Preface

Since their introduction in the pioneering work by Schoenberg [75], splines have become one of the powerful tools in Mathematics [2, 47, 76, 77, 95] and in computer-aided geometric design [22, 27, 46, 48, 57, 103]. In recent decades, splines have served as a source for wavelet [1, 3, 4, 12, 14, 17, 29, 39, 40, 58, 71, 80, 88, 91, 92, 96, 100, 101, 102], multiwavelet [13, 43], and frame constructions [16, 19, 21, 37, 38, 44, 67, 72]. Splines and spline-based wavelets, wavelet packets and frames have been extensively used in signal and image processing applications [5, 6, 11, 15, 18, 23, 24, 26, 31, 32, 51, 53, 54, 65, 81, 85, 87, 89, 90], to name a few.

An excellent survey for state-of-the-art (as of year 1999) on spline theory and applications is given in [86]. This survey motivated us in writing the present textbook. Another motivation was the emergence in recent years of new contributions of splines to wavelet analysis and applications. In addition, we believe that the socalled discrete splines and their applications deserve a systematic exposure.

Discrete splines [30, 52, 60, 61, 62, 63, 68, 77, 93], whose properties mimic the properties of polynomial splines, are the discrete-time counterparts of polynomial splines. They provide natural tools for handling discrete-time signal processing problems and serve as a source for the design of wavelet transforms [9, 10, 55, 69] and frames transforms [16, 20, 105], whose properties perfectly fit signal/image processing applications.

The goal of this book is to provide a universal toolbox accompanied by a MATLAB software for manipulating polynomial and discrete splines, spline-based wavelets, wavelet packets, and wavelet frames in signal/image processing applications. For this, known and new contributions of splines to signal and image processing are described from a unified perspective, which is based on the so-called Zak transform (ZT) [28, 94]. Being applied to B-splines, the ZT produces sets of so-called exponential splines (in Schoenberg [76] sense), which are similar to Fourier exponentials. This approach provides explicit constructions of different types of splines such as interpolating, quasi-interpolating and smoothing splines, best approximation splines and orthonormal bases for spline spaces. Constructions and utilization of various spline wavelets and spline wavelet packets have become straightforward. The ZT of discrete B-splines produces exponential discrete splines.

Coupled with the Lifting scheme [82] of a wavelet transform, the ZT approach utilizes polynomial and discrete splines for the design of versatile library of biorthogonal wavelets, multiwavelets, and wavelet frames (framelets) [9, 10, 12, 13, 14, 16, 17, 19, 20, 21, 105]. Properties of the designed wavelets and framelets, such as symmetry, flat spectra, vanishing moments, and good localization in either time or frequency domains, are valuable for signal/image processing applications. For example, the so-called Butterworth biorthogonal wavelets and wavelet frames, which originate from discrete splines, have proved to be especially efficient in signal/image processing applications. Digital filters, which have been produced during wavelets design process, give birth to subdivision schemes for fast explicit computation of splines values at dyadic and triadic rational points [22, 103, 104], which is needed for interpolation, resampling, and geometric transformations of images.

Periodic exponential splines form orthogonal bases of periodic splines spaces, which are very similar to periodic Fourier exponentials. Representation of periodic splines via orthonormal bases produces the so-called Spline Harmonic Analysis (SHA) [99, 101], which combines approximation abilities of splines with the computational strength of the Fast Fourier transform (FFT). It introduces the harmonic analysis methodology into periodic spline spaces.

SHA enables us to efficiently construct and manipulate different types of splines, wavelets, wavelet packets, and wavelet frames. SHA has paved the way for periodic splines to contribute to solutions of signal/image processing applications [12, 18, 23, 24, 26, 101, 105].

The textbook is divided into two volumes. In Volume I, periodic splines and their diverse signal processing applications are discussed. Volume II deals with nonperiodic splines.

The following topics are explored in Volume I of the book:

Zak transform (ZT): ZT of periodic functions is introduced and its properties are outlined. In particular, they include periodic counterparts of the Poisson summation formulas (for example, [70, 79]). Realizations of the ZT in polynomial and discrete periodic spline spaces result in the SHA, which is presented in detail.

