the actual system can be different. The admissible deviation is generally determined based on parameter variation induced into the system and the variation of characteristics in the system. These admissible deviations can be determined more effectively by using the sensitivity analysis.

In confined aquifer, according to Theis formula, there is

$$s = \frac{Q}{4\pi T} \int_{0}^{\infty} e^{-u} \frac{u}{u} \, du$$

where

$$u = \frac{r^2 \mu^*}{4Tt}$$

So the solution of an aquifer model can be written as follows:

$$h = h(x, y, t; T, \mu^*, Q)$$

where \( h \) represents the water head.

Considering the change of one parameter, for example, \( T \) is seen as variable, when \( T \) has an increment \( \Delta T \), it has

$$h^* = h^*(x, y, t; T + \Delta T, \mu^*, Q)$$

It is assumed that the solutions of the aquifer model depend on parameter \( T \) and \( \mu^* \). \( T, \mu^* \), and \( Q \) are all independent variables. So function \( h^*(x, y, t; T + \Delta T, \mu^*, Q) \) can be expanded to Taylor series. If the value of \( \Delta T \) is very little, the quadratic term and high order terms can be neglected

$$h^*(x, y, t; T + \Delta T, \mu^*, Q) = h(x, y, t; T, \mu^*, Q) + U_T \Delta T$$ (2.89)

$$U_T(x, y, t; T, \mu^*, Q) = \frac{\partial h(x, y, t; T, \mu^*, Q)}{\partial T}$$ (2.90)

Transmissibility coefficient sensitivity \( U_T(x, y, t; T, \mu^*, Q) \) will be represented by \( U_T \).

If sensitivity \( U_T \) and the initial water head are known, the new water head caused by variation \( \Delta T \) of transmissibility coefficient can be calculated by Eq. (2.89).

Similarly, the new water head caused by variation \( \Delta \mu^* \) of water storage coefficient \( \mu^* \) can be calculated by the following equation:

$$h^*(x, y, t; T + \Delta \mu^*, \mu^*, Q) = h(x, y, t; T, \mu^*, Q) + U_{\mu^*} \Delta \mu^*$$ (2.91)
Sensitivity of water storage coefficient \( U_{\mu^*} (x, y, t; T, \mu^*, Q) \) will be represented by \( U_{\mu^*} \).

Equations (2.90) and (2.92) indicate that to a given model, \( U_T \) and \( U_{\mu^*} \) need to be calculated. The response of model under a variety of variation can be calculated easily by (2.89) and (2.91) without re-calculation of model equations.

Sensitivity coefficient can be obtained from Theis formula by the definition of Eqs. (2.90) and (2.92):

\[
U_T = \frac{\partial s}{\partial T} = - \frac{s}{T} + \frac{Q}{4\pi T^2} \exp \left( - \frac{r^2 \mu^*}{4T} \right) \tag{2.93}
\]

\[
U_{\mu^*} = \frac{\partial s}{\partial \mu^*} = - \frac{Q}{4\pi T \mu^*} \exp \left( - \frac{r^2 \mu^*}{4T} \right) \tag{2.94}
\]

If \( \mu^* \) and \( T \) change, respectively, with the variation of \( \Delta \mu^* \) and \( \Delta T \), \( U_T \) and \( U_{\mu^*} \) can be obtained from Eqs. (2.93) and (2.94) can be used in Eqs. (2.89) and (2.91) to obtain the drawdown values. Data show that when \( \Delta \mu^* \) and \( \Delta T \) are no more than 20% of \( \mu^* \) and \( T \), respectively, Eqs. (2.89) and (2.91) is valid.

### 2.7.2.2 Least-Squares Fitting

The purpose of the sensitivity analysis is to obtain the least-squares fitting of actual pumping test data to Theis formula and then to obtain the best value of \( \mu^* \) and \( T \).

