Preface

The dynamical systems theory was shaped by the investigation of stellar and planetary motions. The applications were later extended to isolated mechanisms, electronics and other processes. Analysis was dominated by assumptions of continuity and extension to infinity. At present we face new challenges, such as events with a finite life and models of large dimensions. The processes are interconnected, rather than isolated, and the motions are alternately continuous and discrete. To meet these challenges we need appropriate tools: differential equations with a discontinuous right-hand side, differential equations with impulses and hybrid systems.

Hybrid systems are a recent concept in dynamical systems theory and have important applications. Hybrid dynamical systems have not been defined in a precise way yet. Roughly speaking, we call a model as hybrid if it is a combination of continuous and discrete dynamics. One can say that a system is hybrid if some of the dependent variables satisfy differential, while others - discrete equations. In addition, one may have a variable that satisfies a differential and a discrete equation alternately. More formal and general definitions of hybrid systems are given in [279, 280]. Further information can be found in [55, 80, 96, 97, 101, 128, 145, 194–196, 201, 209, 210, 213, 227, 231, 279, 280, 282, 338].

In the early 80's, K. Cook, S. Busenberg, J. Wiener and S. Shah started developing a new type of differential equations. They called these differential equations with piecewise constant argument. Many interesting results and many applications of this theory have been produced in the last three decades. The existence and uniqueness of solutions, oscillations and stability, integral manifolds and periodic solutions, and numerous other issues have been intensively discussed. Besides the theoretical analysis, various models in biology, mechanics and electronics were introduced through these systems.

The original method of investigation of these equations was based on the reduction to discrete systems. That is, only the values of solutions at moments that are integers or multiples of integers were discussed. Moreover, systems must be *linear* with respect to the values of solutions, if the argument is not deviated. These requirements significantly limit the theoretical depth of investigation, as well as the scope of real world problems that can be modeled using these equations. From the analysis of *MathSciNet*, we found that the number of papers on differential equations with piecewise constant argument published over the last ten and five years is 138 and 69, respectively. This number is 12 in 2009, and 11 in 2010. Thus, it is clear that the rate of publication is quite low and has not changed over the last ten years, and that a new methodological approach is necessary to develop the theory further.

In the Conference on Differential and Difference Equations at the Florida Institute of Technology, 2005, it was proposed [8] to consider non-linear differential equations with a more general type of piecewise constant argument, and equivalent integral equations, as the basis of investigation. In our opinion, this line of thought should provide new direction for the theory. In this book, we will show in detail how this can be done. The reader will see that not only can the scope of problems studied be expanded through this approach, but one can also observe entirely new phenomena and deepen the parallelism with the theory of ordinary differential equations, despite the fact that the systems under discussion are functional differential equations.

Next, a compartmental model of blood pressure distribution is considered. The systemic arterial pressure is assumed to be dominating. The heart contraction moments are supposed to be prescribed, and the cases where they behave periodically and almost periodically are examined.

We also consider the case where the moments are defined recursively, so that chaotic phenomena appear. Additionally, the case where the moments are not prescribed and are variable is investigated.

Finally, a biological model of integrate-and-fire oscillators is examined. The main result is the solution of the synchronization problem. That is, we find conditions that ensure that identical or not quite identical units of the system fire in unison. This problem is of extreme importance for the cardiac pacemaker research. Examples with numerical simulations are provided to validate the theoretical results, and prospects for further investigation are discussed.

The first chapter serves as the introductory part of the book. We present the description of the systems and the main definitions, and outline the literature.

In the following chapter, linear and quasilinear systems with argument functions that are both advanced and delayed, are considered. It is shown that the set of all solutions of the linear homogeneous equation is a finite-dimensional linear space under certain conditions. The fundamental matrix of solutions is built. For quasilinear systems, the integral representation formula is defined. Moreover, basic concepts of stability theory are provided for these equations.

In the third chapter, we prove the reduction principle for differential equations with piecewise constant arguments. Theorems on the existence of integral manifolds, their stability, and the stability of the zero solution are proved.

The fourth chapter is devoted to periodic solutions of perturbed linear systems. The method of the small parameter is used as an instrument in this chapter. Both non-critical and critical cases are investigated. Continuous and differentiable dependence of solutions on initial data is examined to prove the main results. Simulations are provided to illustrate the theoretical analysis.

The stability of nonlinear systems is analyzed in the fifth chapter. We use the Lyapunov-Razumikhin technique, as well as the method of Lyapunov functions, to investigate stability. We apply the obtained theorems to prove the stability of the zero solution of the logistic equation.

Differential equations with state-dependent piecewise constant argument are described in the sixth chapter. We analyze the conditions of existence and uniqueness of the solutions of these systems in a general case, and for quasilinear equations. Then periodic solutions and the stability of the zero solution are discussed. The theory of differential equations with discontinuous right-hand-sides and with variable moments of impulses shows the way to developing the theory of systems with state-dependent piecewise constant argument.

The existence of almost periodic solutions is the main subject of the seventh chapter. Using Bohner type discontinuous almost periodic functions and the technique developed in [117], we prove that the exponential dichotomy is a sufficient condition for the existence of these solutions.

The following chapter deals with the stability of neural networks. The biological explanations of advance and delay arguments are discussed. The methods of Lyapunov functionals and functions are applied to obtain the main results. Several examples are provided to illustrate the theorems.

Blood pressure distribution is investigated in the ninth chapter. A model where the systemic arterial pressure dominates is developed. The existence of periodic, almost periodic motions, stability, and chaotic behavior are examined. We consider both cases where the moments of jumps of the pressure are prescribed, and where they are variable. In the last case the threshold concept is formalized.

The last chapter of the book is devoted to the integrate-and-fire model of biological oscillators. This model is appropriate for cardiac pacemaker analysis, as well as for neural networks research. Mathematical models of integrate-and-fire biological oscillators, as far as we know, were initiated in [260,262]. In this book, a method of analysis of integrate-and-fire models that consist of pulse-coupled biological oscillators is developed. The method is based on a thoroughly constructed map and the technique of investigation of differential equations with discontinuities at non-fixed moments.

Synchronization and existence of periodic motions of identical and not quite identical oscillators are investigated. The second Peskin's conjecture, [262], has been solved. The synchronized regime of identical oscillators admits moments of fire that are equally-distanced. Their discrete dynamics can be represented as a solution of simple differential equations, if needed. Consequently, the model belongs to the class of systems considered in our book.

This book contains very recent results, and any comments and suggestions would be greatly appreciated.

The author would like to thank Duygu Aruğaslan, Cemil Büyükadalı, Mehmet Onur Fen, Mehmet Turan and Enes Yılmaz for discussions of the problems considered in the book and for collaboration on several joint papers.

http://www.springer.com/978-94-91216-02-2

Nonlinear Hybrid Continuous/Discrete-Time Models
Akhmet, M.
2011, XIII, 216 p., Hardcover
ISBN: 978-94-91216-02-2
A product of Atlantis Press