Elements of spline theory and design: Different types of periodic polynomial and discrete splines with equidistant nodes are presented and their properties are outlined. The design of interpolating, smoothing, shift-orthogonal splines becomes straightforward due to the SHA methodology. Constructions of these splines types utilize filters with infinite impulse response (IIR). However, due to the FFT utilization, the computational cost of filtering with IIR is similar to the cost of filtering with finite impulse response (FIR) filters. FIR filtering enables real-time processing. For this purpose, the so-called local quasi-interpolating and smoothing splines, which are constructed by filtering data samples with FIR filters, can be used, [17, 97, 98]. Their properties are close to the properties of global interpolating and smoothing splines. These splines are presented in details in Volume II of this book. However, a couple of examples of quasi-interpolating splines are given in the current volume.

Spline subdivision and signals (images) upsampling: If spline values at grid points are given, the computation of its values between the grid points is called a spline subdivision. We describe SHA-based fast subdivision algorithms, which explicitly derive spline values at dyadic and triadic rational points from the samples taken at integer grid points in one and two dimensions. The computer-aided geometric design is a main field of application for subdivision schemes (for example, see [27, 73]). However, these techniques suit well for signals and images upsampling, to restore sparsely sampled signals and images at intermediate points. These upsampling procedures increase the objects resolution. On the other hand, when data are corrupted by noise, upsampling from a sparse grid can significantly reduce the noise level.

Deconvolution: Deconvolution here means restoration of a signal or an image from blurred sampled data that are typically corrupted by noise. This is an ill-posed problem, which means that even small fluctuations in the input may lead to output instability. Fourier analysis is a good tool in handling the convolution and convolution-related problems, such as inverse of the heat equation and the Cauchy problem for the Laplace equation, to name a few. This is because that Fourier exponentials are the eigenfunctions of the convolution operator. The exponential splines possess a similar property.

Many deconvolution algorithms are based on the Tikhonov regularization scheme [83, 84], where the approximated solution of the deconvolution problem is reduced to minimizing a parameterized functional. The regularization parameter provides a tradeoff between the approximation of the available data and the regularity of the solution. Typically, data are discrete and corrupted by noise but the physical meaning of the problem dictates the solution smoothness. In these setups, splines are a good match. SHA provides a unified computational scheme for finding a stable solution that possesses the required smoothness. The regularization parameter is derived automatically from the evaluation of the relative contributions of the coherent signal and the noise in the data [18]. Note that construction of a smoothing spline can be regarded as a special case of the regularization algorithm when the data are discrete and noised and blurring is not present.

Design of spline wavelets and wavelet packets: Constructions of different types of spline wavelets and wavelet packets in an explicit form and fast implementation of the corresponding transforms using FFT are described. The spline wavelets construction stems from the two-scale relations between exponential splines from different resolution scales, which are simple and the computation complexity of which is practically independent of the splines order. The Fourier spectra of spline wavelets from different resolution scales partitions the frequency domain in a logarithmic way. Spline wavelet packets, which generalize wavelets, are designed by using the same two-scale relations. The Fourier spectra of spline wavelet packets from the j-th resolution scale split the frequency domain into 2^j bands of approximately equal width. The shapes of the magnitude spectra of the spline wavelet packets tend to be rectangular as the splines order increases. Therefore, the wavelet packet transform of a signal produces a number of the signal representations, which are associated with different sets of frequency bands. An optimal (up to a certain cost function) representation is achieved by the so-called Best Basis algorithm [41, 74]. On the other hand, spline wavelet packets provide a versatile collection of waveforms with different spans and different frequency contents. This collection can be used as a dictionary for the Matching Pursuit framework [45, 59, 66].

Deconvolution revisited: When a signal/image is restored from blurred sampled data affected by noise, it is worthwhile to keep in mind that relative contributions of a coherent signal and noise are different in different frequency components of the data. The deconvolution problem then can be solved separately in different frequency bands, while regularization parameters are found according to the signal-to-noise ratio in each band. This approach significantly extends the adaptation abilities and the method robustness. Practically, this scheme is implemented via the utilization of orthonormal spline wavelet packets. SHA provides a unified computational scheme for their design, a fast implementation of the algorithm and an explicit representation of the solutions. An optimal set of wavelet packets is selected by the application of the Best Basis algorithm. Then, the equation is solved separately for each wavelet packet transform coefficients block [24].

