

# Contents

<b>1</b>	<b>Introduction</b> . . . . .	1
	References . . . . .	5
<b>Part I Noncommutative Geometric Spaces</b>		
<b>2</b>	<b>Finite Noncommutative Spaces</b> . . . . .	9
	2.1 Finite Spaces and Matrix Algebras . . . . .	9
	2.1.1 Commutative Matrix Algebras . . . . .	13
	2.1.2 Noncommutative Matrix Algebras . . . . .	13
	2.2 Noncommutative Geometric Finite Spaces . . . . .	18
	2.2.1 Morphisms Between Finite Spectral Triples . . . . .	23
	2.3 Classification of Finite Spectral Triples . . . . .	25
	Notes . . . . .	28
	References . . . . .	29
<b>3</b>	<b>Finite Real Noncommutative Spaces</b> . . . . .	31
	3.1 Finite Real Spectral Triples . . . . .	31
	3.1.1 Morphisms Between Finite Real Spectral Triples . . . . .	33
	3.2 Classification of Finite Real Spectral Triples . . . . .	35
	3.3 Real Algebras and Krajewski Diagrams . . . . .	42
	3.4 Classification of Irreducible Geometries . . . . .	45
	Notes . . . . .	46
	References . . . . .	47
<b>4</b>	<b>Noncommutative Riemannian Spin Manifolds</b> . . . . .	49
	4.1 Clifford Algebras . . . . .	49
	4.1.1 Representation Theory of Clifford Algebras . . . . .	53
	4.2 Riemannian Spin Geometry . . . . .	55
	4.2.1 Spin Manifolds . . . . .	56
	4.2.2 Spin Connection and Dirac Operator . . . . .	59
	4.2.3 Lichnerowicz Formula . . . . .	63

4.3	Noncommutative Riemannian Spin Manifolds:	
	Spectral Triples . . . . .	64
4.3.1	Commutative Subalgebra . . . . .	70
	Notes . . . . .	71
	References . . . . .	73
<b>5</b>	<b>The Local Index Formula in Noncommutative Geometry . . . . .</b>	<b>75</b>
5.1	Local Index Formula on the Circle and on the Torus . . . . .	75
5.1.1	The Winding Number on the Circle . . . . .	75
5.1.2	The Winding Number on the Torus . . . . .	77
5.2	Hochschild and Cyclic Cohomology . . . . .	81
5.3	Abstract Differential Calculus . . . . .	85
5.4	Residues and the Local $(b, B)$ -Cocycle . . . . .	90
5.5	The Local Index Formula . . . . .	93
	Notes . . . . .	96
	References . . . . .	98
 <b>Part II Noncommutative Geometry and Gauge Theories</b>		
<b>6</b>	<b>Gauge Theories from Noncommutative Manifolds . . . . .</b>	<b>103</b>
6.1	'Inner' Unitary Equivalences as the Gauge Group . . . . .	103
6.1.1	The Gauge Algebra . . . . .	106
6.2	Morita Self-Equivalences as Gauge Fields . . . . .	107
6.2.1	Morita Equivalence . . . . .	107
6.2.2	Morita Equivalence and Spectral Triples . . . . .	112
6.3	Localization . . . . .	114
6.3.1	Localization of Gauge Fields . . . . .	117
	Notes . . . . .	118
	References . . . . .	119
<b>7</b>	<b>Spectral Invariants . . . . .</b>	<b>121</b>
7.1	Spectral Action Functional . . . . .	121
7.2	Expansions of the Spectral Action . . . . .	123
7.2.1	Asymptotic Expansion . . . . .	123
7.2.2	Perturbative Expansion in the Gauge Field . . . . .	125
7.A	Divided Differences . . . . .	130
	Notes . . . . .	131
	References . . . . .	134
<b>8</b>	<b>Almost-Commutative Manifolds and Gauge Theories . . . . .</b>	<b>137</b>
8.1	Gauge Symmetries of AC Manifolds . . . . .	137
8.1.1	Unimodularity . . . . .	139
8.2	Gauge Fields and Scalar Fields . . . . .	141
8.2.1	Gauge Transformations . . . . .	143

8.3	The Heat Expansion of the Spectral Action . . . . .	144
8.3.1	A Generalized Lichnerowicz Formula . . . . .	144
8.3.2	The Heat Expansion . . . . .	148
8.4	The Spectral Action on AC Manifolds . . . . .	151
	Notes . . . . .	157
	References . . . . .	157
<b>9</b>	<b>The Noncommutative Geometry of Electrodynamics . . . . .</b>	<b>159</b>
9.1	The Two-Point Space . . . . .	159
9.1.1	The Product Space . . . . .	160
9.1.2	$U(1)$ Gauge Theory . . . . .	162
9.2	Electrodynamics . . . . .	163
9.2.1	The Finite Space . . . . .	164
9.2.2	A Non-trivial Finite Dirac Operator . . . . .	165
9.2.3	The Almost-Commutative Manifold . . . . .	166
9.2.4	The Spectral Action . . . . .	167
9.2.5	The Fermionic Action . . . . .	168
9.2.6	Fermionic Degrees of Freedom . . . . .	170
9.A	Grassmann Variables, Grassmann Integration and Pfaffians . . . . .	171
	Notes . . . . .	172
	References . . . . .	173
<b>10</b>	<b>The Noncommutative Geometry of Yang–Mills Fields . . . . .</b>	<b>175</b>
10.1	Spectral Triple Obtained from an Algebra Bundle . . . . .	175
10.2	Yang–Mills Theory as a Noncommutative Manifold . . . . .	179
10.2.1	From Algebra Bundles to Principal Bundles . . . . .	179
10.2.2	Inner Fluctuations and Spectral Action . . . . .	180
10.2.3	Topological Spectral Action . . . . .	183
	Notes . . . . .	183
	References . . . . .	184
<b>11</b>	<b>The Noncommutative Geometry of the Standard Model . . . . .</b>	<b>185</b>
11.1	The Finite Space . . . . .	185
11.2	The Gauge Theory . . . . .	189
11.2.1	The Gauge Group . . . . .	189
11.2.2	The Gauge and Scalar Fields . . . . .	191
11.3	The Spectral Action . . . . .	194
11.3.1	Coupling Constants and Unification . . . . .	200
11.3.2	The Higgs Mechanism . . . . .	201
11.4	The Fermionic Action . . . . .	205
	Notes . . . . .	211
	References . . . . .	212

<b>12 Phenomenology of the Noncommutative Standard Model</b> . . . . .	213
12.1 Mass Relations . . . . .	213
12.1.1 Fermion Masses . . . . .	213
12.1.2 The Higgs Mass . . . . .	214
12.1.3 The Seesaw Mechanism . . . . .	215
12.2 Renormalization Group Flow . . . . .	216
12.2.1 Coupling Constants . . . . .	216
12.2.2 Renormalization Group Equations . . . . .	218
12.2.3 Running Masses . . . . .	219
12.3 Higgs Mass: Comparison to Experimental Results . . . . .	222
12.4 Noncommutative Geometry Beyond the Standard Model . . . . .	224
Notes . . . . .	227
References . . . . .	228
<b>Subject Index</b> . . . . .	231
<b>Notation Index</b> . . . . .	235



<http://www.springer.com/978-94-017-9161-8>

Noncommutative Geometry and Particle Physics

van Suijlekom, W.

2015, XVI, 237 p. 28 illus., 2 illus. in color., Hardcover

ISBN: 978-94-017-9161-8