\( T \) and \( \mu^* \) are changed by \( \Delta T \) and \( \Delta \mu^* \). The new drawdown value \( s^* \) can be calculated by the following equation:

\[
s^* = s + U_T \Delta T + U_{\mu^*} \Delta \mu^* \tag{2.95}
\]

The actual drawdown measured at time \( t \) is represented by \( s_c(t) \). It is assumed that appropriate estimate can be made for \( \mu^* \) and \( T \), then \( s(t) \) is the drawdown obtained from Theis formula by these parameters. The preliminary estimate value of \( \mu^* \) and \( T \) can be changed by \( \Delta \mu^* \) and \( \Delta T \). The new drawdown value \( s^* \) is calculated by Eq. (2.95). The errors dropped to minimum with the aid of the following error function, and the better fitting results compared with actual pumping test data will be obtained.
\[ E(\Delta T, \Delta \mu^*) = \sum_{i=1}^{n} [s_c(t_i) - s^*(t_i)]^2 \]

\[ = \sum_{i=1}^{n} [s_c(t_i) - s(t_i) - U_T(t_i)\Delta T - U_{\mu'}(t_i)\Delta \mu^*]^2 \]

\[ = \sum_{i=1}^{n} [s_c(t_i) - s(t_i)]^2 - 2\Delta T \sum_{i=1}^{n} U_T(t_i)[s_c(t_i) - s(t_i)] + 2\Delta \mu^* \sum_{i=1}^{n} U_{\mu'}(t_i)[s_c(t_i) - s(t_i)] \]

\[ - 2\Delta \mu^* \sum_{i=1}^{n} U_{\mu'}(t_i)[s_c(t_i) - s(t_i)] + \sum_{i=1}^{n} [U_{\mu'}^2(t_i) \Delta \mu^2] \]

\[ + 2U_T(t_i)U_{\mu'}(t_i)\Delta \mu^* \Delta T + U_{\mu'}^2(t_i)\Delta T^2 \]

(2.96)

\( t_i \) represents any moment, when a test value of drawdown can be obtained. The error function is determined by the square sum of difference between measured values \( s_c \) and \( s^* \). Sensitivity coefficient \( U_T \) and \( U_{\mu'} \) are based on \( t_i \).

The first-order derivative of \( \Delta T \) and \( \Delta \mu^* \) are selected, and are equal to zero. Then the errors are minimum values. The equations of \( \Delta T \) and \( \Delta \mu^* \) are as follows:

\[ \frac{\partial E(\Delta T, \Delta \mu^*)}{\partial \Delta T} = -2 \sum_{i=1}^{n} U_T(t_i)[s_c(t_i) - s(t_i)] + 2\Delta \mu^* \sum_{i=1}^{n} U_{\mu'}(t_i)U_T(t_i) \]

\[ + 2\Delta T \sum_{i=1}^{n} U_{\mu'}^2(t_i) = 0 \]

(2.97)

\[ \frac{\partial E(\Delta T, \Delta \mu^*)}{\partial \Delta \mu^*} = \mu^* - 2 \sum_{i=1}^{n} U_{\mu'}(t_i)[s_c(t_i) - s(t_i)] + 2\Delta \mu^* \sum_{i=1}^{n} U_{\mu'}(t_i) \]

\[ + 2\Delta T \sum_{i=1}^{n} U_{\mu'}(t_i)U_T(t_i) = 0 \]

(2.98)

\( \Delta T \) can be obtained by Eq. (2.97):

\[ \Delta T = \left\{ \sum_{i=1}^{n} \{ U_T(t_i)[s_c(t_i) - s(t_i)] \} \Delta \mu^* \right\} / \sum_{i=1}^{n} U_{\mu'}^2(t_i) \]

(2.99)
\[ \Delta \mu^* \] can be obtained by Eq. (2.98):

\[
\Delta \mu^* = \left( \sum_{i=1}^{n} [U_T^2(t_i)] \sum_{i=1}^{n} \{ U_{\mu^*}(t_i) [s_c(t_i) - s(t_i)] \} - \left\{ \sum_{i=1}^{n} [U_{\mu^*}(t_i) U_T(t_i)] \right\} \right) \\
\left( \sum_{i=1}^{n} \{ U_T(t_i) [s_c(t_i) - s(t_i)] \} \right) / \left( \sum_{i=1}^{n} [U_T^2(t_i)] \right) \\
\left( \sum_{i=1}^{n} [U_{\mu^*}(t_i)] \sum_{i=1}^{n} [U_T^2(t_i)] - \left\{ \sum_{i=1}^{n} [U_{\mu^*}(t_i) U_T(t_i)] \right\} \right)^2
\]

\[ (2.100) \]

The best value (namely the best fitting results) of \( \Delta \mu^* \) can be obtained from Eq. (2.100). By substitution of \( \Delta \mu^* \) into Eq. (2.99), the best fitting value of \( \Delta T \) can be found.

