# Chapter 2
# Some Continuous Distributions

In this chapter several basic properties of some useful univariate distributions will be discussed. These properties will be useful for our characterization problems.

## 2.1 Beta Distribution

A random variable X is said to have a BE(m, n) distribution if its pdf $f_{m,\ n}(x)$ is of the following form.

$$f_{m\cdot n}(X) = \frac{1}{B(m,n)} x^{m-1}(1-x)^{n-1}, 0 < x < 1, m > 0, n > 0. \quad (2.1.1)$$

Mean $= \frac{m}{m+n}$ and variance $= \frac{mn}{(m+n)^2(m+n+1)}$.
The moment generating function $M_{m,\ n}(t)$ is
$M_{m,\ n}(t) = F(m, m + n, t)$, where

$$F(a, b, x) = 1 + \frac{a}{b}x + \frac{a(a+1)}{b(b+1)}\frac{x^2}{2!} + \cdots$$

The characteristic function $\phi_{m.n}(t) = F(m, m+n, it)$
The pdfs of BE(3, 3), BE(4, 6) and BE(4, 9) are given in Fig. 2.1.
If m = 1/2 and n = 1/2, then BE(1/2, 1.2) is the arcsine distribution.
If X is distributed as BE(m, n), then 1-x is distributed as BE(n, m).

 
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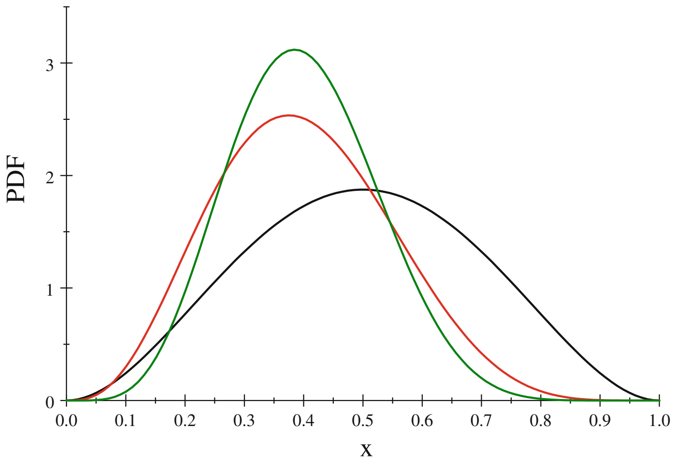


**Fig. 2.1** BE(3, 3) *Black*, BE(4, 6) *Red* and BE(6, 9) *Green*

## 2.2 Cauchy Distribution

A random variable X is said to have a Cauchy $(\mathrm{CA}(\mu,\sigma))$ distribution with location parameter $\mu$ and scale parameter $\sigma$ if the pdf $(\mathrm{f_c}(\mathrm{x},\mu,\sigma))$ is of the following form.

$$\mathrm{f_c}(\mathrm{x},\mu,\sigma)=\frac{1}{\pi\sigma\left(1+\left(\frac{x-\mu}{\sigma}\right)^2\right)},\ -\infty<x<\mu<\infty,\sigma>0. \tag{2.1.2}$$

The Fig. 2.2 gives the pdfs of CA(0, 1), CA(0, 2) and CA(0, 5).

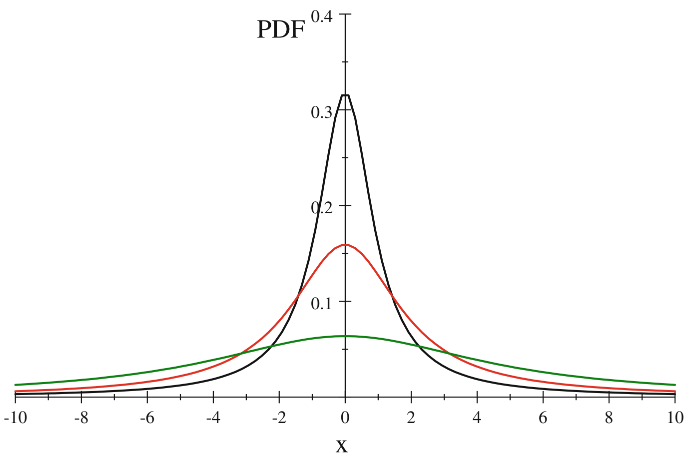


**Fig. 2.2** CA(0, 1) *Black*, CA(0, 2) *Red* and CA(0, 5) *Green*

The mean of CA$(\mu, \sigma)$ does not exist. The median and the mode are equal to μ. The cdf $F_c(x, \mu, \sigma)$ is

$$F_c(x, \mu, \sigma) = \frac{1}{2} + tan^{-1}(\frac{x-\mu}{\sigma}) \quad (2.1.3)$$

If $X_1, X_2, \ldots, X_n$ are n independent CA$(\mu, \sigma)$, then $S_n = X_1 + X_2 + \cdots + X_n$ is distributed as CA$(n\mu, n\sigma)$..

If $X_1$ and $X_2$ are distributed as normal with mean = 0 and variance = 1, the X/Y is distributed as CA(0, 1).

If X is CA(0, 1), then $2X/(1-X^2)$ is distributed as CA(0, 1).

The pdf $f_{gc}(x, \mu, \sigma)$ of generalized Cauchy (GCA$(\mu, \sigma)$) is given by

$$f_{gc}(x, \mu, \sigma) = \frac{\Gamma(n)}{\sigma\sqrt{\pi}\Gamma(n-\frac{1}{2})} \frac{1}{(1+(\frac{x-\mu}{\sigma})^2)^n}, n \geq 1, -\infty < \mu < x < \infty, \sigma > 0. \quad (2.1.4)$$

For n = 1, the mean does not exist.