If an original signal to be restored from convolution with a bandlimited kernel is highly inhomogeneous and the discrete output is strongly noised, a method, which can be characterized as a Regularized Matching Pursuit (RMP) method, produces good results. RMP is a greedy algorithm, which uses orthonormal spline wavelet packet dictionaries. The main distinction from the conventional Matching Pursuit is utilization of two different dictionaries. One dictionary, which consists of discrete-time signals, is used to test discrete data. The second dictionary, which consists of spline wavelet packets, approximates the continuous solution.

The regularization is achieved by replacing the orthogonal projections of the data onto the dictionary elements, which are used in the conventional Matching Pursuit, by oblique projections.

The above-listed regularized deconvolution methods turn into denoising methods when the input data is not blurred.

Acoustic classification and target detection: The spline wavelet packet transforms, which produce multiple splits of the frequency domain of a signal, can be used to reveal frequency bands, which are characteristic either for the signal or for a class of signals. Determining such bands is important when the processing goal is to detect the presence or arrival of objects of a certain type or to classify objects of different types via the analysis of their acoustic signatures. Typically, signals emitted by mechanical objects have a quasi-periodic structure. Such acoustic signals contain only a few dominating bands in the frequency domain, whose general disposition remains stable with respect to varying conditions. The combination of the inherent energies in these frequency bands can form an acoustic signature of the object. An efficient way to determine the characteristic bands is to apply an orthogonal spline wavelet packet transform to the signal and calculate energies in the blocks of the transform coefficients [7, 25, 26]. This approach is illustrated in the book on detection of the arrival of a vessel of a certain type by analyzing hydro-acoustic signals recorded by a hydrophone. However, the above approach is applicable to processing other types of quasi-periodic signals such as biomedical signals [8].

Discrete splines: The space of periodic discrete splines, which is a subspace of the periodic signals space is described. Properties of the discrete splines mirror properties of polynomial splines. The Zak transform applied to the discrete B-splines produces exponential discrete splines, which serve as a source for the construction of a discrete version of the SHA [68, 69]. The discrete SHA simplifies manipulations with the discrete splines. In particular, it provides explicit expressions for interpolating and smoothing discrete splines in one and two dimensions and provides fast algorithms for calculation of the discrete splines values from grid samples. This is a useful tool for upsampling signals and images.

Discrete splines wavelets and wavelet packets: Similarly to the polynomial splines case, the wavelet and wavelet packet transforms are introduced to the discrete splines space. These transforms are based on the relations between the exponential discrete splines from different resolution scales. Practically, the transforms of periodic signals are implemented by multirate filtering signals by two-channel filter banks with the downsampling factor of 2 (critically sampled filter banks). The filtering implementation is accelerated by switching to the polyphase representation of signals and filters and using the FFT. The field of applications of the discrete splines wavelet and wavelet packet transforms is, generally, the same as the field of their polynomial counterparts. In particular, the discrete splines version of the RMP is outlined.

Design of biorthogonal wavelets: The polynomial and discrete splines may contribute to wavelet analysis in another way. They are a source for a family of filters, which generate biorthogonal wavelets, whose properties are valuable for signal processing. Although these wavelets originate from splines, they, unlike the spline wavelets, do not belong to spline spaces. Design of biorthogonal wavelets and efficient implementation of the transforms of signals is carried out through the so-called Lifting scheme [82]. The idea is to split the signal into even and odd subarrays. Then, the even subarray is filtered using some *prediction* filter in order to predict the odd subarray. The predicted subarray is extracted from the original odd subarray. The difference array is filtered by an *update* filter and it is used to update the even subarray in order to eliminate aliasing. These operations are then applied to the updated even subarray and so on. Thus, multiscale wavelet decomposition is achieved. Reconstruction is implemented in the reverse order. The key point is a proper choice of the prediction and update filters. Naturally, odd samples can be predicted from midpoint values of either polynomial or discrete splines, which interpolate or quasi-interpolate the even samples of the signal. In this way, a number of linear phase IIR and FIR prediction filters are designed. Being properly modified, they are used for the update step as well. By using these filters, a diverse library of biorthogonal wavelets is constructed [9, 12]. Exclusive properties are demonstrated by the so-called Butterworth wavelets, which originate from discrete interpolatory splines. They are related to the Butterworth filters [64], which are widely used in signal processing.