The values of \( \Delta T \) and \( \Delta \mu^* \) can be used to correct the first estimated values of \( T \) and \( \mu^* \). The improved values of \( T \) and \( \mu^* \) are used in the program of least-squares fitting in order to obtain the new values of \( \Delta T \) and \( \Delta \mu^* \). This cycle continues until \( \Delta T \) and \( \Delta \mu^* \) are little enough to be negligible. The best values after iteration of \( j \) times can be obtained by the following equations:

\[ T_j = T_{j-1} + \Delta T_{j-1} \quad (2.101) \]

\[ \mu_{j-1}^* = \mu_{j-1}^* + \Delta \mu_{j-1}^* \quad (2.102) \]

The program block diagram of the calculation of hydrogeological parameters of the aquifer by sensitivity analysis is shown in Fig. 2.47.

If the initial values of \( T \) and \( \mu^* \) are particularly poor, the program may not be convergent. Available data indicate that good convergence can be obtained even for the case that the initial values of \( \mu^* \) and \( T \) are less or greater for two orders of magnitude.

From the above, the best transmissibility coefficient and water storage coefficient are obtained from least-squares fitting by using sensitivity analysis in order to fit Theis formula by actual pumping test data automatically. That method can be applied in more complicated hydrogeological conditions.

### 2.7.3 Hydrogeological Parameter Optimization Based on Numerical Method and Optimization Method Coupling Model

The key question of reversing hydrogeologic parameters by numerical method is to obtain a set of parameters which can objectively represent the hydrogeological characteristics of the actual aquifer. The test standard is the error between water table (head) of every node calculated by mathematical model and the actually measured water table (head).
2.7 Other Methods for Hydrogeological Parameters Calculation

Input $r$, $Q$, $t(n)$, $s(n)$ and control variable $\varepsilon_T$, $\varepsilon_\mu$

$j = 1$

When $j = 1$, calculate $u(t_i)$, $W(u(t_i))$, $s_j(t_i) = \frac{Q}{4a_0T} W(u(t_i))$

When $j \neq 1$, calculate $s_j(t_i) = s_{j-1}(t_i) + U_{t_j}(t_i) \Delta T_{j-1} + U_{\mu_j}(t_i) \Delta \mu_{j-1}$

Calculate the sensitive coefficient $U_{t_j}(t_i)$ and $U_{\mu_j}(t_i)$

$i = i + 1$

$i = n$?

Calculate $E(j) = \sum_{i=1}^{n} [s_j(t_i) - s_{j}(t_i)]^2$

Calculate $\Delta \mu_j$, $\Delta T_j$

$\Delta \mu_j \leq \varepsilon_\mu$, $\Delta T_j \leq \varepsilon_T$?

Input $j$, $T$, $\mu^*$, $E$

$j = j + 1$

$T_j = T_{j-1} + \Delta T_{j-1}$, $\mu_j^* = \mu_{j-1}^* + \Delta \mu_{j-1}$

End

1. Output $j$, $T$, $\mu^*$, $E$
2. Output the hole number $r, \theta, \varepsilon_T, \varepsilon_\mu, T, \mu^*$
3. Output $s(i), s_j(i)$ and $s(i-s_j(i)) (i=1,2,...n)$

Fig. 2.47 The program block diagram of the calculation of hydrogeological parameters of the aquifer by sensitivity analysis
When using the error standard of the least square method, the identification of parameters can be formulated as the optimization problem as follows:

Objective function:

\[
\min E(k_1^*, k_2^*, \ldots, k_n^*) = \frac{1}{C_3} \sum_{i=1}^{J} \sum_{j=1}^{N} \omega_{ij} \left[ H_j(t_i) - H_0^j(t_i) \right]^2
\]  

(2.103)

Constraints:

\[
\alpha_i \leq k_i \leq \beta_i \quad (i = 1, 2, \ldots, n)
\]  

(2.104)

where \( E \) is the objective function; \( k_1^*, k_2^*, \ldots, k_n^* \) are a group of optimal parameters; \( J \) is the total number of observation periods; \( N \) is the total number of observation points; \( H_j(t_i) \) is the calculated water table of node \( j \) at \( t_i \); \( H_0^j(t_i) \) is the measured water table of node \( j \) at \( t_i \); \( \omega_{ij} \) is the weight factor. The higher the precision is, the greater \( \omega_{ij} \) is; \( k_i \) is the \( i \)th parameter of any group; \( \alpha_i \) is the lower limit of the \( i \)th parameter; \( \beta_i \) is the higher limit of the \( i \)th parameter.