For n > 1, the mean = μ, the median = μ and the odd moments are zero.

For n > 1,

$$E(X^m) = \frac{\Gamma(\frac{m+1}{2})\Gamma(n-\frac{m+1}{2})}{\Gamma(\frac{1}{m})\Gamma(n-\frac{1}{m})} \text{for m even}, m < 2n-1, m > 1.$$

## 2.3 Chi-Squared Distribution

A random variable X is said to have a Chi-squared (CH$(\mu, \sigma, n)$) distribution with location parameter $\mu$ and scale parameter $\sigma$ if the pdf ($f_{ch}(x, \mu, \sigma, n)$) is of the following form.

$$f_{ch}(x, \mu, \sigma, n) = \frac{1}{2^{n/2}\Gamma(\frac{n}{2})}(\frac{x-\mu}{\sigma})^{\frac{n}{2}-1} e^{-(\frac{x-\mu}{2\sigma})}, n > 1, -\infty < \mu < x < \infty, \sigma > 0.$$

The parameter n is known as degrees of freedom.

For n > 1, Mean = $\mu + n\sigma$, *and variance* $= 2n\sigma^2$.

The moment generating function $M_{CH}$ (t) is

$$M_{CH}(t) = e^{\mu t}(1-2\sigma t)^{-n/2}, t < \frac{1}{2\sigma}.$$

The Fig. 2.3 gives the pdfs of The CH(0, 1, 4), CH(0, 1, 10) and CH(0, 1, 20).

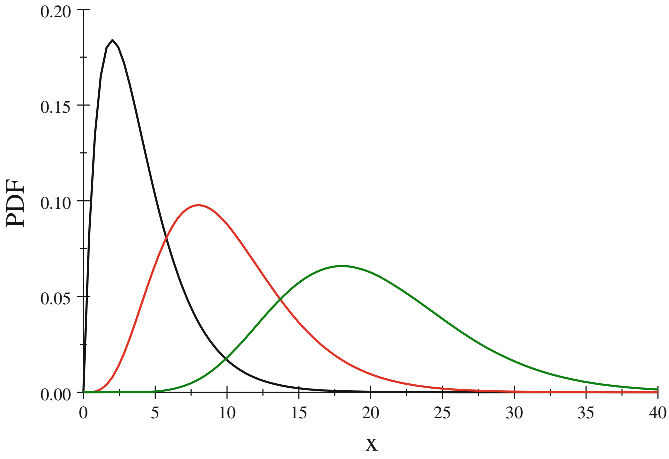


**Fig. 2.3** The CH(0, 1, 4)-Black, CH(0, 1, 10)-red and CH(0, 1, 20)-green

If $X_i$, i = 1, 2, …, n are n independent CH(0, 1, $n_i$). i = 1, 2, …, n, random variables then $S_k = X_1 + X_2 + \cdots + X_k$, then $S_k$ is distributed as CH(0, 1, m), where $m = n_1 + n_2 + \cdots + n_K$. If X is standard normal (N(0, 1)), then $X^2$ is distributed as CH(0, 1, 1).

## 2.4 Exponential Distribution

A random variable X is said to have a exponential $(E(\mu, \sigma))$ distribution with location parameter $\mu$ and scale parameter $\sigma$ if the pdf $(f_e(x, \mu, \sigma))$ is of the following form.

$$f_e(x, \mu, \sigma) = \frac{1}{\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)}, \ -\infty < \mu < x < \infty.$$

The exponential distribution E(0, 1) is known as standard exponential.

The Fig. 2.4 gives the pdfs of E(0, 1), E(0, 2) and E(0, 5).

The cdf $F_e(x, \mu, \sigma)$ is given by

$$F_e(x, \mu, \sigma) = 1 - e^{-\left(\frac{x-\mu}{\sigma}\right)}, \ -\infty < \mu < x < \infty.$$

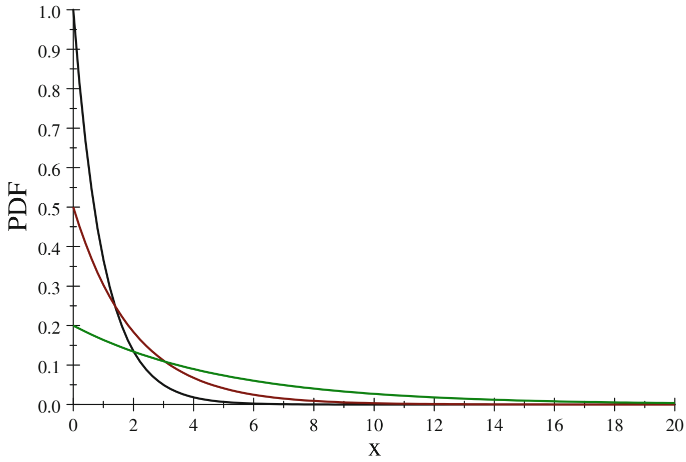


**Fig. 2.4** E(0, 1) *Black*, E(0, 2) *Red* and E(0, 5) *Green*

The moment generating function $M_{ex}(t)$

$$M_{ex}(t) = (1 - \sigma t)^{-1} e^{-\mu t}$$

Mean = $\mu + \sigma$ and Variance = $\sigma^2$.

If $X_i$, i = 1, 2, …, n are i.i.d. exponential with $F(x) = 1 - e^{-x/\sigma}, x \geq 0, \sigma > 0$, and $S(n) = X_1 + X_2 + \ldots + X_n$, then pdf $f_{S(n)}$ (x) of S(n) is

$$f_{S(n)}(x) = \frac{1}{\sigma} e^{-x/\sigma} \frac{(x/\sigma)^{n-1}}{\Gamma(n)}, x \geq 0, \sigma > 0.$$

This is a gamma distribution with parameters n and $\sigma$.