Data compression is an important application of the wavelet analysis. The spline-based biorthogonal wavelets proved to be highly efficient in achieving highquality low-bitrate compression. This application is discussed in detail in Volume II.

Wavelet frames (framelets): Recently frames or redundant expansions of signals have attracted considerable interest from researchers working in signal processing although one particular class of frames, the Gabor systems, has been applied and investigated since 1946 [49]. As the requirement of one-to-one correspondence between the signal and its transform coefficients is dropped, there is more freedom to design and implement frame transforms.

Here we present some key features of framelet transforms and application to image restoration.

Design and implementation: Wavelet transforms use critically sampled filter banks. On the other hand, wavelet frame transforms are implemented by application of oversampled perfect reconstruction (PR) pairs of filter banks. It means that the number of channels in the filter banks exceeds the downsampling factor. Moreover, translations of the filters, that constitute such filter banks, form wavelet frames in the signal space [33, 42, 56]. Generally, the synthesis filter bank in the PR pair differs from the analysis filter bank. In the case when both filter banks are the same, the corresponding frame is tight. Tight frames can be regarded as redundant counterparts of orthogonal bases. The design of a variety of three- and four-channel PR filter banks, which generate tight frames in the space of periodic signals, is described. The filter banks comprise one low-pass, one high-pass and either one or two band-pass filters. All these filters are derived from the spline-based prediction filters, which were used for the design of biorthogonal wavelet transforms. In addition, the so-called semi-tight frames are introduced, where the low- and high-pass filters in the synthesis filter bank are the same as in the analysis filter bank, while the bandpass filters are different. Periodic setting, which enables the utilization of a wide range of IIR filter banks with a relaxation of the tightness requirement, provides a number of additional opportunities. Properties such as symmetry, interpolation, flat spectra combined with fine time-domain localization of framelets as well as a high number of vanishing moments can be easily achieved. The transforms implementation is reduced to application of the direct and the inverse FFT.

Image restoration: A valuable advantage that redundant representations hold is their ability to restore missing and incomplete information, which is based on the prior assumption that a frame expansion of a given signal/image is sparse. In principle, only part of the samples/pixels is needed to (near) perfect object restoration. This approach, which is a variation of the *Compressive Sensing* methodology (see, for example, [36]), has proved to be extremely efficient for image restoration.

In practice, this approach is implemented via minimization of a parameterized functional where the sparse representation is reflected in the $l1$ norm of the frame transform coefficients. The $\|\cdot\|_1$ minimization does not have an explicit solution and can be resolved only by iterative methods. The so-called *split Bregman iteration* (SBI) scheme [50] provides a fast and stable algorithm for that. Variations of this scheme and its application to image restoration using wavelet frames are described in [34, 35], to mention a few. A variety of impressive results on image restoration have been reported in the last couple of years. A survey is given in [78] while a recent development is described in [34].

Due to applications diversity, it is important to have a library of wavelet frames in order to select a frame that fits best to a specific task. Forward and inverse transforms in iterative algorithms are repeated many times: therefore, members in this library must have fast and stable transforms implementation. Waveforms symmetry with the availability of vanishing moments is also important in order to avoid distortions when thresholding is used.

The designed family of the spline-based wavelet frames perfectly meet these requirements. A number of experiments on image restoration, where performance of different frames is compared with each other, is described. Their diversity enabled a frame to be selected that best fits to each specific application. In particular, in most of the experiments semi-tight frames outperform tight frames.

The framelets construction is presented together with a fast computational scheme in [19–21].

All the presented methods are accompanied by Matlab codes. A guide to the software is given in Appendix.

The authors thank Steve Legrand for thorough proofreading and his comments, which resulted in many corrections of the text.