There are many methods of solving the optimization problems. The workload of the commonly used trial method is huge because it lacks a convergence criteria in every repeated trial calculation. The parameter in this process is blind and waste of time especially when there are a large number of unknown parameters. Hydrogeological parameter optimization based on optimization method can overcome the shortcomings of the trial method. There are many optimization methods and one of the direct unconstrained optimization methods—the stepped up simplex method will be introduced here.

### 2.7.3.1 The Basic Principle of the Advanced Simplex Method

The basic the principle of the advanced simplex method is: calculate the objective function value \( E \), respectively at \( n + 1 \) simplex vertexes in \( E^n \) and compare them to determine the worst point, the second worst point and good points. Judge the approximate trend of function variation from the size relationship of the points. Choose reflection, extension, compression and so on to structure new simplex under different circumstances until the function value of simplex vertexes reach to the required minimum value. Then that set of parameter values are the optimal parameter values. The simplex in \( E^n \) means a polyhedron with \( n + 1 \) vertexes. If the edge-length equals to each other, the simplex is called the regular simplex. The following point is given in \( n \)—dimension space:

\[
K_0 = (K_0^0, K_0^1, \ldots, K_0^n)^T
\]
where $K^0$ is a vertex of a regular simplex with the edge-length of $a$. Set

$$p = \frac{\sqrt{n+1} + n - 1}{n\sqrt{2}} a$$  \hspace{1cm} (2.105)$$

$$q = \frac{\sqrt{n+1} - 1}{n\sqrt{2}} a$$  \hspace{1cm} (2.106)$$

The rest $n$ vertexes:

$$K^i = (K_1^i, K_2^i, \ldots, K_n^i)^T \quad (i = 1, 2, \ldots, n)$$  \hspace{1cm} (2.107)$$

Construct as follows:

$$K_j^i = \left( K_j^0 + q \right) \quad (j \neq i)$$  \hspace{1cm} (2.108)$$

$$K_i^i = \left( K_i^0 + p \right)$$  \hspace{1cm} (2.109)$$

Namely:

$$K^1 = \left( K_1^0 + p, K_2^0 + q, \ldots, K_n^0 + q \right)^T$$

$$K^2 = \left( K_1^0 + q, K_2^0 + q, \ldots, K_n^0 + q \right)^T$$

$$\ldots$$

$$K^n = \left( K_1^0 + q, K_2^0 + q, \ldots, K_n^0 + p \right)^T$$  \hspace{1cm} (2.110)$$

then, $K_1^0, K_2^0, \ldots, K_n^0$ constitute a regular simplex with the edge-length of $a$.

2.7.3.2 Iterative Steps of the Advanced Simplex Method

1. The initial simplex is constructed with the given initial point $K^0$. The vertexes are assumed as $K^1, K^2, \ldots, K^n$. The permissible error $e > 0$, then calculate:

$$E_i = f(K^2), \quad (i = 1, 2, \ldots, n).$$

Set:

$$E_i = f(K_i^i) = \min \{ f(K^0), f(K^1), \ldots, f(K^n) \}$$  \hspace{1cm} (2.111)$$

$$E_h = f(K^h) = \min \{ f(K^0), f(K^1), \ldots, f(K^n) \}$$  \hspace{1cm} (2.112)$$

where $K^i$ and $K^h$ are called the best and worst points of the simplex.
If the worst point $K^h$ is removed, the rest $n$ vertexes $K^0, K^1, \ldots, K^{h-1}, K^{h+1}, \ldots, K^n$ constitute the simplex of $(n - 1)$-dimension space. Its center is:

$$K^f = \frac{1}{n} \left( \sum_{j=0}^{n} K^j - K^h \right) \quad (2.113)$$

2. Reflection: $K^h$ is reflected to $K'$ in the center of $K^f$.

$$K' = K^f + \alpha (K^f - K^h) \quad (2.114)$$

Among which $\alpha > 0$ is reflection coefficient and $\alpha = 1$ generally. Because $K^h$ is the worst point and through reflection there will be:

$$E_r < E_h \quad (2.115)$$

then point $K'$ better than $K^h$ can be obtained, just like shown in Fig. 2.48.