If $X_1$ and $X_2$ are independent exponential random variables with scale parameters $\sigma_1$ and $\sigma_2$, then $P(X_1 < X_2) = \frac{\sigma_2}{\sigma_1 + \sigma_2}$.

If $X_i$, i = 1, 2, …, n are n independent exponential random variable with $F(x) = 1 - e^{-x/\sigma}, x \geq 0, \sigma > 0$. Let m(n) = min $\{X_1, \ldots, X_n\}$ and

M(n) = max$\{X_1, \ldots, X_n\}$, $F_{(m)}$ be the cdf of m(n) and $F_{(M)}$ be the cdf of M(n), then

$$\begin{aligned} 1 - F_{(m)}(x) &= P(X_1 > x, X_2 > x, \ldots X_n > x) \\ &= e^{-nx/\sigma} \end{aligned}$$

$$F_{(M)}(x) = P(X_1 < x. X_2 < x, \ldots, X_n < x) = (1 - e^{-x/\sigma})^n.$$

**Memoryless Property**. P(X > s+t|X > t) P(X > s).

$$\begin{aligned}\mathrm{P(X > s + t|X > t)} &= \frac{P(X > s + t, X > t}{P(X > t)}) \\ &= \frac{e^{-(s+t-2\mu)/\sigma}}{e^{-(t-\mu)/\sigma}} \\ &= e^{-(s-\mu)/\sigma} \\ &= P(X > s)\end{aligned}$$

## 2.5 F-Distribution

A random variable X is said to have F distribution F(m, n) with numerator degree of degrees of freedom m and numerator degrees of freedom n if its pdf $f_F$ (x, m, n) is given by

$$\mathrm{f_F(x, m, n)} = \frac{\Gamma(\frac{m+n}{2})(\frac{m}{n})^{\frac{m}{2}}x^{(m-2)/2}}{\Gamma(\frac{m}{2})\Gamma(\frac{n}{2})(1+\frac{mx}{n})^{(m+n)/2}}, \mathrm{x > 0, m\ > 0, n > 0.}$$

The cdf $F_F$(x, m, m) is given by

$$\mathrm{F_F(x, m, n)} = I_{\frac{mx}{m+n}}(\frac{m}{2}, \frac{n}{2}),$$

where = $I_x(a, b) = \int_0^x u^{a-1}(1-u)^{b-1}du$ is the incomplete beta function.

The Fig. 2.5 gives the pdfs of F(5, 5), F(10, 1)) and F(10, 20).

Mean = $\frac{n}{n-2}$, n > 2 and variance = $\frac{2n^2(m+n-2)}{m(m-2)^2(n-4)}$, n > 4.

The characteristic function $\phi_F\ (\mathrm{m \cdot n}) = \frac{\Gamma(\frac{m+n)}{2}}{\Gamma(\frac{n}{2})}\mathrm{U}(\frac{m}{2}, 1-\frac{n}{2}, -\frac{m}{n}it)$, where U(a, b, x) is the confluent hypergeometric function of the second kind.

If $U_1$ and $U_2$ are independently distributed as chi-squared distribution with m and n degrees of freedom, then X = (n/m) ($U_1/U_2$) is distributed as F with cdf $F_F$ (m, n).

If X is distributed as Beta (m/2, n/2), then $\frac{nX}{m(1-X)}$ is distributed as F with cdf $F_F$ (m, n).

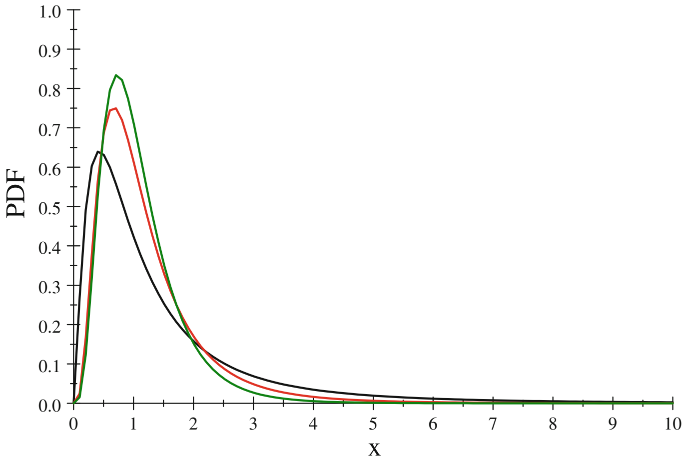


**Fig. 2.5** F(5, 5) *Black*, F(10, 10) *Red* and F(10, 20) *Green*

## 2.6 Gamma Distribution

A random variable X is said to have gamma distribution GA $(a, b)$ if its pdf $f_{ga}(a, b, x)$ is of the following form.

$$f_{ga}(a, b, x) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b}, \mathrm{x} \geq 0, a > 0, b > 0.$$

Mean = $ab$ and variance = $ab^2$.

The Fig. 2.6 give the pdfs of GA(2, 1), GA(5, 1) and GA(10, 1).

The moment generating function M(t) is

$$\mathrm{M(t)} = (1 - bt)^{-a}, \mathrm{t} < 1/\mathrm{b}.$$

The characteristic function $\phi_{ga}(t) is\ \phi_{ga}(t) = (1 - ibt)^{-a}$.