Tel Aviv, Israel, November 2013 — Amir Z. Averbuch
Valery A. Zheludev
Jyväskylä, Finland — Pekka Neittaanmaki

References

1. P. Abry, A. Aldroubi, Designing multiresolution analysis-type wavelets and their fast algorithms. J. Fourier Anal. Appl. **2**(2), 135–159 (1995)
2. J.H. Ahlberg, E.N. Nilson, J.L. Walsh, *The Theory of Splines and Their Applications* (Academic Press, New York, 1987)
3. A. Aldroubi, M. Eden, M. Unser, Discrete spline filters for multiresolutions and wavelets of l_2. SIAM J. Math. Anal. **25**(5), 1412–1432 (1994)
4. A. Aldroubi, M. Unser, Families of multiresolution and wavelet spaces with optimal properties. Numer. Funct. Anal. Optim. **14**(5–6), 417–446 (1993)
5. O. Amrani et al., Symmetric interpolatory framelets and their erasure recovery properties. Int. J. Wavelets Multiresolut. Inf. Process. **5**(4), 541–566 (2007)
6. M. Antonini et al., Image coding using wavelet transform. IEEE Trans. Image Process. **1**(2), 205–220 (1992)
7. A. Averbuch et al., A wavelet packet algorithm for classification and detection of moving vehicles. Multidimens. Syst. Signal Process. **12**(1), 9–31 (2001)
8. A. Averbuch, I. Kozlov, V. Zheludev, Wavelet-packet-based algorithm for identification of quasi-periodic signals, ed. by A.F. Laine, M.A. Unser, A. Aldroubi. Wavelets: Applications in Signal and Image Processing IX, vol. 4478 of Proc. SPIE, 353–360 (2001)

9. A. Averbuch, A.B. Pevnyi, V. Zheludev, Biorthogonal Butterworth wavelets derived from discrete interpolatory splines. IEEE Trans. Signal Process. **49**(11), 2682–2692 (2001)
10. A. Averbuch, A.B. Pevnyi, V. Zheludev, Butterworth wavelet transforms derived from discrete interpolatory splines: recursive implementation. Signal Process. **81**(11), 2363–2382 (2001)
11. A. Averbuch, V. Zheludev, Image compression using spline based wavelet transforms ed. by A. Petrosian, F. Meyer, in *Wavelets in Signal and Image Analysis: From Theory to Practice*, (Kluwer Academic Publishers, Dordrecht, 2001) p. 341–376
12. A. Averbuch, V. Zheludev, Construction of biorthogonal discrete wavelet transforms using interpolatory splines. Appl. Comput. Harmon. Anal. **12**(1), 25–56 (2002)
13. A. Averbuch, V. Zheludev, Lifting scheme for biorthogonal multiwavelets originated from Hermite splines. IEEE Trans. Signal Process. **50**(3), 487– 500 (2002)
14. A. Averbuch, V. Zheludev, Splines: a new contribution to wavelet analysis. ed. by J. Levesley, I.J. Anderson, J.C. Mason, Algorithms for Approximation IV: in *Proceedings of the 2001 International Symposiu*m, (University of Huddersfield, 2002) p. 314–321
15. A. Averbuch, V. Zheludev, A new family of spline-based biorthogonal wavelet transforms and their application to image compression. IEEE Trans. Image Process. **13**(7), 993–1007 (2004)
16. A. Averbuch, V. Zheludev, Wavelet and frame transforms originated from continuous and discrete splines. ed. by J. Astola, L. Yaroslavsky, Advances in Signal Transforms: Theory and Applications, (Hindawi Publishing Corporation, New York, 2007) p. 1–56
17. A. Averbuch, V. Zheludev, Wavelet transforms generated by splines. Int. J. Wavelets Multiresolut. Inf. Process. **5**(2), 257–291 (2007)
18. A. Averbuch, V. Zheludev, Spline-based deconvolution. Signal Process. **89**(9), 1782–1797 (2009)
19. A. Averbuch, V. Zheludev, T. Cohen, Interpolatory frames in signal space. IEEE Trans. Signal Process. **54**(6), 2126–2139 (2006)
20. A. Averbuch, V. Zheludev, T. Cohen, Tight and sibling frames originated from discrete splines. Signal Process. **86**(7), 1632–1647 (2006)
21. A. Averbuch, V. Zheludev, T. Cohen, Multiwavelet frames in signal space originated from Hermite splines. IEEE Trans. Signal Process. **55**(3), 797–808 (2007)
22. A. Averbuch et al., Ternary interpolatory subdivision schemes originated from splines. Int. J. Wavelets Multiresolut. Inf. Process. **9**(4), 611–633 (2011)
23. A. Averbuch, V. Zheludev, M. Khazanovsky, Deconvolution by matching pursuit using spline wavelet packets dictionaries. Appl. Comput. Harmon. Anal. **31**(1), 98–124 (2011)
24. A. Averbuch et al., Block based deconvolution algorithm using spline wavelet packets. J. Math. Imaging Vision. **38**(3), 197–225 (2010)
25. A. Averbuch et al., Acoustic detection and classification of river boats. Appl. Acoustics **72**(1), 22–34 (2011)
26. A. Averbuch et al., Wavelet-based acoustic detection of moving vehicles. Multidimens. Syst. Signal Process. **20**(1), 55–80 (2009)
27. R. Bartels, J. Beatty, B. Barsky, *An Introduction to Splines for Use in Computer Graphics and Geometric Modeling* (Morgan Kaufmann Publishers, San Mateo, CA, 1987)
28. M.J. Bastiaans, Gabor's Expansion and the Zak Transform for Continuoustime and Discrete-Time Signals, ed. by Y.Y. Zeevi, R. Coifman, Signal and Image Representation in Combined Spaces, number 7 in Wavelet Anal. Appl. (Academic Press, San Diego, CA, 1998) pp. 23–69
29. G. Battle, A block spin construction of ondelettes. I. lemarié functions. Comm. Math. Phys. **110**(4), 601–615 (1987)
30. M.G. Ber, Natural discrete splines and the averaging problem. Vestnik Leningrad Univ. Math. **23**(4), 1–4 (1990)

