1. The suggested book deals with the stability of linear and nonlinear vector neutral type functional differential equations. Equations with neutral type linear parts and nonlinear causal mappings are also considered. Explicit conditions for the exponential, absolute and input-to-state stabilities are derived. Moreover, solution estimates for the considered equations are established. These estimates provide the bounds for regions of attraction of steady states. The main methodology presented in the book is based on a combined usage of the recent norm estimates for matrix-valued functions with the following methods and results: the generalized Bohl-Perron principle, the integral version of the generalized Bohl-Perron principle and the positivity conditions for fundamental solutions to scalar neutral equations. We also apply the so-called generalized norm. A significant part of the book is devoted to the generalized Aizerman problem.

2. Neutral type functional differential equations (NDEs) naturally arise in various applications, such as control systems, mechanics, nuclear reactors, distributed networks, heat flows, neural networks, combustion, interaction of species, microbiology, learning models, epidemiology, physiology, and many others. The theory of functional differential equations has been developed in the works of V. Volterra, A.D. Myshkis, N.N. Krasovskii, B. Razumikhin, N. Minorsky, R. Bellman, A. Halanay, J. Hale and other mathematicians. The problem of stability analysis of various neutral type equations continues to attract the attention of many specialists despite its long history. It is still one of the most burning problems because of the absence of its complete solution. The basic method for the stability analysis is the method of the Lyapunov type functionals. By that method many very strong results are obtained. We do not consider the Lyapunov functionals method because several excellent books cover this topic. It should be noted that finding the Lyapunov type functionals for vector neutral type equations is often connected with serious mathematical difficulties, especially in regard to nonautonomous and nonlinear equations. On the contrary, the stability conditions presented in the suggested book are mainly formulated in terms of the determinants and eigenvalues of auxiliary matrices.
dependent on a parameter. This fact allows us to apply the well-known results of the theory of matrices to the stability analysis.

3. The aim of the book is to provide new tools for specialists in the stability theory of functional differential equations, control system theory and mechanics. This is the first book that:

(i) gives a systematic exposition of the approach to stability analysis of vector neutral type functional differential equations which is based on estimates for matrix-valued functions allowing us to investigate various classes of equations from the unified viewpoint;

(ii) contains a solution of the generalized Aizerman problem for NDEs;

(iii) presents the generalized Bohl-Perron principle for neutral type systems and its integral version;

(iv) suggests explicit stability conditions for semilinear equations with linear neutral type parts and nonlinear causal mappings.

The book is intended not only for specialists in the theory of functional differential equations, but for anyone interested in various applications who has had at least a first year graduate level course in analysis.

I was very fortunate to have fruitful discussions with the late Professors M.A. Aizerman, M.A. Krasnosel’skii, A.D. Myshkis, A. Pokrovskii and A.A. Voronov, to whom I am very grateful for their interest in my investigations.

4. The book consists of 9 chapters.

Chapter 1 is of preliminary character. In that chapter we present standard facts, mainly from the theory of Banach and ordered spaces, required in further consideration. In addition, we establish norm estimates for operators of the forms

\[ f \to \int_0^t dR_0(s) f(t - s) \]

and

\[ f \to \int_0^t dR(t, s) f(t - s) \quad (t \geq 0), \]

where \( R_0(s) \) and \( R(t, s) \) are real matrix valued functions having bounded variations in \( s \). These estimates play an essential role in the book.

In Chap. 2 we accumulate norm estimates for matrix-valued functions and bounds for the eigenvalues of matrices. In particular, we suggest estimates for the resolvents, powers of matrices as well as for matrix exponentials. We also present bounds for the spectral radius and investigate perturbations of eigenvalues. The material of this paper is systematically applied in the rest of the book chapters.

Chapter 3 deals with linear autonomous and time-variant vector difference-delay
equations with continuous time. Under various assumptions, norm estimates for the Cauchy (resolvent) operators of the considered equations are derived. In the sequel chapters these estimates give us stability conditions for the neutral type functional differential equations.

In Chap. 4 we consider vector linear differential-delay equations (DDEs). We derive estimates for the $L^p$- and $C$-norms of the Cauchy operators of autonomous and time-variant differential-delay equations. These estimates enable us to investigate neutral type equations.

Chapter 5 is devoted to vector linear autonomous NDEs. We derive estimates for the $L^p$- and $C$-norms of the characteristic matrix functions and fundamental solutions of the considered equations, as well as establish stability conditions for autonomous NDEs.

In Chap. 6 we investigate linear vector time-variant (non-autonomous) neutral type functional differential equations. In particular, we extend the Bohl-Perron principle to a class of neutral type functional differential equations. Namely, it is proved that the homogeneous equation is exponentially stable, provided the corresponding non-homogeneous equation with the zero initial condition and an arbitrary bounded free term has a bounded solution. We also establish the integral version of the generalized Bohl-Perron principle for NDEs, i.e. it is shown that the homogeneous equation is exponentially stable, if the corresponding non-homogeneous equation with the zero initial condition and an arbitrary bounded free term from $L^p([0,1],C^n)$ has a solution belonging to $L^p(0,\infty)$. As applications of these principles, the stability conditions for time-variant NDEs close to autonomous systems are derived. In addition, we investigate time-variant systems with small principal operators and obtain stability conditions independent of delay in the non-autonomous case.

Chapter 7 deals with nonlinear vector equations having linear autonomous neutral type parts and nonlinear causal mappings. Explicit conditions for the exponential, absolute and input-to-state stabilities are derived.

Chapter 8 is concerned with scalar nonlinear neutral type functional differential equations with autonomous linear parts. We derive explicit absolute $L^2$-stability conditions in terms of the norms of the Cauchy operators to the linear parts and the Lipschitz constants of the nonlinearities. In addition, we consider the generalized Aizerman problem.

In Chap. 9 we discuss certain properties of the characteristic values of autonomous vector NDEs. In particular, bounds for characteristic values and perturbations results are derived.
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