If a = 1, b = 1 then we GA(1, 1) is an exponential distribution and if a is a positive integer, then GA(a, b) is an Erlang distribution.

If b = 1, then we call GA $(a, b)$ as the standard gamma distribution.

If $X_1$ and $X_2$ are independent gamma random variables then the random variables $X_1 + X_2$ and $\frac{X_1}{X_1 + X_2}$ are mutually independent.

If $X_1$, $X_2$, .., $X_n$ are n independently distributed as GA(a, b), then S(n) = $X_1 + X_2 + \cdots + X_n$ is distributed as GA(na, b).

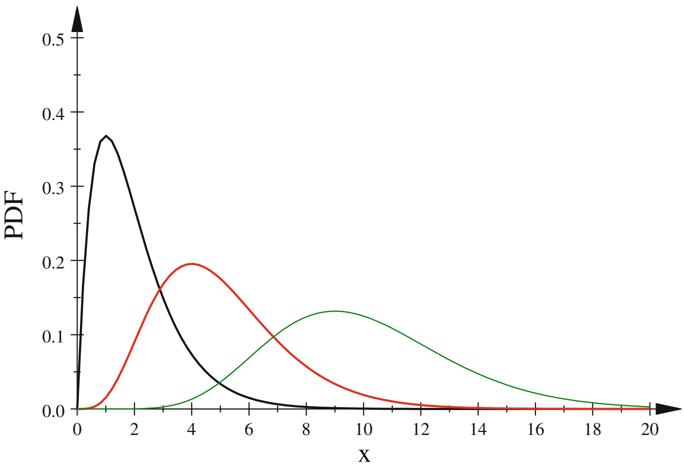


**Fig. 2.6** GA(2, 1) *Black*, GA(5, 1) *Red* and GA(10, 1) *Green*

## 2.7 Gumbel Distribution

A random variable X is said to have Gumbel $(GU(\mu, \sigma))$ distribution with location parameter μ and scale parameter $\sigma$ if its pdf, $f_{ig}$ $(x, \mu, \sigma)$ is of the following form

$$f_{gu}(x, \mu, \sigma) = \frac{1}{\sigma} e^{-\frac{x-\mu}{\sigma}} e^{-e^{-\frac{x-\mu}{\sigma}}}, -\infty < \mu < x < \infty, \sigma > 0.$$

Gumbel distribution is also known as Type I extreme (maximum) value distribution. The cdf $F_{gu}(x, \mu, \sigma)$ is of the following form

$$F_{gu}(x, \mu, \sigma) = e^{-e^{-\frac{x-\mu}{\sigma}}}, -\infty < \mu < x < \infty, \sigma > 0.$$

Mean = $\mu + \gamma\sigma$, where $\gamma$ is Euler's constant.
Median = $\mu - \ln(ln2)\sigma$.
Variance = $\frac{\pi^2\sigma^2}{6}$.
The Fig. 2.7 gives the pdfs of GU(0, 1/2), GU(0, 1) and GU(0, 2).
If X is distributed as E(0, 1), then $\mu - \sigma lnX$ is distributed as $GU(\mu, \sigma)$).
If X is distributed as GU(0, 1), then $Y = e^{-X}$ is distributed as E(0, 1).

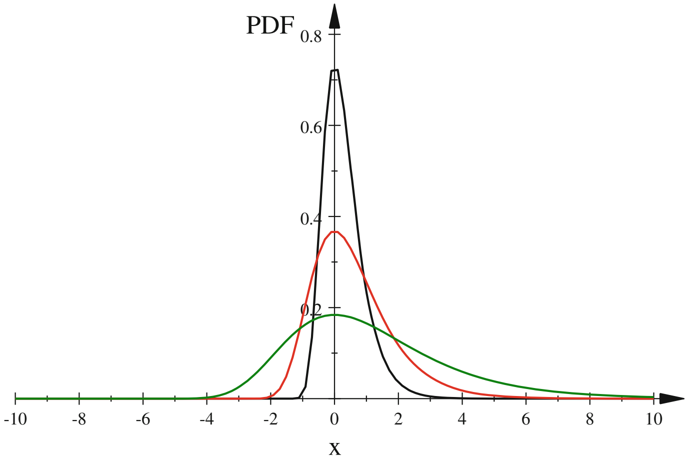


**Fig. 2.7** GU(0, 1/2) *Black*, GU(0, 1) *Red* and GU(0, 2) *Green*

## 2.8 Inverse Gaussian (Wald) Distribution

A random variable X is said to have Inverse Gaussian $(\mathrm{IG}(\mu, \sigma))$ distribution with parameters μ and $\lambda$ if its pdf

$f_{ig}(x, \mu, \lambda)$ is of the following form

$$f_{ig}(x, \mu, \lambda) = \left(\frac{\lambda}{2\lambda x^3}\right)^{\frac{1}{2}} e^{-\frac{\lambda}{2x}\left(\frac{x-\mu}{\mu}\right)^2}, 0 < \mu < x < \infty, \lambda > 0.$$

Mean $= \mu$ and variance $= \frac{\mu^3}{\lambda}$.
The Fig. 2.8 gives the pdfs of IG(1, 10), IG(1, 2) ans IG(1, 3).