31. O. Bernard et al., Variational B-spline level-set method for fast image segmentation in 2008 5th IEEE International Symposium on Biomedical Imaging: From Nano to Macro. Proceedings, p. 177–180, IEEE, 2008
32. T. Blu et al., Sparse sampling of signal innovations. IEEE Signal Process. Mag. **25**(2), 31–40 (2008)
33. H. Bölcskei, F. Hlawatsch, H.G. Feichtinger, Frame-theoretic analysis of oversampled filter banks. IEEE Trans. Signal Process. **46**(12), 3256–3268 (1998)
34. J. Cai et al., Image restoration: total variation, wavelet frames, and beyond. J. Amer. Math. Soc. **25**(4), 1033–1089 (2012)
35. J. Cai, S. Osher, Z. Shen, Split Bregman methods and frame based image restoration. Multiscale Model. Simul. **8**(2), 337–369 (2009/2010)
36. E. Candes et al., Stable signal recovery from incomplete and inaccurate measurements. Comm. Pure Appl. Math. **59**(8), 1207–1223 (2006)
37. C.K. Chui, W. He. Compactly supported tight frames associated with refinable functions. Appl. Comput. Harmon. Anal. **8**(3), 293–319 (2000)
38. C.K. Chui, W. He, J. Stöckler, Compactly supported tight and sibling frames with maximum vanishing moments. Appl. Comput. Harmon. Anal. **13**(3), 224–262 (2002)
39. C.K. Chui, J.-Z. Wang, On compactly supported spline wavelets and a duality principle. Trans. Amer. Math. Soc. **330**(2), 903–915 (1992)
40. A. Cohen, I. Daubechies, J.-C. Feauveau, Biorthogonal bases of compactly supported wavelets. Comm. Pure Appl. Math. **45**(5), 485–560 (1992)
41. R.R. Coifman, V.M. Wickerhauser, Entropy-based algorithms for best basis selection. IEEE Trans. Inform. Theory **38**(2), 713–718 (1992)
42. Z. Cvetković, M. Vetterli, Oversampled filter banks. IEEE Trans. Signal Process. **46**(5), 1245–1255 (1998)
43. W. Dahmen et al., Biorthogonal multiwavelets on the interval: Cubic Hermite splines. Constr. Approx. **16**(2), 221–259 (2000)
44. I. Daubechies et al., Framelets: MRA-based constructions of wavelet frames. Appl. Comput. Harmon. Anal. **14**(1), 1–46 (2003)
45. G. Davis, S. Mallat, Z. Zhang, Adaptive time-frequency decompositions with matching pursuits. SPIE J. Opt. Eng. **33**(7), 2183–2191 (1994)
46. P. Davis, B-splines and geometric design. SIAM News **29**(5) (1997)
47. C. de Boor, *A Practical Guide to Splines* (Springer, New York, 1978)
48. P. Dierckx, *Curves and Surface Fitting with Splines* (Oxford University Press, New York, 1993)
49. D. Gabor, Theory of communications. J. Inst. Electr. Eng. **93**, 429–457 (1946)
50. T. Goldstein, S. Osher, The split Bregman method for L1-regularized problems. SIAM J. Imaging Sci. **2**(2), 323–343 (2009)
51. H.S. Hou, H. C. Andrews, Cubic splines for image interpolation and digital filtering. IEEE Trans. Acoust. Speech Signal Process. **26**(6), 508–517 (1978)
52. K. Ichige, M. Kamada, An approximation for discrete B-splines in time domain. IEEE Signal Process. Lett. **4**(3), 82–84 (1997)
53. S. Jonić et al., An optimized spline-based registration of a 3D CT to a set of C-arm images. Internat. J. Biomed. Imaging 2006(47197), p. 12 (2006)
54. M. Kamada, K. Toraichi, R.E. Kalman, A smooth signal generator based on quadratic B-spline functions. IEEE Trans. Signal Process. **43**(5), 1252–1255 (1995)
55. V.A. Kirushev, V.N. Malozemov, A.B. Pevnyi, Wavelet decomposition of the space of discrete periodic splines. Math. Notes **67**(5–6), 603–610 (2000)
56. J. Kovacevic, P.L. Dragotti, V.K. Goyal, Filter bank frame expansions with erasures. IEEE Trans. Inform. Theory **48**(6), 1439–1450 (2002)
57. B. Kvasov, *Methods of Shape-Preserving Spline Approximation* (World Scientific, River Edge, NJ, 2000)

