Researching on the Dynamic Dependence Structure of Liquidity

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Abstract The dependence structure of liquidity is ubiquitous which is described as cooperating of fluctuation about liquidity index between different financial markets simultaneously, especially in extreme cases such as financial crisis. This paper attempts to show the characteristic of liquidity dependence structure of stock market in China and the United States by time-varying copula function. The results indicate that the liquidity dependence structure is steady on the whole although the tail correlation probability increased slightly when the sub-prime mortgage crisis broke out. Research shows that dependence structure is impressionable to domestic regulation and policy although the life of "policy effect" is transitory.

Keywords Dependence structure · Liquidity · Policy · Time-varying copula function

1 Introduction

The correlative relationship is increasingly complex between financial markets which cause the different kind of liquidity dependence structures along with financial globalization. A lot of literatures have showed that this correlation between financial variables significantly enhanced, especially in the financial crisis or other extreme cases.

Chordia et al. [1] study daily changing of liquidity in stock and bond market and find the common factors for liquidity volatility. Zeng and Luo [2] show that the correlative relationship of liquidity between stock and bond market indicate that daily liquidity volatility in bond market is ahead of stock market about three days

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and no lead-lag relationship for monthly liquidity. Boyson et al. [3] show that the lack of liquidity of the stock market is the main factor that cause liquidity fund market fluctuate. Liang and Xu [4] research the leverage effect and correlation between yield and liquidity in ShangHai and ShenZhen stock markets based on asymmetric SV model. Kaul and Stefanescu [5] study cooperation in the foreign exchange market by intra and daily data and point out that this cooperation is susceptible to foreign reserve and financing demand. Cheung and Lo [6] study the spillover effect of liquidity of fifty financial markets and point out that there is an obvious synergistic effect across markets. Lien et al. [7] indicate that the nonlinear dependence structure present asymmetrical time-varying characteristic between spot and future markets. Correlation coefficient test is main method to study dependence structure, such as King and Wadhwani [8], which analyze the changing of dependency level around stock market crash in 1987 and point out that correlation between Europe and the United States stock market was enhanced after the crisis. Scholars have focused on dynamic dependence structure, especially in extreme cases as the volatility aggravated after 1990's. Lai et al. [9] research the dependency between exchange rate and stock market by GJR–GARCH model which show that there is a asymmetry threshold co-integration. Similar literature, such as Zhang et al. [10] and Chen [11], show that there is a dynamic relationship of yield for a long time between Chinese and American stock market by co-integration analysis or GARCH model. Since the copula function was introduced to the financial research, it has been widely applied, and become one of the main ways to study correlation between financial markets. Patton [12] put forward four kinds of time-varying copula function based on different distribution functions. According to the result of Patton [12], Wang et al. [13], Jiang et al. [14] focus on the dynamic correlative relationship between different financial markets and get similar results that there is some dependence structure, especially in extreme case.

At present, the literatures about liquidity dependence structure usually focus on domestic market or heterogeneous markets across countries, such as spot and futures market, stock and bonds market and so on. However, there is few literatures studying liquidity dependence structure between homogeneous markets across countries. There are four methods used to research function. Compared with copula function, the other methods exist some disadvantages which empirical test may be deflected from reasonable value or inaccuracy if there are characteristics of non-linearity, conditional heteroscedasticity, and non-normal distribution in sample data. While dynamic copula function gives a satisfying description to microscopic characteristics of dynamic nonlinear correlation across financial markets, it is adaptive to overcome the trouble that distribution function of financial variables is inconclusive according to the sample data. Considering the core role of the stock market and bilateral trade dependency, this paper focus on liquidity dependence structure characteristics between China and the U.S stock market based on the time-varying copula function.

2 Time-Varying Copula Function

Differences from constant copula function, the parameters of time-varying copula function are dynamic over time, so it could describe the micro dynamic characteristics of financial variables. According to the contribution of Patton [12], there are four kinds of time-varying copula function: time-varying normal copula function, time-varying T copula function, time-varying rotated-Gumbel copula function and time-varying Symmetrized Joe-Clayton copula function [12].