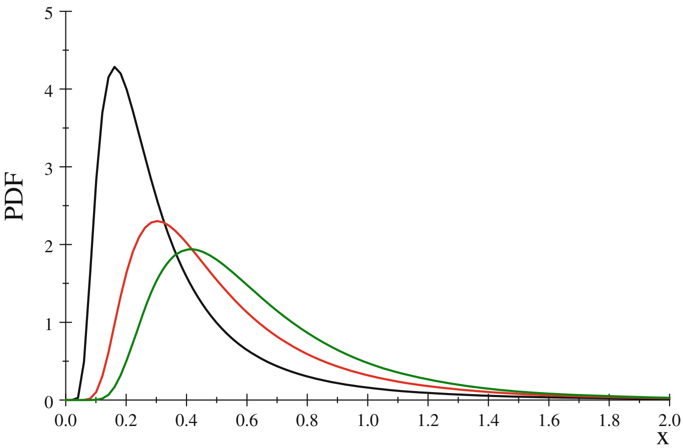


**Fig. 2.8** IG(1, 1) *Black*, IG(1, 2) *Red* and IG(1, 3) *Green*

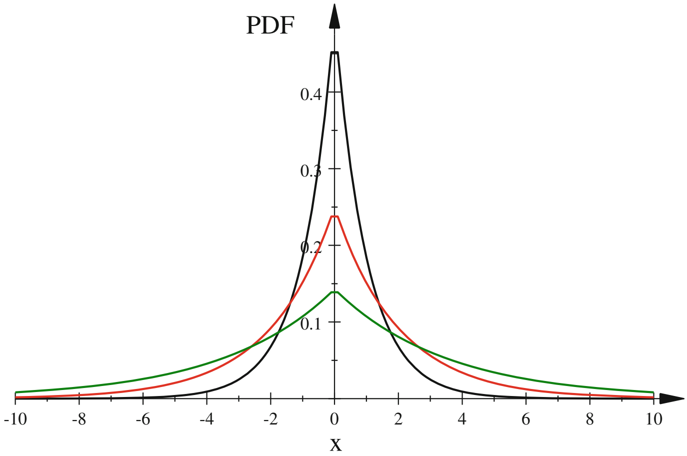


**Fig. 2.9** LP(0, 1) *Black*, LP(0, 2) *Red* and LP(0, 3.5) *Green*

If X is distributed as IG $(\mu, \lambda)$ then αX is distributed as IG $(\alpha\mu. \alpha\lambda)$.
If X is distributed as IGv $(1, \lambda)$, then X is known as Wald distribution.

## 2.9 Laplace Distribution

A random variable X is said to have Laplace $(LP(\mu, \sigma))$ distribution with location parameters μ and scale parameter $\lambda$ if its pdf

$f_{lp}(x, \mu, \lambda)$ is of the following form

$$f_{lp}(x, \mu, \lambda) = \frac{1}{2\sigma} e^{-\left|\frac{x-\mu}{\sigma}\right|}, \infty < x < \mu < \infty, \sigma > 0.$$

Mean $= \mu$ and variance $= 2\sigma^2$.
The Fig. 2.9 gives the pdfs of LP(0, 1), LP(0, 2) and LP(0, 3.5).
The moment generating function is $M_{lp}(t)$ is

$$M_{lp}(t) = \frac{e^{\mu t}}{1 - \sigma^2 t^2}.$$

The characteristic function $\phi_{lp}(t)$ is

$$\phi_{lp}(t) = \frac{e^{i\mu t}}{1+\sigma^2 t^2}.$$

*If* X and Y are independent E(0, 1), then X-Y is LP(0, 1).
*If* X is LP $(\mu, \sigma)$, then kX is LP $(k\mu, k\sigma)$.
*If* X is LP(0, 1), then |X| is E(0, 1).

## 2.10 Logistic Distribution

A random variable X is said to have Logistic $(\mathrm{LG}(\mu, \sigma))$ distribution with location parameters μ and scale parameter $\lambda$ if its pdf

$f_{lg}(x, \mu, \sigma)$ is of the following form

$$\mathrm{f_{lg}}(x, \mu, \sigma) = \frac{1}{\sigma}\frac{e^{-\frac{x-\mu}{\sigma}}}{(1+e^{-\frac{x-\mu}{\sigma}})^2}, -\infty < \mu < x < \infty, \sigma > 0.$$

Mean = μ and variance $= \frac{\pi^2\sigma^2}{3}$.
Moment generating function $\mathrm{M_{lg\ (t)}}$ is

$$\mathrm{M_{lg}(t)} = e^{\mu t}\Gamma(1+\sigma t)\Gamma(1-\sigma t), t < \frac{1}{\sigma}.$$

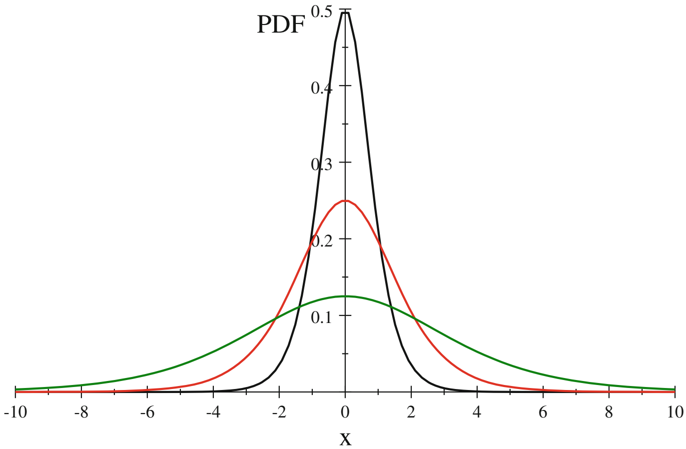


**Fig. 2.10** LG(0, 1/2) *Black*, LG(0, 1) *Red* and LG(0, 2) *Green*

The characteristic function $\phi_{lg}(t) is$

$$\phi_{lg}(t) = e^{i\mu t}\Gamma(1 + i\sigma t)\Gamma(1 - i\sigma t).$$

The Fig. 2.10 gives the pdfs of LG(0, 1/2), LG(0, 1) and LB(0, 2).

