Leibniz, Philosopher Mathematician and Mathematical Philosopher

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Of the numerous constants in Leibniz's philosophy, stretching from his intellectually formative years in Leipzig and Jena through to the mature writings of the *Monadology* conceived largely in Hanover and Berlin, few are as remarkable as his conviction that a firm understanding of the concepts of unity and infinity ultimately provide the key to developing sound metaphysics. When he famously wrote to Gilles Filleau des Billettes in December 1696 that his fundamental considerations rest on two things, namely unity and infinity[1], he simultaneously situated his metaphysics on the one side in the philosophical tradition of Aristotle and Thomas Aquinas and on the other side in the new mechanistic approach of Galileo and Descartes. For infinity and the intimately related concept of continuity, while having an ancient philosophical tradition of their own, were for him always also the key to harmonizing or at least connecting the three central pillars of his system: mathematics, physics, and indeed metaphysics itself. Even before his progression from mathematical novice, who had probably read little more on the subject than Daniel Schwenter's *Erquickstunden*, and some of Girolamo Cardano's *Practica arithmeticae generalis*[2], to becoming one of the most productive mathematical thinkers of his day, Leibniz recognized that mathematics was the foundation on which modern scientific and technological advances leading to an improvement of the human condition firmly rested[3]. Avoiding the negative theological consequences of Cartesianism and atomism, his metaphysics would ultimately conform more radically to the conceptual foundations of the mathematical sciences than the philosophical positions of any of his contemporaries.

[1] Leibniz to Des Billettes, 4/14 December 1696, Leibniz 1923 (cited hereafter as 'A') I, 13, 90: "Mes Meditations fondamentales roulent sur deux choses, Sçavoir sur l'unité, et sur l'infini."

[2] See Hofmann 1974, pp. 3–4.

[3] See Leibniz, De republica literaria, A VI, 4, 432, 438.

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N. B. Goethe et al. (eds.), *G.W. Leibniz, Interrelations between Mathematics and Philosophy,* Archimedes 41, DOI 10.1007/978-94-017-9664-4_2

1 Rivers and Motion

Evidence of Leibniz's recognition of this fundamental role of mathematics is to be found in his earliest extant piece of writing on the theory of motion. In *De rationibus motus*, drawn up under remarkable circumstances in 1669, he describes mathematics metaphorically as the source of the mixed sciences, on which so much useful knowledge depends:

> We therefore set out the foundations of motion, such as they are in the pure state of nature, for they are neither strengthened by demonstrations, nor embellished by logical conclusions, though these conclusions be infinite and manifest; for the sources of the arts, though they are accustomed because of a certain aridity and simplicity to displease the fastidious, flow from thence through continuous descent to the richest rivers of the sciences, and finally as if into an ocean of uses and applications.[4]

It is surely no accident that this metaphor was conceived during Leibniz's first encounter with recent scientific work from France and England on the theory of motion. In August 1669, he spent three weeks in the spa town of Bad Schwalbach in Hesse accompanying his patron Johann Christian von Boineburg, who regularly spent part of the summer taking waters away from the stresses and strains of court life in Mainz. While he was there, Leibniz was lent the recently-published April 1669 issue of the *Philosophical Transactions* by a friend of Boineburg's, the Kiel law professor Erich Mauritius. Geographically far removed from the meetings of the leading scientific community in Europe, he was able to read that the discovery of the laws of motion was a central concern to members of the Royal Society of London. The April issue contained Christiaan Huygens's contribution to the debate, which controversially had been suppressed from earlier publication, because the publisher, Henry Oldenburg, had considered Huygens's rules to be largely identical to those of the English mathematician Christopher Wren[5].