2.1 Time-Varying N-Copula Function

Distribution function and correlation coefficient equation of time-varying N-copula function is respectively

$$N - Copula : C(u_1, u_2, \ldots, u_N; \rho) = \Phi_p\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \ldots, \Phi^{-1}(u_N)\right)$$
$$\rho_{N,t} = \Lambda\left(\omega_N + \beta_N \rho_{t-1} + \alpha_N \times \tfrac{1}{10}\sum_{j=1}^{10} \Phi^{-1}\left(u_{t-j}\right)\Phi^{-1}\left(v_{t-j}\right)\right.$$

$\Phi_p(\cdot)$ is standard normal distribution function; $\Phi^{-1}(\cdot)$ is inverse function of $\Phi_p(\cdot)$. $\Lambda(x) = (1 - e^{-x})/(1 + e^{-x})$ is a modified Logistic transformation which make $\rho_{N,t}$ lie in [−1, 1]. Time-varying N-copula function is satisfactory to describe symmetrical correlation, but is not sensitive to upper and lower tail. In particular, the correlation coefficient equal 0 if $\rho_{N,t} < 1$, otherwise equal 1.

2.2 Time-Varying T-Copula Function

$$T - Copula : C(u_1, u_2, \ldots, u_N; \rho, v) = T_{p,v}\left(T_v^{-1}(u_1), T_v^{-1}(u_2), \ldots, T_v^{-1}(u_N)\right)$$
$$\rho_{T,t} = \Lambda\left(\omega_T + \beta_T \rho_{t-1} + \alpha_T \times \tfrac{1}{10}\sum_{j=1}^{10} T^{-1}\left(u_{t-j}; v\right)T^{-1}\left(v_{t-j}; v\right)\right)$$

$T_{p,v}(\cdot)$ is standard t distribution function with degree of freedom v; $T_v^{-1}(\cdot)$ is inverse function. Time-varying T copula function describe not only symmetrical correlation but also upper and lower tail correlation which can be represented as

$$\lambda^U = \lambda^L = 2\left(1 - t_{v+1}\left(\sqrt{v+1}\sqrt{1-\rho}/\sqrt{1+\rho}\right)\right)$$

2.3 Time-Varying RG-Copula Function

We can get time-varying RG-Copula function through transforming Gumbel-copula function, that is

$$\mathrm{RG}-Copula: C(u,v|\rho)=\exp\Big(-((-\ln(1-u))^{\rho}+(-\ln(1-v))^{\rho})^{1/\rho}\Big)$$
$$\rho_{RG,t}=\Lambda'\left(\omega_{RG}+\beta_{RG}\rho_{t-1}+\alpha_{RG}\times\tfrac{1}{10}\sum_{j=1}^{10}\left|u_{t-j}-v_{t-j}\right|\right)$$

Time-varying RG-copula function can describe asymmetric dependency and is sensitive to lower tail correlation but not sensitive upper tail, namely $\lambda^U=2-2^{1/\rho}, \lambda^L=0$. In practice, it is used to research correlation in prosperous market.

2.4 Time-Varying SJC-Copula Function

$$\begin{aligned}\mathrm{SJC}-Copula: C\left(u,v|\tau_t^U,\tau_t^L\right)=&0.5\left(C_{JC}\left(u,v\middle|\tau_t^U,\tau_t^L\right)\right.\\&\left.+C_{JC}\left(1-u,1-v\middle|\tau_t^U,\tau_t^L\right)+u+v-1\right)\end{aligned}$$

C_{JC} is distribution function of Joe-Clayton Copula function, expressed as

$$C_{JC}\left(u,v\middle|\tau_t^U,\tau_t^L\right)=1-\left\{1-\left[\left(1-(1-u)^K\right)^{-\gamma}+\left(1-(1-v)^K\right)^{-\gamma}-1\right]^{-1/\gamma}\right\}^{1/K}$$

Among them, $\gamma=-\frac{1}{\log_2\left(\tau_t^L\right)}$, $K=\frac{1}{\log_2\left(2-\tau_t^U\right)}$. τ_t^U and τ_t^L express characteristics of correlation of upper and lower tails respectively which absolute value less than 1. Function of τ_t^U and τ_t^L is

$$\tau_t^U=\Lambda''\left(\omega_U+\beta_U\tau_{t-1}^U+\alpha_U\times\frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j}-v_{t-j}\right|\right)$$
$$\tau_t^L=\Lambda''\left(\omega_L+\beta_L\tau_{t-1}^L+\alpha_L\times\frac{1}{10}\sum_{j=1}^{10}\left|u_{t-j}-v_{t-j}\right|\right)$$