Let $X_{i,}$, i = 1, 2, …, n are independent and identically distributed as LP(0, 1), then Y = $X_1X_2 \ldots X_n$ is distributed as LG(0, 1).

If X and Y are independent GU $(\mu, \sigma), then\, X - Y$ is LG(0, 1).

If X is LG($(\mu, \sigma)$ then kX is LG $(\mu k, k\sigma)$.

If X and Y are independent and E(0, 1), then $\mu - \sigma \ln(\frac{X}{Y})$ is LG $(\mu, \sigma)$.

## 2.11 Lognormal Distribution

A random variable X is said to have Lognormal $(\mathrm{LN}(\mu, \sigma))$ distribution with location parameters μ and scale parameter $\sigma$ if its pdf

$f_{\ln}(x, \mu, \sigma)$ is of the following form

$$f_{\ln}(x, \mu, \sigma) = \frac{1}{x\sigma\sqrt{(2\pi)}} e^{-\frac{1}{2}(\frac{\ln(x-\mu)}{\sigma})^2}, x > 0, > 0, \mu > 0, \sigma > 0.$$

Mean = $e^{\mu + \frac{\sigma^2}{2}}$

Variance = $(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$.

Moment generating function $M_{ln}(t)$ is

$$M_{ln}(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} e^{n\mu + \frac{n^2\sigma^2}{2}}.$$

The characteristic function $\phi_{ln}(t) is$

$$\phi_{ln}(t) = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} e^{n\mu + \frac{n^2\sigma^2}{2}}$$

The Fig. 2.11 gives the pdfs of LN(0, 1/2), LN(0, 1) and LN(0, 2).

If X is distributed as normal with location parameter $\mu$ and scale parameter $\sigma$, *then* $e^X$ is distributed as LN $(\mu, \sigma)$.

If X is distributed as LN $(\mu,\ \sigma)$, *then* $\ln X$ *is distributed as* normal with location parameter $\mu$ and scale parameter $\sigma$.

If $X_i$, i = 1, 2, …, n are independent and identically distributed as LN $(\mu,\ \sigma)$, *then* Y = $X_1X_2 \ldots X_n$ is distributed as LN $(n\mu, \sigma\sqrt{n})$.

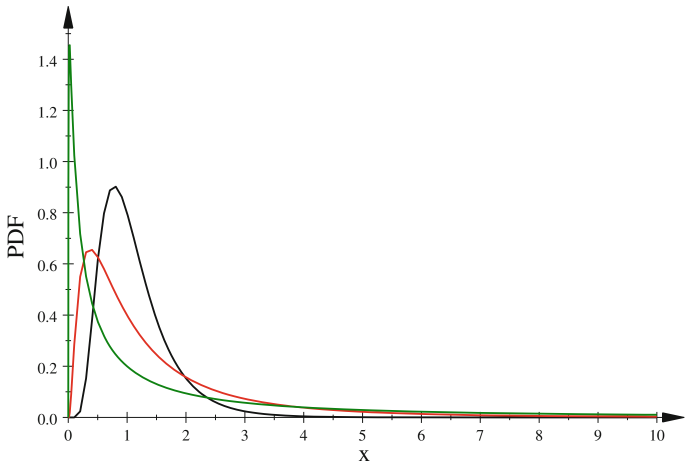


**Fig. 2.11** LN(0, 1/2) *Black*, LN(0, 1) *Red* and LN(0, 2) *Green*

## 2.12 Normal Distribution

A random variable X is said to have normal $(N(\mu, \sigma))$ distribution with location parameters μ and scale parameter $\sigma$ if its pdf

$f_n(x, \mu, \sigma)$ is of the following form

$$f_n(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < \mu < x < \infty, \sigma > 0.$$

$$Mean = \mu \; and \; variance = \sigma^2.$$

$$The \; moment \; generating \; function \; M_n(t) is$$

$$M_n(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}.$$

The characteristic function $\phi_n(t)$ is

$$\phi_n(t) = e^{i\mu t - \frac{\sigma^2 t^2}{2}}.$$

The Fig. 2.12 gives the pdfs of N(0, 1/2), N(0, 1) and N(0, 2).

If $X_i$ is N $(\mu_i, \sigma_i)$, i = 1, 2, …, n and $X_i$'s are independent, then for any $\alpha_i$, i = 1, 2, …, n, $\sum_{i=1}^n \alpha_i X_i$ is N( $\sum_{i=1}^n \alpha_i \mu_I$, $\surd(\sum_{i=1}^n \alpha_i^2 \sigma_i^2)$).

If X is normal and X = $X_1 + X_2$, where $X_1$ and $X_2$ are independent, then both $X_1$ and $X_2$ are normal.