During the next two years, Leibniz would adopt quite a different approach to work on the laws of motion than that which he found while reading the *Philosophical Transactions* in the idyllic surroundings of Bad Schwalbach.[6] But for the moment it was primarily the geography of the region itself which arrested the young philosopher's mind. Bad Schwalbach lies on a tributary of the river Aar, which flows into the river Lahn. Its waters flow into those of Germany's great mercantile artery, the Rhine, and they in turn flow into the North Sea. Leibniz's surroundings suggested a metaphor for the relations between theory and practice, mathematics,

[4] Leibniz, *De rationibus motus* § 7, A VI, 2, 160: "Ordiamur igitur proferre Fundamenta motuum, qualia sunt in puro naturae statu, neque tamen munita demonstrationibus, neque ornata consectariis, infinitis tamen illis et illustribus; nam fontes artium, ut ariditate quadam et simplicitate delicatis displicere solent, ita decursu perpetuo in uberrima scientiarum flumina, denique quoddam, velut mare usus ac praxeos excrescunt."

[5] Christiaan Huygens's 'Regulae de motu corporum ex mutuo impulsu' were published under the English title 'A Summary Account of the Laws of Motion' in *Philosophical Transactions* Huygens 1669, pp. 927–928. (Oldenburg's account of the controversy is printed pp. 925–9277.) The laws had been published before in French in the *Journal des Sçavans*, 18 March 1669, pp. 22–24.

[6] See Garber 1995, especially pp. 273–281; Beeley 1999, pp. 134–135.

the mathematical sciences, and their applications which would hold true throughout the whole of his philosophical career. Indeed, as I will seek to show in the course of this chapter, the river metaphor only begins to acquire its full significance towards the end of his life, when so many of his theories and ideas contained in his innumerable drafts, memoranda, and letters were falling into place. Just as there is remarkable constancy in the theoretical importance of the concepts of unity and infinity in his thought, so, too, in the systematic importance he attaches to mathematics. And as I also hope to demonstrate, the concept of infinity serves as a vital link between the two.

2 Three Central Considerations: Utility, Conciliation and Rigour

It cannot be emphasized enough that among seventeenth-century and early eighteenth-century Enlightenment philosophers Leibniz stands out as having thought more than any other about the need to provide a rational account of the successful application of mathematics in the mixed sciences such as mechanics, optics, and navigation, which he expressed so evocatively in the river metaphor of 1669. Three central considerations motivated him in this.

First, it was driven by his conviction, already expressed in his youth, that the sole aim of philosophy consists in improving the lives of people and that the mathematical sciences must necessarily play a decisive role in achieving this aim[7]. The utilitarian justification of philosophy was of course widely propagated in the seventeenth century and was for instance written into the charter of the Royal Society. But for Leibniz its importance stretched beyond general claims of mechanistic and experimental science to the explanatory core of his hypothesis. In effect, the extraordinary success of the mathematical sciences needed, in his view, to be explained metaphysically. No other thinker had succeeded in providing such an explanation before him.

Second, accommodating the mathematical sciences on a structural level was part of Leibniz's broader conciliatory approach, often descriptively couched in terms of divine economy, by means of which he sought to evince the highest possible degree of veracity of his own hypothesis, be it the *Hypothesis physica nova* of his youth or the *Monadology* of his philosophical maturity[8]. All philosophical and scientific tradition contained in his view something of value as part of the heritage of human culture and learning[9]. But for Leibniz it was more than simply taking account of philosophical and scientific tradition which he set out to achieve. As he writes on

[7] See Leibniz, *Hypothesis physica nova*, conclusio, A VI, 2, 257.

[8] See Leibniz, *Hypothesis physica nova*, § 59, A VI, 2, 255; Leibniz to Arnauld, [29 September/9 October 1687], A II, 2, 249–250; Leibniz, *Reponse aux reflexions contenues dans la seconde Edition du Dictionnaire Critique de M. Bayle*, Gerhardt 1875-90 (cited hereafter as 'GP') IV, 568.

[9] See Leibniz, *Nouveaux Essais* III, 9, § 9, A VI, 6, 336–337.

one occasion, "it is no small indication of the truth of our hypothesis that it harmonizes all"[10]. If central tenets of ancient learning and modern scientific thought could be comfortably accommodated in his philosophy, there could be no better evidence for its truth.