3. Extension: after reflection not only does Eq. (2.115) hold, but there is a further step:

$$E_r < E_l \quad (2.116)$$

which indicates that $K'$ is better than $K^f$. Thus, the reflection direction is an effective direction of reducing the function value. So the simplex is being extended in this direction. Set:

$$E_e = K^f + (K' - K^f)$$

Among which $\gamma > 1$ is extension coefficient and generally $\gamma = 2$. If:

$$E_e < E_h \quad (2.117)$$

Fig. 2.48 Schematic diagram of iterative steps of the advanced simplex method
then $K^h$ is replaced by $K^c$ and the rest $n$ vertexes are unchanged. A new simplex is constructed just like shown in Fig. 2.48(1). Turn to step 6.

If Eq. (2.111) holds and Eq. (2.117) does not, $K^h$ is replaced by $K^r$ to construct a new simplex, just like shown in Fig. 2.28(2). Turn to step 6.

4. Compression: if Eq. (2.116) does not hold, namely the reflection point $K^r$ is not better than the best point $K^f$ of the original simplex, there are two circumstances:

   (1) When $j \neq h$, set $E_r \leq E_j$, which means the reflection point $K^r$ is not worse than all the rest vertexes except the worst point $K^h$. Then $K^h$ is still replaced by $K^r$ to construct a new simplex. Turn to step 6.

   (2) If for every $K^h$, there is:

\[
E_r > E_j
\]

then the reflection produces a new bad point. The simplex is being compressed in this direction. There are two cases as shown in Fig. 2.48(3).

In the first case, if

\[
E_r > E_h
\]

(2.118)

namely the reflection point is worse than the worst point of the original simplex, $K^c$ is abandoned. Compress vector $K^h - K^f$:

\[
K^c = K^f + \beta(K^h - K^f)
\]

among which $0 < \beta < 1$ is compression coefficient and generally $\beta = 0.5$.

In the second case, if Eq. (2.118) does not hold, compress vector $K^r - K^f$:

\[
K^c = K^f + \beta(K^r - K^f)
\]

Discrimination of whether the compression point $K^c$ is worse than the worst point of the original simplex $K^h$ is necessary, namely whether the following equation holds

\[
E_c > E_h
\]

If it holds, the compression point $K^c$ is abandoned and turn to step 5. If not, $K^h$ is replaced by $K^c$ to construct a new simplex and turn to step 6.

5. Decreasing the edge-length: the best point $K^d$ of the origin simplex remains unchanged and the rest vertexes are compressed to $K^d$ for a half distance, namely:

\[
K^i = \frac{1}{2}(K^i + K^d), \quad i = 0, 1, 2, \ldots, n
\]
A new simplex is obtained and the edge-length is half of the edge-length of the origin simplex. Turn to step 6.

6. Discrimination

\[
\left\{ \frac{1}{n+1} \sum_{i=0}^{n} (E_i - E_i)^2 \right\}^{1/2} \leq \varepsilon
\]

whether or not the inequality holds. If it holds, then stop calculation and \( K^* = K^l \). If not, return to step 1.

### 2.7.3.3 Application

When fitting the optimal parameters by the advanced simplex method and the finite element program, the advanced simplex method is the main program. After determining the optimizing direction and a set of parameters, the subroutine needs to be called, as Fig. 2.49 shows. Whether the parameters optimized by the iterative steps of the advanced simplex method coincidence the required upper and lower limit range. If they coincide, finite element subroutine is called to calculate the water table of the nodes and the function value \( E \). Return to the main program after consummation. Compare the size of the function value of every vertex of the simplex to determine the next lookup direction until the optimal vertex is found, namely the optimal parameters.