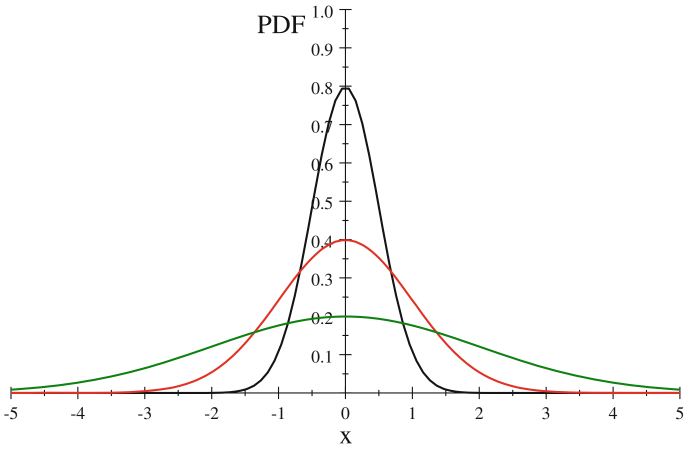


**Fig. 2.12** N(0, 0.5) *Black*, N(0, 1) *Red* and N(0, 2) *Green*

## 2.13 Pareto Distribution

A random variable X is said to have Pareto (PA(α, β)) distribution with parameters α and β if its pdf $f_{pa}(x, \alpha, \beta)$ is of the following form

$$f_{pa}(x, \alpha, \beta) = \frac{\beta \alpha^{\beta}}{x^{\beta+1}}, x > \alpha > 0, \beta > 0.$$

Mean $= \frac{\alpha\beta}{\beta-1}, \beta > 1$ and variance $= \frac{\alpha^2\beta}{(\beta-1)^2(\beta-2)}. \beta > 2.$

The characteristic function $\phi_{pa}(t)$ is given by

$$\phi_{pa}(t) = \beta(-i\alpha t)^{\beta}\Gamma(-\beta, -i\alpha t).$$

The Fig. 2.13 gives the pdfs of PA(1, 1/2), PA(1, 1) and PA(1, 20).

If $X_1, X_2, \ldots, X_n$ are n independent PA(α, β), then

$2\beta \ln(\frac{\prod_{i=1}^{n} X_i}{\alpha^n})$ is distributed as CH(0, 1, n).

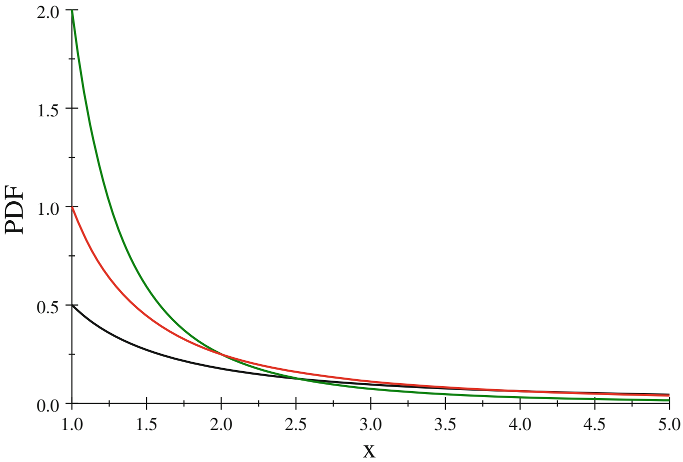


**Fig. 2.13** PA(0, 1/2) *Black*, PA(1, 1) *Red* and PA(1, 2) *Green*

## 2.14 Power Function Distribution

A random variable X is said to have power function (Po(α, β, δ)) if its pdf $f_{po}(\alpha, \beta, \delta)$ is of the following form

$$f_{po}(\alpha,\beta,\delta) = \frac{\delta}{\beta-\alpha}\left(\frac{x-\alpha}{\beta-\alpha}\right)^{\delta-1}, -\infty < \alpha < x < \beta < \infty, \delta > 0$$

Mean = $\alpha + \frac{\delta}{\delta+1}(\beta-\alpha)$ and variance = $\frac{\delta(\beta-\alpha)^2}{(\delta+1)^2(\delta+2)}$.

The Fig. 2.14 gives the pdfs of Po(0, 1, 3), Po(0, 1, 4) and Po(0, 1, 4).

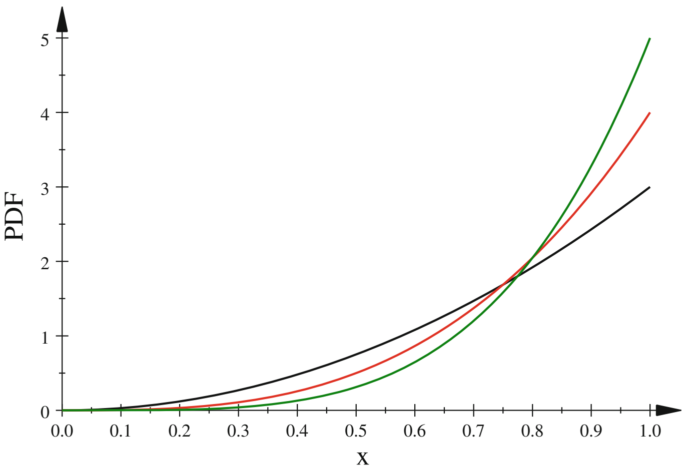


**Fig. 2.14** Po(0, 1, 3) *black*, Po(0, 1, 4) *red* and Po(0, 1, 4) *green*

If $\delta = 1, then\, PO(\alpha, \beta, 1)$ becomes a uniform (U(α, β)) with pdf $f_{un}(x, \alpha, \beta)$ as

$$F_{un}(x, \alpha, \beta) = \frac{1}{\beta - \alpha}, \; -\infty < \alpha < \beta < \infty.$$

## 2.15 Rayleigh Distribution

A random variable X is said to have Rayleigh (RA(μ, σ)) with location parameter μ and scale parameter σ if its pdf $f_{ra}(x, \mu, \sigma)$ is of the following form

$$f_{ra}(x, \mu, \sigma) = \frac{x - \mu}{\sigma^2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \; -\infty < \mu < x < \infty, \sigma > 0.$$

Mean $= \mu + \sigma\sqrt{\frac{\pi}{2}}$ and variance $= \frac{4-\pi}{2}\sigma^2$;
Moment generating function $M_{ra}(t)$ is

$$M_{ra}t = e^{\mu t}\left(1 + \sigma t e^{-\frac{\sigma^2 t^2}{2}} \sqrt{\frac{\pi}{2}}\left(\text{erf}\left(\frac{\sigma t}{\sqrt{2}}\right) + 1\right)\right)$$

The Fig. 2.15 gives the pdfs of RA(0, 1/2), RA(0, 1) and RA(0, 2).