Third, Leibniz was convinced that mathematical minds were able to introduce rigour into metaphysics which would otherwise be lacking, particularly as concerns our fundamental knowledge of the natural world. At the same time, he rejected the use of *mos geometricus* or geometrical method in philosophy, as this is found for example in Baruch de Spinoza's *Ethics*, on the grounds that such an approach represented an unsuitable methodological crossing of disciplinary borders[11]. When Pierre Bayle suggested in his *Remarques sur le systeme de M. Leibniz* that mathematicians who get involved in philosophical matters do not achieve much, Leibniz retorted that, on the contrary, they necessarily achieve more than most philosophers, "because they are accustomed to reason with precision"[12].

It is not necessary here to rehearse the many instances in which mathematical reasoning informed Leibniz's philosophical investigations. Much of his work on universal character and combinatorics sought to achieve a more exact, that is to say semantically unambiguous determination of the world around us, which would lead ultimately to the growth of knowledge, by starting out from basic elements or simple propositions. It suffices to say that already in his *Dissertatio de arte combinatoria*, first published in1666, the basic concept of the mechanistic philosophy, that all larger things are composed of smaller ones, be they atoms or molecules, was seen as providing the metaphysical basis for the universal application of combinatorics, since the fundamental relation of the latter, that of whole to part, is seen as having a direct correspondent in reality[13]. Even after Leibniz had disassociated himself from atomism, to which he was at times attracted[14], and was reintroducing the concept of substantial form into his philosophy, that is to say around 1679, he still described combinatorics as "a kind of metaphysical geometry"[15]. What is important here is that Leibniz for profound metaphysical and theological reasons rejected the reduction of nature philosophically to a mathematical model, for this would entail among other things the absolute necessity of human action. Nonetheless, he consistently employed from the time of his early writings on the theory of motion onward

[10] Leibniz, *Hypothesis physica nova* § 59, A VI, 2, 252: "Hypothesis nostra non parvo veritatis indicio omnes conciliat." See Beeley 1996, pp. 223–234.

[11] See Leibniz, *Recommandation pour instituer la science générale*, A VI, 4, 705.

[12] Leibniz to Masson, after [1]/12 October, 1716, GP VI, 628: "Il pretend que les Mathematiciens qui se mêlent de Philosophie, n'y reussissent gueres: au lieu qu'il semble qu'ils devroient reussir le mieux, étant accoustumés à raisonner avec exactitude."

[13] See Leibniz, *Dissertatio de arte combinatoria* § 34, A VI, 1, 187; Leibniz to Gallois, end of 1672, A II, 1 (2006), 354; Leibniz, *De arte inveniendi combinatoriae*, A VI, 4, 332; Beeley 1999, p. 138.

[14] See Arthur 2003.

[15] Leibniz, *De arte inveniendi combinatoria*, A VI, 4, 332: "Combinatoria agit quodammodo de Entium configuratione, seu coordinatione, nullo respectu habito loci, est quasi Geometria Metaphysica."

a model of the core of nature—albeit one which underwent a radical transformation from quasi-materialism to idealism by means of the concept of force—which had its origins in essentially mathematical concepts. Conversely his model of nature was able to facilitate the harmonization of mathematics and nature to a degree not found among any of his contemporaries. This essentially mathematical core is the reason why we find so many references in his letters and papers to “arcana rerum”[16], “arcana naturae”[17], “interieur de la nature”[18], and the like.

3 Metaphysics of Discovery and Explanation

The relationship between mathematics and model of nature in Leibniz is complex, as it comprises three components which do not always sit comfortably alongside one another.

First, there are genuinely metaphysical concepts which he postulates as part of the fundamental architectonics, most readily apparent in the hypothesis that basic material structures are replicated or folded into infinity, but also evident in questions concerning the composition of motion. The monadological concept of the existence of worlds within worlds *ad infinitum* always took its source from theoretical deliberations on infinite divisibility of the continuum[19].

Second, there is a heuristic aspect to the interrelationship, because we are led to expect precisely the agreement between the explanatory model and scientific discovery, that is to say between the *explanans* and the *explanandum* which is ultimately evinced. Thus for example the law of continuity, which as Leibniz never ceases to tell us[20], he first introduced into the republic of letters, is able to exclude false theories of motion, such as those originally proposed by Descartes and later elaborated by some of his followers. At the same time, the law of continuity derives from the principle of reason and is thus already written into the very constitution of the created world[21].