---

**Fig. 2.49** Subdiagram of parameter adjusting

1. Give a set of parameter: \( K_1, K_2, \cdots, K_n \)

2. \( \alpha_i \leq K_i \leq \beta_i \) (\( i = 1, 2, \cdots, n \))

3. No \( \Rightarrow \) Stop and adjust the parameters

4. Calculate the water level of nodes by calling finite element subroutine

5. Calculate: 

\[
E = \sum_{i=1}^{l} \sum_{i=1}^{M_i} c_{ij} \left[ H_i(t_i) + H_j^b(t_i) \right]^2
\]
When the aquifer parameters of the calculation area are divided into several zones, the parameters of each zone can be called, respectively. In the end, the parameters of the whole zone will be called comprehensively.

### 2.8 Case Study

1. Object: multiple-hole unsteady and steady pumping tests are conducted in situ to know about the site requirement, and to figure out the experimental method and information collection of steady and unsteady flow pumping tests, additionally to calculate the hydrogeological parameters by various theories.
2. The plane arrangement of dewatering wells is present as below.

![Diagram](image)

3. The requirement of pumping test site
   - Once the pumping test starts, any tester should keep on duty without any absence. The water table and water discharge should be measured and recorded timely.
   - Each team should take turns 15 min earlier to leave enough time for preparation. During the time for overlap, two teams should measure the water table together at the same location. Adjustment for the measuring tape also should be taken.
   - Before pumping, natural static water table should be measured (the accuracy is best for mm).
   - Starting pumping, the duration for measuring should be 1’, 2’, 3’, 4’, 6’, 8’ 10’, 15’, 20’, 25’, 30’ 40’, 50’, 60’, 90’, 120’... (afterward measure once in each 30’); measuring: water table and water discharge in main well, water table in observation wells. (means min)
   - Each team collects and analyzes the data, including:
     1. Main well: $Q-t$ curve, $s-t$ curve; Observation wells: $s-t$ curve;
     2. All wells: $s-lg t$ curve;
     3. All wells: $lg s-lg t$ curve;
In case of emergency occurrence, timely report should be informed to instruction teacher. The pump should be stopped if necessary. Or the power is cut down by accident; the water table in recovery duration should be measured right away.

The recorded data could not be changed if not necessary.

4. Make the report the experimental summary, including:

(1) objects and requirement of pumping tests;
(2) pumping method and procedure;
(3) the main results of pumping tests;
(4) treatment of abnormal circumstance during experiments and quality assessment and conclusions.

5. Draw the comprehensive resultant curves by pumping tests.

6. Use steady and unsteady flow method (fitting curve method, linear graphic method, water table recovery method) to calculate the hydrogeological parameters of aquifer.

7. Attach resultant table of pumping tests.

Summary of aquifer parameter results

<table>
<thead>
<tr>
<th>Calculation method</th>
<th>Data</th>
<th>Aquifer parameter</th>
<th>Data</th>
<th>Aquifer parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K$ (m/day)</td>
<td>$T$ (m$^2$/day)</td>
<td>$\alpha$ (m$^2$/day)</td>
</tr>
<tr>
<td>Steady flow</td>
<td>Single well</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Multiple wells</td>
<td></td>
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<td></td>
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<tr>
<td>Unsteady flow</td>
<td>Fitting curve method</td>
<td>Observation well 2</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Observation well 3</td>
<td></td>
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<tr>
<td></td>
<td>Linear graphic method</td>
<td>Observation well 2</td>
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<td></td>
<td></td>
<td>Observation well 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Water table recovery method</td>
<td>Observation well 2</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Observation well 3</td>
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<td>Recommendation value</td>
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</table>
2.9 Exercises

1. What parameters reflect the hydrological properties of aquifer?
2. What is the main task of pumping test?
3. What is the steady flow pumping test? And how about unsteady flow pumping test?
4. What is the difference between fully penetrated well pumping test and partial penetrated well pumping test?
5. How to collect and analyze the data of pumping test?
6. What is the object of water pressure test? How to collect and analyze the data?
7. How to measure the groundwater table, flow direction and flow velocity?
8. According to steady pumping test, what parameters can be obtained? What methods can be used to calculate hydraulic conductivity?
9. How to calculate the influence radius by steady pumping test?
10. According to unsteady pumping test, what parameters can be obtained?
11. How to estimate the coefficient of transmissibility and well loss constant from multi-water discharge test?
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