58. P.G. Lemarié. Ondelettes à localisation exponentielle. J. Math. Pures Appl. (9), **67**(3), 227–236 (1988)
59. S. Mallat, Z. Zhang, Matching pursuits with time-frequency dictionaries. IEEE Trans. Signal Process. **41**(12), 3397–3415 (1993)
60. V.N. Malozemov, N.V. Chashnikov, Discrete periodic splines with vector coefficients and geometric modeling. Doklady Math. **80**(3), 797–799 (2009)
61. V.N. Malozemov, A.B. Pevnyi, Discrete periodic B-splines. Vestn St. Petersburg Univ. Math. **30**(4), 10–14 (1997)
62. V.N. Malozemov, A.B. Pevnyi, Discrete periodic splines and their numerical applications. Comput. Math. Math. Phys. **38**(8), 1181–1192 (1998)
63. V.N. Malozemov, A.N. Sergeev, Discrete nonperiodic splines on a uniform grid, ed. by N.N. Uraltseva, in *Proceedings of the St. Petersburg Mathematical Society, Vol. VIII*, vol. 205 of Amer. Math. Soc. Transl. Ser. 2, p. 175–187 (2002) First published in Trudy Sankt-Peterburgskogo Matematicheskogo Obshchestva, Vol. 8 (2000), 199–213 (Russian)
64. A.V. Oppenheim, R.W. Schafer, *Discrete-time signal processing*, 3rd edn. (Prentice Hall, New York, 2010)
65. D. Pang, L.A. Ferrari, P.V. Sankar, B-spline FIR filters. Circ. Syst. Sig. Proc. **13**(1), 31–64 (1994)
66. Y.C. Pati, R. Rezaiifar, P.S. Krishnaprasad, Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition, in *Conference Record of the Twenty-Seventh Asilomar Conference on Signals, Systems & Computers*, vol. 1, p. 40–44, (1993)
67. A.P. Petukhov, Symmetric framelets. Constr. Approx. **19**(2), 309–328 (2003)
68. A.B. Pevnyi, V. Zheludev. On the interpolation by discrete splines with equidistant nodes. J. Approx. Theory **102**(2), 286–301 (2000)
69. A.B. Pevnyi, V. Zheludev. Construction of wavelet analysis in the space of discrete splines using Zak transform. J. Fourier Anal. Appl. **8**(1), 59–83 (2002)
70. M. Pinsky, *Introduction to Fourier Analysis and Wavelets* (Brooks Cole, 2002)
71. G. Plonka, M. Tasche, On the computation of periodic spline wavelets. Appl. Comput. Harmon. Anal. **2**(1), 1–14 (1995)
72. A. Ron, Z. Shen, Compactly supported tight affine spline frames in $L^2\mathbb{R}^d$. Math. Comp. **67**(221), 191–207 (1998)
73. M. Sabin, *Analysis and Design of Univariate Subdivision Schemes, Geometry and Computing*, vol. 6 (Springer, Berlin, 2010)
74. N. Saito, R.R. Coifman, Local discriminant bases and their applications. J. Math. Imaging Vision **5**(4), 337–358 (1995)
75. I.J. Schoenberg, Contributions to the problem of approximation of equidistant data by analytic functions. Quart. Appl. Math. **4**, 45–99, 112–141 (1946) Parts A and B
76. I.J. Schoenberg, *Cardinal Spline Interpolation* (SIAM, Philadelphia, PA, 1973)
77. L.L. Schumaker, *Spline Functions: Basic Theory* (Wiley, New York, 1981)
78. Z. Shen, Wavelet frames and image restorations. in *Proceedings of the International Congress of Mathematicians*, ed. by R. Bhatia, vol. 6. (Hindustan Book Agency, New Delhi, 2010), p. 2834–2863
79. E. Stein, G. Weiss. *Introduction to Fourier Analysis on Euclidean Spaces* (Princeton University Press, Princeton, 1971)
80. J.-O. Strömberg, A modified Franklin system and higher-order spline systems of Rn as unconditional bases for Hardy spaces, in *Conference on Harmonic Analysis in Honor of Antoni Zygmund,* vol. 1, 2 (Chicago, Ill., 1981) (Wadsworth, Belmont, CA, 1983) pp. 475–494
81. H. Sugiyama, M. Kamada, R. Enkhbat. Image compression by spline wavelets orthogonal with respect to weighted sobolev inner product. Advanced Modeling and Optimization **10**(2), 163–175 (2008)