Third, there is the explanatory component itself, in which Leibniz on the basis of his model of nature seeks to provide metaphysical reasons for discoveries in contemporary mathematical—and indeed biological—science, which are often expressed in terms of divine benevolence. These explanations are essentially grounded in his metaphysics, for example in the concept that God is a perfect geometer or

[16] Leibniz, *Rationale fidei catholicae*, A VI, 4, 2321. See Beeley 2004, pp. 29–30.

[17] Leibniz, *Praefatio ad libellum elementorum physicae*, A VI, 4, 2003; Leibniz to Carcavy, beginning of November 1671, A II, 1 (2006), 288.

[18] Leibniz to Oldenburg, end of October 1676?, A II, 1 (2006), 380.

[19] See Leibniz to Queen Sophie Charlotte, [20]/31 October 1705, GP VII, 560.

[20] See for example Leibniz, *Tentamen anagogicum*, GP VII, 279; Leibniz, *Justification du calcul des infinitesimales par celuy de l’algebre ordinaire*, Gerhardt 1849-63 (cited hereafter as ‘GM’) IV, 105; Leibniz to De Volder, 1699, GP II, 192.

[21] See Duchesneau 1994.

that the continuum constitutes the base of God's reason. Here, not only are metaphysical grounds given for the scientific discoveries themselves, but also—as a kind of divine payback—post hoc justification for the very hypothesis that is obtained.

The beginning of this interrelationship between mathematics and nature can be observed already in Leibniz's first philosophical system which emerged during his employment as privy councillor in Mainz in the early 1670s. The concept of point which he developed in his *Theoria motus abstracti* mirrored precisely the concept of conatus, thereby allowing a direct correlation between lines and motions, mathematics and phoronomy. It is to this early work of Leibniz which I now turn.

4 Metaphysics and Mathematics in Leibniz's Early Philosophy

Leibniz presents his concept of point in *Theoria motus abstracti* through a series of negations by means of which he sets it apart from the defining characteristics laid down by Aristotle and Hobbes. Just as the Aristotelian conception of unextended indivisible is rejected, so, too, the diametrically opposing Hobbesian conception of an infinitely small extended divisible whose parts and quantity simply do not enter any calculation:

> A point is not that whose part is nothing, nor that whose part is not considered; rather, it is that whose extension is nothing or that whose parts are without distance, whose magnitude is inconsiderable, indeterminable, smaller than that whose relation to another sensible magnitude can be expressed except as infinity, smaller than that which can be given. Moreover this is the foundation of the method of Cavalieri, through which its truth is plainly demonstrated when certain rudiments or beginnings of lines and figures are imagined to be smaller than any giveable line or figure."[22]

By conceiving points in this way, Leibniz believed he would be able to overcome the labyrinth of the continuum, whose composition had vexed philosophers since ancient times. The paradoxes which arose when attempting to compose a continuum out of true indivisibles– as atomism had sought to do at least implicitly—vanished once the categorical distinction between point and extension was largely negated[23]. Both now consisted of parts, the only difference being that the parts of point were considered to be "indistant" (partes indistantes). It was for this reason that the young Leibniz was convinced that he was also able to save Cavalieri's method for determining quadratures and cubatures from critics like the Austrian mathematician

[22] Leibniz, *Theoria motus abstracti*, fund. praed. § 5, A VI, 2, 265: "Punctum non est, cujus pars nulla est, nec cujus pars non consideratur; sed cujus extensio nulla est, seu cujus partes sunt indistantes, cujus magnitudo est inconsiderabilis, inassignabilis, minor quam quae ratione, nisi infinita ad aliam sensibilem exponi possit, minor quam quae dari potest: atque hoc est fundamentum Methodi Cavalerianae, quo ejus veritas evidenter demonstratur, ut cogitentur quaedam ut sic dicam rudimenta seu initia linearum figurarumque qualibet dabili minora."

[23] See Beeley 1996, pp. 235−261. This interpretation is accepted by Arthur, although he confusingly calls Leibniz's actually divided points "indivisibles". See Arthur 2009, pp. 12−17.

and astronomer Paul Guldin who had attacked it on account of the implied composition of extension from indivisibles[24]. Significantly, the characterization of the magnitude of point as being smaller than any magnitude which can be given negates the absolute and thus opens up the possibility of quantitative relations between points themselves.