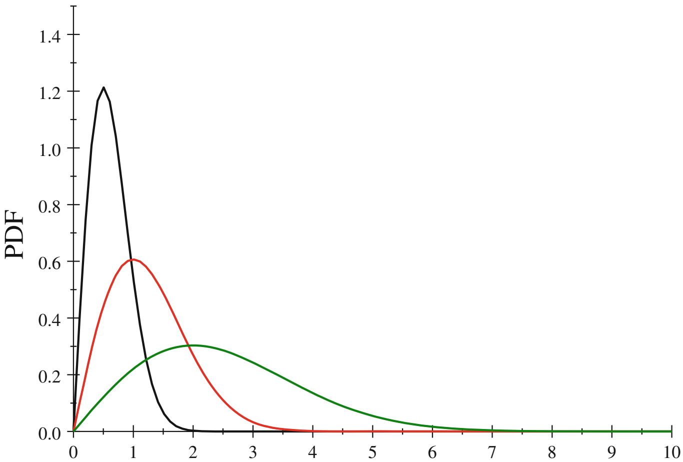


**Fig. 2.15** RA(0, 1/2) *Black*, RA (0, 1) *Red* and RA(0, 2) *Green*

## 2.16 Student's t-Distribution

A random variable X is said to have Students t-distribution ST(n)) with n degrees of freedom, if its pdf $f_{st}$ $(x,\ n)$ is as follows.

$$\mathrm{f_{st}(x, n)} = \frac{1}{\sqrt{n}}\frac{1}{B(n/2, 1/2)}(1+\frac{x^2}{n})^{-(n+1)/2}, \ -\infty < t < \infty, n \geq 1.$$

Mean = 0 if n > 1 and is not defined for n = 1.
Variance = $\frac{n}{n-2}, n > 2.$
The Fig. 2.16 gives the pdf of ST(!), ST(4) and ST(16) (Fig. 2.16).

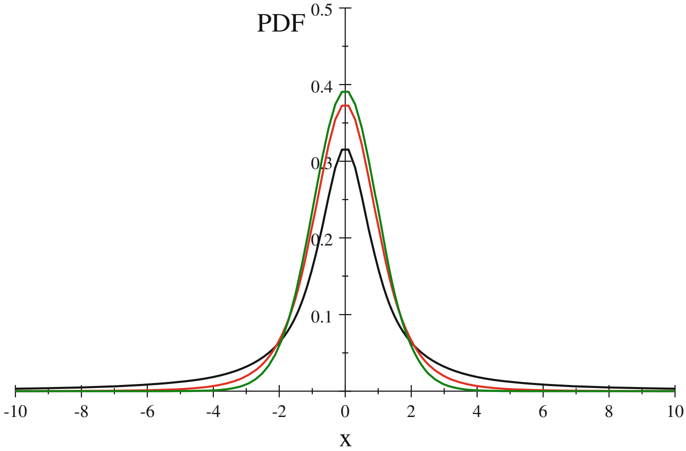


**Fig. 2.16** ST(1) *Black*, ST(4) *Red* and ST(16) *Green*

## 2.17 Weibull Distribution

A random variable is said to have Weibull WB(x, μ, σ, δ) with location parameter μ, scale parameter σ and shape parameter δ if its pdf $f_{Wb}$(x, μ, σ, δ) is of the following form.

$$\mathrm{f_{Wb}}(\mathrm{x}, \mu, \sigma, \delta) = \frac{\delta}{\sigma}(\frac{x-\mu}{\sigma})^{\delta-1}e^{-(\frac{x-\mu}{\sigma})_{\delta}}, \ -\infty < \mu < x < \infty, \sigma > 0, \delta > 0.$$

Mean = $\mu + \Gamma(1+\frac{1}{\delta})$ and variance = $\sigma^2((\Gamma 1+\frac{2}{\delta}) - (\Gamma(1+\frac{1}{\delta}))^2)$.

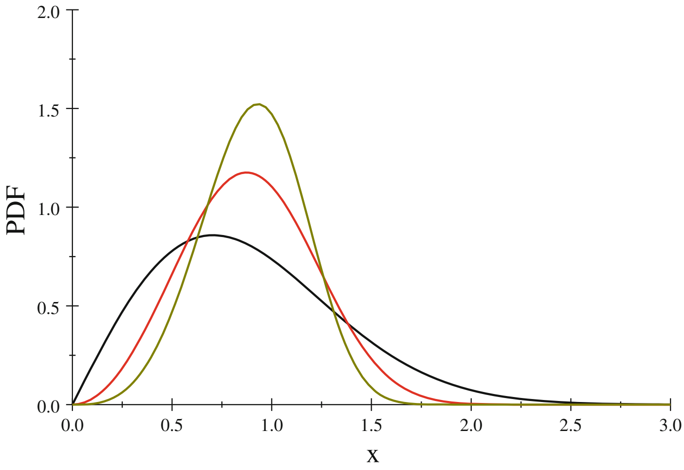


**Fig. 2.17** WB(0, 1, 1) *Black*, WB(0, 1, 3) *red* and WB(0, 1, 4) *green*

The Fig. 2.17 gives the pdfs of B(0, 1, 1), WB(0, 1, 3) and B(0, 1, 4).

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