3 Empirical Analysis

3.1 Liquidity Index and Descriptive Statistics

Liquidity index mainly includes Amihud illiquidity index, Amivest liquidity ratio and Hui–Heubel liquidity ratios in the order-driven market. Among them, Amivest liquidity ratio can eliminate the influence of the exchange rate, therefore this paper will use a modified Amivest liquidity ratios to measure index of stock market liquidity. That is

$$liQ_t = \frac{\left(p_{t,\max} + p_{t,\min}\right) \times Vol_t}{2\left(p_{t,\max} - p_{t,\min}\right)} \times 10^{-8} \tag{1}$$

liQ_t is liquidity at time t; the highest and the lowest daily price are $p_{t,\max}$ and $p_{t,\min}$ respectively; Vol_t indicates the trading volume. Compared with Amivest liquidity rate, the average of the lowest and highest price is used to calculate price volatility in (1) instead of opening or closing price. So liQ_t is accurate to describe degree of deviation from the average price due to the impact of the volume.

Liquidity of CSI 300 and S&P 500 index is used instead of the overall stock market liquidity for simplification. Sample data is a total of 1896 from January 4, 2005 to December 25, 2012 obtained from Wind database.

Descriptive statistics results as shown in Table 1. Kurtosis of liquidity sequences of S&P 500 and CSI 300 index are higher than 3 and skewness are 1.4639 and 1.6780 respectively, which mean that there is a situation of thick tail and high peak in any distribution. The same evidence is in J-B test and Figs. 1 and 2 that fluctuations are tend to gather in two liquidity sequences as large fluctuations appear usually after large fluctuations and vice versa. So liquidity sequences do not obey the normal distribution. The two liquidity sequences are stationary and no unit root according to the ADF test. Further, in combination with ARCH test and LBQ (Ljung–Box Q) test, we find that there are conditional heteroskedasticity under 5 % significance level and self-correlation in two liquidity sequences.

3.2 Model of Marginal Distribution

According to the results of Descriptive statistics, AR(q)-GARCH model are selected to construct marginal distribution model for two liquidity sequences under Gaussian, student t and SkewT distribution respectively.

After searching the satisfactory lag intervals for endogenous within the scope of 1 to 20, q = 5 are optimal for the three kinds of AR(q)-GARCH (1, 1) model. As shown in Table 2, the AIC value is the smallest of three under SkewT distribution which not only is characteristic of thick tail but also could describe skewness

Table 1 Descriptive statistics

	Mean	Standard deviation	Skewness	Kurtosis	ADF test	J-B test	ARCH effect	LBQ test
CSI 300	2762.046	1932.960	1.4639	5.9229	−10.1446	1348.515	367.5641	388.7817
					h = 1	h = 1,	h = 1	h = 1
					(0.0010)	(0.0010)	(0.0000)	(0.0000)
S&P 500	1055.074	700.584	1.3780	6.1069	−11.3971	2216.323	161.8389	542.9253
					h = 1	h = 1,	h = 1	h = 1
					(0.0010)	(0.0037)	(0.0000)	(0.0000)