In *Theoria motus abstracti*, Leibniz defines the phoronomic concept of conatus analogously to the concept of point as the beginning or end of motion. Just as point is ontologically the limit of a line, so conatus is ontologically the limit of a motion. But points fulfilled this function already within the Aristotelian tradition, where they are conceived as being true indivisibles. The decisive theoretical development which Leibniz proposes in his early philosophy is that through infinite replication points can be seen to compose lines, just as through infinite forward replication conatus can be seen to compose motion: “Conatus is to motion as point is to space, or as one to infinity, it is namely the beginning or end of motion.”[25]

Conatus is therefore clearly distinguished from rest, which as Leibniz emphasizes does not stand in relation to motion as point does to space, but rather in the relation of nothing to the number one[26]. Leibniz always believed that there could be no true state of rest in nature[27]. Everything on his view is constantly in flux, even if imperceptibly so; conatus is thus something like an infinitely small motion, endowed with its own direction. This belief was theoretically founded in his conviction that motion cannot be rationally defined in terms of its contrary, and provided him with one of his strongest reasons for rejecting the laws of motion propounded by Huygens and Wren. According to Huygens’s first law of motion, a hard body in motion colliding with an equally hard body at rest would lose all its motion and transfer that motion to the body hitherto at rest[28]. Leibniz explicitly negates the idea that a resting body can cause another body to lose its motion[29].

As already mentioned, Leibniz’s early concept of point allowed him to postulate the existence of quantitative relationships on a non-quantitative level of a kind which to a remarkable extent anticipated the quantitative relationships he postulated in his subsequent work on the calculus. The most thorough exposition of his concept of point at this time is to be found in the section of his theory of abstract motion entitled “Predemonstrable foundations” (fundamenta praedemonstrabilia). Drawing on the third of these foundations, which denies either in space or body the existence

[24] See Mancosu 1996, pp. 50–56.

[25] Leibniz, *Theoria motus abstracti*, fund. praed. § 10, A VI, 2, 265: “Conatus est ad motum, ut punctum ad spatium, seu ut unum ad infinitum, est enim initium finisque motus.”

[26] Leibniz, *Theoria motus abstracti*, fund. praed. § 6, A VI, 2, 265: “Quietis ad motum non est ratio quae puncti ad spatium, sed quae nullius ad unum.”

[27] See Leibniz, *De rationibus motus* § 12, A VI, 2, 161; *De mundo praesenti*, A VI, 4, 1511; *Nouveaux Essais*, préface, A VI, 6, 53, 56.

[28] Huygens 1669, p. 927: “1. Si Corpori quiescenti duro aliud aequale Corpus durum occurrat, post contactum hoc quidem quiescet, quiescenti vero acquiretur eadem quae fuit in Impellente celeritas.”

[29] See Leibniz, *De rationibus motus* § 12, A VI, 2, 161: “Quies nullius rei causa est, seu corpus quiescens alii corpori nec motum tribuit, nec quietem, nec directionem, nec velocitatem.”

of a minimum, where minimum is understood as that which has neither part nor magnitude, he argues in the fourteenth foundation for the interdependency of his concepts of point, temporal moment and conatus:

> But whatever moves at all is not at any time in one place while it moves, not even in a certain instant or minimum time, since what moves in time strives in an instant or begins or ceases to move, that is, it changes its place. Nor is it necessary to say that to strive in a time smaller than any time which can be given is assuredly to be in a minimal space, for there can be no minimal part of time or else there would be a minimal part of space. Then what covers the length a line in time covers the length of a line smaller than any line which can be given or a point in a time smaller than any time which can be given; and in an absolutely minimal time an absolutely minimal part of space, such as which cannot be the case according to foundation 3.[30]

This interdependency of space, time, and motion allows Leibniz to develop a further principle, namely that one point can be larger than another just as one conatus can be larger than another, on the necessary assumption of a *tertium comparationis*: that each temporal instant is equal to another.

