

# Preface

This monograph came out of my thesis work under the supervision of my Ph.D. advisor Mario Bonk during my graduate studies at the University of Michigan, Ann Arbor, and later at the University of California, Los Angeles. It focuses on the dynamics, more specifically ergodic theory, of some continuous branched covering maps on the 2-sphere, called expanding Thurston maps.

More than 15 years ago, Mario Bonk and Daniel Meyer became independently interested in some basic problems on quasisymmetric parametrization of 2-spheres, related to the dynamics of rational maps. They joined forces during their time together at the University of Michigan and started their investigation of a class of continuous (but not necessarily holomorphic) maps modeling a subclass of rational maps. These maps belong to a bigger class of continuous maps on the 2-sphere studied by William P. Thurston in his famous characterization theorem of rational maps (see [DH93]). As a result, Mario Bonk and Daniel Meyer called their maps *expanding Thurston maps*. Related studies were carried out by other researchers around the same time, notably Peter Haïssinsky and Kevin Pilgrim [HP09], and James W. Cannon, William J. Floyd, and Walter R. Parry [CFP07].

By late 2010, Mario Bonk and Daniel Meyer had summarized their findings in a reader-friendly arXiv draft [BM10] entitled *Expanding Thurston maps*, which they initially intended to publish in the AMS Mathematical Surveys and Monographs series. In order to make the material even more accessible, they decided later to expand their draft. This led to a long delay for the final published version [BM17] with almost twice the size of [BM10].

I was introduced to expanding Thurston maps by Mario Bonk soon after I joined in the graduate program at the University of Michigan. I quickly got deeply fascinated by this subject due to the connections to geometry, analysis, combinatorics, and dynamical systems.

I finished my first project on the periodic points and properties of the measures of maximal entropy of expanding Thurston maps under the supervision of Mario Bonk (later appeared in [Li13], see Chap. 4) after we moved to Los Angeles. I then decided to continue working on the ergodic theory of expanding Thurston maps, on which I eventually wrote my thesis.

This monograph covers investigations on the measures of maximal entropy, and more generally, equilibrium states of expanding Thurston maps, and their relations to the periodic points and the preimage points. In order to study the equilibrium states, the theory of thermodynamical formalism for Hölder continuous potentials is established in our context (see Chap. 5). The study of equidistribution results also leads to a close investigation on the expansion properties of our dynamical systems (see Chap. 6) and the discovery of some large deviation results (see Chap. 7).

This monograph is also intended to serve as a basic reference for the theory of thermodynamical formalism in our context. The applications to the study of the dynamical zeta functions were also kept in mind when this monograph was being prepared. As such, complex-valued function spaces are used whenever they do not introduce too much complication.

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