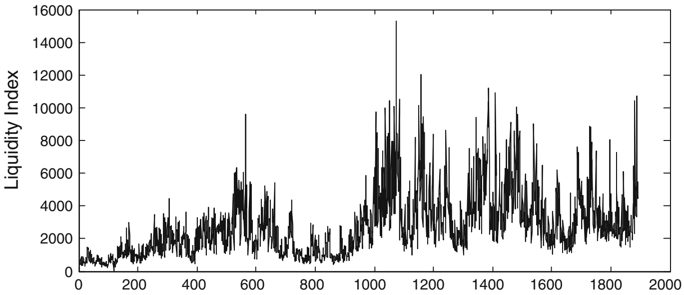


Fig. 1 Liquidity sequence of csi 300 index

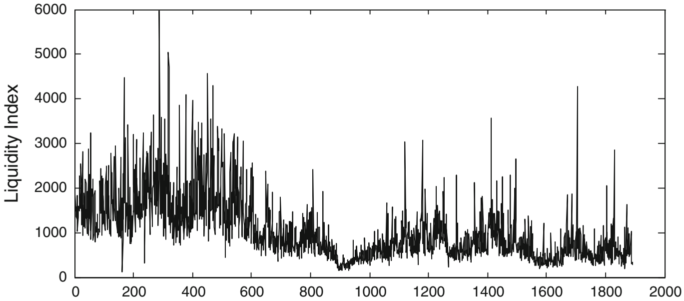


Fig. 2 Liquidity sequence of S&P 500 index

Table 2 AIC Value

	AR(5)-GARCH(1, 1)-N	R(5)-GARCH(1, 1)-t	R(5)-GARCH(1, 1)-SkewT
liQ_{sp}	28,536.9714	28,027.4380	27,621.3792

Compared with student t distribution. The function and parameter estimation of AR (q)-GARCH(1, 1)-SkewT are shown as (2) and Table 3. The AR(q)-GARCH(1, 1)-SkewT will be written as

$$\begin{aligned} liQ_{i,t} &= f\left(liQ_{i,t-1}, liQ_{i,t-2}, \ldots, liQ_{i,t-q}\right) + e_{i,t} \\ e_{i,t} &= \sqrt{h_{i,t}}\epsilon_{i,t}, \epsilon_{i,t} \sim SkT(v, \lambda) \\ h_{i,t} &= \omega_{i,t} + \alpha e_{i,t-1}^2 + \beta h_{i,t-1} \end{aligned} \tag{2}$$

To prepare for modeling by copula function, we standardize residual sequence of AR(5)-GARCH(1, 1)-SkewT and transform it into probability integral. Residual sequence would be uniform distribution from 0 to 1 after transforming of probability integral according to the result of K-S test that means test would accept the null hypothesis and could matching the two residual sequences by copula function.

Table 3 Parameter estimation of AR(5)-GARCH(1, 1)-SkewT

c_0	c_1	c_2	c_3	c_4	c_5	ω	α	β	v	λ	LL	K-S test
0.423	0.190	0.246	0.226	0.017	0.018	1677.245	0.159	0.841	5.578	0.446	−13,889.3	h = 0
(0.227)	(0.022)	(0.018)	(0.020)	(0.163)	(0.150)	(738.566)	(0.030)	(0.026)	(0.473)	(0.033)		

3.3 Parameter Estimation and Characteristics of Dynamic Dependence Structure

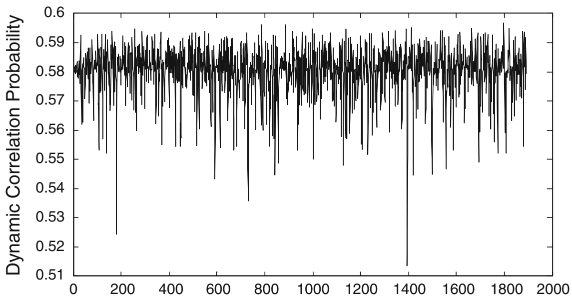
In this section, we will match the two residual sequences by time-varying N-copula, time-varying T-copula, time-varying RG-copula and time-varying SJC-copula function respectively. Then, the most optimal copula function will be used to dependence structure of CSI 300 and S&P 500 index based on AIC information criterion and logarithmic likelihood estimation.

In Table 4, we find that the value of AIC information criterion is the smallest but logarithmic likelihood estimation is the biggest in parameter estimation of T-copula, so T-copula is the most optimal of four. The general dynamic dependence structure of liquidity sequences of CSI 300 and S&P 500 index based on T-copula function is shown in Fig. 3, while tail dynamic dependence structure is in Fig. 4. As shown in Table 4, the value of β is 0.3035, which indicates that the volatility is intensive and no memo ability with correlation coefficient changing, that means there a "corrective effect" between any two conjoint correlation coefficients. The same evidence is shown by LBQ test of Table 5 and Fig. 3. In addition, the freedom parameter DoF is 18.1418 so that the characteristic of the joint fat-tail is ambiguous.