> No-one can easily deny the inequality of conatus, for this follows from the inequality of points. […] Therefore in a given instant the stronger conatus covers more spacethan the weaker, but no conatus can pass through more than a point or a part of space smaller than can be expressed in one instant; otherwise it could pass through an infinite line in time. Therefore one point is larger than another.[31]

Implicitly, the concepts of point, instant, and conatus are understood in *Theoria motus abstracti* as being infinitely small in relation to their corresponding quantitatively extensive concepts of space, time, and motion. Already in Mainz, the infinite difference between point and line, instant and time, as well as between conatus and motion, precludes on Leibniz's view that the one can be expressed in terms of the other.

[30] Leibniz, *Theoria motus abstracti*, fund. praed. § 14, A VI, 2, 265−266: "Sed et omnino quicquid movetur non est unquam in uno loco dum movetur, ne instanti quidem seu tempore minimo; quia quod in tempore movetur, in instanti conatur, seu incipit desinitque moveri, id est locum mutare: nec refert dicere, quolibet tempore minore quam quod dari potest, conari, minimo vero esse in loco: non enim datur pars temporis minima, alioquin et spatii dabitur. Nam quod tempore absolvit lineam, tempore minore quam quod dari potest, absolvet lineam minorem quam quae dari potest seu punctum; et tempore absolute minimo partem spatii absolute minimam, qualis nulla est *per fund*. 3."

[31] Leibniz, *Theoria motus abstracti*, fund. praed. 18, A VI, 2, 266−267: "Conatu[u]m inaequalitatem nemo facile negaverit, sed inde sequitur inaequalitas punctorum. […] Ergo instanti dato fortiori spatii absolvet, quam tardior, sed quilibet conatus non potest percurrere uno instanti plus quam punctum, seu partem spatii minorem, quam quae exponi potest; alioquin in tempore percurreret lineam infinitam: est ergo punctum puncto majus."

5 Infinity, Error and Utility

Even on the basis of his still largely rudimentary knowledge of mathematics, if we are to believe his own accounts, Leibniz draws conclusions concerning the infinite which anticipate to a remarkable degree later work where he has a much more sophisticated conceptual framework in mathematics at his disposal. This is particularly evident in his treatment of curves. Although he at times allows a metaphysical distinction between a circle and its inscribed or described polygon in his writings of the early 1670s, Leibniz crucially argues that curves can be treated in the way established in Archimedes, because the resulting error is "smaller than can be expressed by us"[32] or, as he also puts it, because the resulting error is "imperceptible"[33]. After outlining a series of special problems such as the construction of a cylinder from mere rectilinear bodies or the construction of circular motion from a number of rectilinear motions, which appear to require mechanical solutions, he writes in *Theoria motus abstracti*:

> Even if these and other problems cannot be solved through abstract concepts of motion in bodies considered absolutely, they can nonetheless easily be explained in sensible bodies, namely under the assumption of an insensible ether, on account of which no sensible error disturbs our reasons, which suffices to explain phenomena. For sure, nature […] and art solve these problems quite differently from the geometer, namely mechanically by means of motions that are not continuous but actually interrupted; just as when geometers describe the quadratrix by points, and Archimedes the circle by polygons, through the removal of error nothing will disturb the phenomena.[34]

As this passage suggests, Leibniz occasionally in his early work allows phenomenal considerations to run into theoretical considerations when talking about what might be called infinitely small quantities. But we should not allow this apparent conflation to obscure the importance of his remarks, formulated as they are, some 2 years before the beginning of his momentous stay in Paris. Leibniz was already clear at this time that only the existence of an imperceptible error could explain the applicability of mathematical reasoning to our understanding of natural phenomena. In this sense there must according to his metaphysics be an insignificant difference between what is mathematically exact and that which is natural and therefore

[32] Leibniz, *Pacidius philalethi*, A VI, 3, 569, var. (reconstructed): " Quemadmodum polygonum regulare infinitorum laterum pro circulo metaphysice haberi non potest, tametsi in Geometria pro circulo habeatur, ob errorem minorem quam ut a nobis exprimi possit." Cf. Leibniz, *De infinite parvis*, A VI, 3, 434.

