Contents

1 Calculus of Variations on Fibre Bundles ................................. 1
   1.1 Infinite Order Jet Formalism ..................................... 1
   1.2 Variational Bicomplex ............................................ 7
       1.2.1 Cohomology of the Variational Bicomplex .............. 9
   1.3 Lagrangian Formalism ............................................. 12

2 Noether’s First Theorem ................................................ 17
   2.1 Lagrangian Symmetries ............................................. 17
   2.2 Gauge Symmetries: Noether’s Direct Second Theorem .......... 20
   2.3 Noether’s First Theorem: Conservation Laws .................. 23

3 Lagrangian and Hamiltonian Field Theories .............................. 27
   3.1 First Order Lagrangian Formalism ............................... 27
   3.2 Cartan and Hamilton–De Donder Equations ...................... 30
   3.3 Noether’s First Theorem: Energy-Momentum Currents ......... 32
   3.4 Conservation Laws in the Presence of a Background Field .... 34
   3.5 Covariant Hamiltonian Formalism ............................... 36
   3.6 Associated Lagrangian and Hamiltonian Systems ............. 41
   3.7 Noether’s First Theorem: Hamiltonian Conservation Laws .... 47
   3.8 Quadratic Lagrangian and Hamiltonian Systems ............. 49

4 Lagrangian and Hamiltonian Nonrelativistic Mechanics ................ 59
   4.1 Geometry of Fibre Bundles over $\mathbb{R}$ ........................ 60
   4.2 Lagrangian Mechanics. Integrals of Motion .................... 63
   4.3 Noether’s First Theorem: Energy Conservation Laws .......... 67
   4.4 Gauge Symmetries: Noether’s Second and Third Theorems ...... 71
   4.5 Non-autonomous Hamiltonian Mechanics ....................... 73
   4.6 Hamiltonian Conservation Laws: Noether’s Inverse First
       Theorem .......................................................... 80
   4.7 Completely Integrable Hamiltonian Systems ................... 84
5 Global Kepler Problem ............................................. 93

6 Calculus of Variations on Graded Bundles ......................... 103
  6.1 Grassmann-Graded Algebraic Calculus ......................... 103
  6.2 Grassmann-Graded Differential Calculus ...................... 106
  6.3 Differential Calculus on Graded Bundles ...................... 109
  6.4 Grassmann-Graded Variational Bicomplex ..................... 121
  6.5 Grassmann-Graded Lagrangian Theory ......................... 127
  6.6 Noether’s First Theorem: Supersymmetries ................... 129

7 Noether’s Second Theorems ........................................ 135
  7.1 Noether Identities: Reducible Degenerate
      Lagrangian Systems ........................................ 136
  7.2 Noether’s Inverse Second Theorem ............................ 145
  7.3 Gauge Supersymmetries: Noether’s Direct Second Theorem ... 148
  7.4 Noether’s Third Theorem: Superpotential ..................... 152
  7.5 Lagrangian BRST Theory ...................................... 155

8 Yang–Mills Gauge Theory on Principal Bundles ....................... 163
  8.1 Geometry of Principal Bundles ................................ 163
  8.2 Principal Gauge Symmetries ................................... 171
  8.3 Noether’s Direct Second Theorem: Yang–Mills Lagrangian ... 173
  8.4 Noether’s First Theorem: Conservation Laws ................. 175
  8.5 Hamiltonian Gauge Theory ..................................... 177
  8.6 Noether’s Inverse Second Theorem: BRST Extension .......... 179

9 SUSY Gauge Theory on Principal Graded Bundles ..................... 183

10 Gauge Gravitation Theory on Natural Bundles ....................... 189
  10.1 Relativity Principle: Natural Bundles ....................... 189
  10.2 Equivalence Principle: Lorentz Reduced Structure .......... 191
  10.3 Metric-Affine Gauge Gravitation Theory ..................... 194
  10.4 Energy-Momentum Gauge Conservation Law ................... 197
  10.5 BRST Gravitation Theory ..................................... 199

11 Chern–Simons Topological Field Theory ............................ 201

12 Topological BF Theory ........................................... 207

Glossary .......................................................... 211

Appendix A: Differential Calculus over Commutative Rings .......... 213

Appendix B: Differential Calculus on Fibre Bundles .................. 227
Noether's Theorems
Applications in Mechanics and Field Theory
Sardanashvily, G.
2016, XVII, 297 p., Hardcover
A product of Atlantis Press