Table 4 Parameter estimation

	T-copula	N-copula	RG-copula	SJC-copula	
				τ_t^U	τ^L
ω	1.3122	1.3093	2.9163	2.9979	2.1880
β	−0.3035	−0.3120	−1.7005	−1.2038	0.9865
α	0.2190	0.2266	−0.4067	−1.5358	−1.4041
c	18.1418				
AIC	−6346.2394	−6336.1201	−4339.5429	−4046.5999	
LL	3177.120	3171.945	2172.728	2029.300	

Fig. 3 General dynamic dependence structure



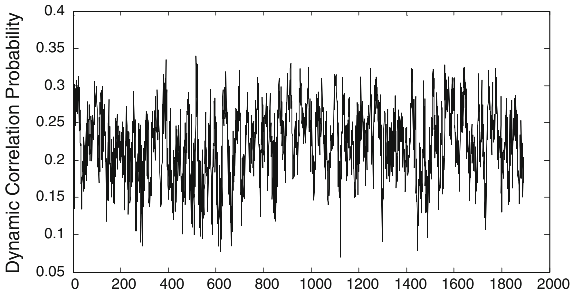


Fig. 4 Dynamic dependence structure in tail

Table 5 Statistical description of dependence structure

	Mean	Standard deviation	Skewness	Kurtosis	LBQ test
ρ	0.5803	0.0081	6.4712	−0.4021	h = 1
λ	0.2220	0.0478	2.7508	−0.2568	h = 1

3.4 Results

The mean and standard deviation are 0.5803 and 0.0081, so the degree of joint fluctuations of two liquidity sequences is moderate and stable in general. However, different from the popular opinion that tail correlation would be enhanced in extreme case, especially in lower tail, the mean of dynamic tail correlation coefficient is 0.222 and lower than general dynamic correlation coefficient in our paper. The dynamic correlation coefficient fluctuates around mean and is asymmetric in Figs. 3 and 4. In order give a specific analysis, we assign a realistic date to each scale of abscissa respectively from left to right in Figs. 3 and 4 which is January 2005, October 2005, August 2006, June 2007, April 2008, January 2009, October 2010, June 2011 March 2012 and December 2012.

The key segments are these sample data which is from 150 to 190, 650 to 840, 1400 in Fig. 3 and from 800 to 1000 in Fig. 4. Matching these sample data with realistic date, the reason of sudden decrease of general dependent probability is illustrated by following. Firstly, the liquidity of stock market was lower and trend of volatility was indeterminate as most of the investors took wait-and-see strategy before equity division reform while liquidity fluctuated increasingly in the U.S from April 2005 to September 2005. Secondly, general dependent probability weakened momentarily and fluctuated violently due to the stamp duty improved in China and subordinated debt crisis in the U.S after June 2007. Finally, there was the most severe fluctuation around 1400 of abscissa in Fig. 3 as the rate of bilateral stamp duty was enhanced by securities and futures commission in China but liquidity was reposeful in the U.S at the same time. Although the tail correlation liquidity increased slightly from 800 to 1000 of abscissa as subordinated debt crisis impacted, dependent structure of tail is stable on the whole. Based on the above analysis, general liquidity dependence structure is more sensitive to policy and emergency than macroeconomic situation. We call the impact of policy and emergency as "policy effect" which have no effect on the stock market of other countries and play a role for a short time.

4 Conclusion

After matching the two residual sequences by time-varying N-copula, time-varying T-copula, time-varying RG-copula and time-varying SJC-copula function respectively, the time-varying T-copula function is satisfying that means the correlation between stock markets in China and the U.S is symmetrical. On the whole, the mean of general dynamic correlation probability is 0.5803, while tail correlation probability is lower than general dynamic correlation coefficient, 0.222 in this paper, which is different from the popular opinion. Dynamic evolution of correlation probability indicate that general dynamic correlation probability fluctuate within the [0.5132, 0.5934] densely and tail dynamic correlation probability within [0.0742, 0.3488].

There is no obvious trend in general dynamic correlation probability as the estimate of β and LBQ test results are −0.3035 and h = 1 respectively which means that there are no memo ability and self-correlation. The kurtosis of distribution of general dynamic correlation probability is −0.4021. The reason could be interpreted that policy and emergency would influence domestic liquidity of stock market significantly but slightly in other countries. This interference of policy and emergency is transiently.

The results of empirical analysis show that the tail correlation is symmetric distribution and possibility is lower than general correlation which liquidity rise and fall at the same time in two stock markets. There are two factors that generate these results above. One is macro economy which would strengthen the dynamic correlation probability. The other is policy and emergency which would weaken it.

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