[33] Leibniz to Jean Chapelain (?), first half of 1670, A II, 1 (2006), 87; Leibniz, *Theoria motus abstracti*, usus, A VI, 2, 273; *De rebus in scientia mathematica tractandis*, A VI, 4, 379; Beeley 1999, p. 140.

[34] Leibniz, *Theoria motus abstracti*, usus, A VI, 2, 273: "Etsi haec aliave solvi non possent ex abstractis motus rationibus in corporibus absolute consideratis; in sensibilibus tamen, assumto saltem Aethere insensibili, facile explicari potest qua ratione efficiatur ut nullus error sensibilis rationes nostras turbet, quod phaenomenis sufficit. Nimirum longe aliter Natura […] et Ars haec problemata solvit, quam Geometra; mechanice scilicet, motibus non continuis, sed revera interruptis; uti Geometriae describunt quadratricem per puncta, et Archimedes quadrat Circulum per Polygona, spreto errore nihil phaenomena turbaturo."

somewhat less than exact. Precisely this kind of approximation came into play when working with the infinite in mathematics. In a letter to Jean Chapelain, written in the first half of 1670, he reveals the results of his latest deliberations. Claiming that no curvilinear figure is imaginable which cannot be rationally expressed, he suggests that any error contained in the expression can be made less than what can be perceived. And this, he argues, is sufficient for practical purposes:

> In general, I maintain, as exemplified by the Elements of Euclid, that no figure can be contrived which cannot be explicated according to a prescribed method (if one does not mind subjecting oneself to laborious operations such as Archimedes did in the measurement of the circle, and also Ludolph van Ceulen, who has proceeded much further) in such a way that no error can be perceived, which is sufficient in practice.[35]

The fundamental significance of these remarks is clear: when working with the infinite, procedures can be pursued indefinitely; we can in principle increase the level of accuracy as much as we want. Almost incidentally, Leibniz refers to the long-windedness of the method of exhaustion employed by Archimedes in his work on the dimension of the circle, and suggests that Ludolph van Ceulen had substantially furthered our knowledge through his calculation of pi in *Van den Circkel* (1596). As is well known, Leibniz himself began work on determining the best possible approximation to pi already during his stay in Paris, although he did not publish details of these investigations until 1682.

The conflation of phenomenal and theoretical considerations during the Mainz period is not as extraordinary as might at first appear. Just as mathematics mirrors physics in respect of the elemental quanta point and conatus, so, too, does it do this in respect of procedures involving the infinite. When Leibniz sets out his model of nature in the *Hypothesis physica nova*, he postulates the existence of an infinitely replicated structure of bubbles (bullae) which combines the fundamental ideas of corpuscular theory with the infinite divisibility of matter. Nor does such a structure preclude the possibility of causal explanation. When considering the eventual causal contribution of events on a lower level to those on a higher level, the young philosopher suggests that they are of no consequence because they do not affect our phenomena. If there are worlds within worlds into infinity, as the micrographic investigations of men, such as Robert Hooke and Marcello Malpighi implied, they were for Leibniz at this time largely autonomous worlds from a causal point of view:

> And although according to the observations of micrographers there are continuously some things smaller than others, the same relations will always obtain. Since aqueous bubbles compared to air bubbles are like earth bubbles and air bubbles compared to ether bubbles have the same relation, nothing prevents the possibility of there being another ether, of which we have no suspicion, and which is loftier than the ether we have recognized through reasoning and experiment to the same extent that water is loftier than earth and air is loftier

[35] Leibniz to Jean Chapelain (?), first half of 1670, A II, 1 (2006), 87: "Omnino inquam, quemadmodum enim post extantia Euclidis Elementa, nulla est excogitabilis figura, quae non praescripta methodo (si quem non taedeat diuturnis subjectionibus operam impendere, uti in dimensione Circuli fecit Archimedes, et qui multo longius progressus est, Ludolphus a Colonia) solvi ita possit, ut error sit insensibilis, quod in praxi sufficit."

http://www.springer.com/978-94-017-9663-7

G.W. Leibniz, Interrelations between Mathematics and Philosophy
Goethe, N.B.; Beeley, P.; Rabouin, D. (Eds.)
2015, X, 210 p. 27 illus., Hardcover
ISBN: 978-94-017-9663-7