82. W. Sweldens, The lifting scheme: a custom-design construction of biorthogonal wavelets. Appl. Comput. Harmon. Anal. **3**(2), 186–200 (1996)
83. A.N. Tikhonov, Solution of incorrectly formulated problems and the regularization method. Soviet Math. Dokl. **4**(4), 1035–1038 (1963)
84. A.N. Tikhonov, V.Y. Arsenin, *Solutions of Ill-Posed Problems* (Wiley, New York, 1977)
85. K. Toraichi et al., Two-dimensional spline interpolation for image reconstruction. Pattern Recognit. **21**(3), 275–284 (1988)
86. M. Unser, Splines: a perfect fit for signal and image processing. IEEE Signal Process. Mag. **16**(6), 22–38 (1999) IEEE Signal Processing Society's 2000 magazine award
87. M. Unser, Splines: in A perfect fit for medical imaging, ed. by M. Sonka, J.M. Fitzpatrick, Medical Imaging 2002: Image Processing, of Proc. SPIE, vol. 4684 (2002) p. 225–236
88. M. Unser, A. Aldroubi, M. Eden, On the asymptotic convergence of Bspline wavelets to Gabor functions. IEEE Trans. Inform. Theory **38**(2) 864–872 (1992)
89. M. Unser, A. Aldroubi, M. Eden, B-spline signal processing. I. Theory. IEEE Trans. Signal Process. **41**(2), 821–833 (1993) IEEE Signal Processing Society's 1995 best paper award
90. M. Unser, A. Aldroubi, M. Eden, B-spline signal processing. II. Efficiency design and applications. IEEE Trans. Signal Process. **41**(2), 834–848, (1993)
91. M. Unser, A. Aldroubi, M. Eden. A family of polynomial spline wavelet transforms. Signal Process. **30**(2), 141–162 (1993)
92. M. Unser, P. Thévenaz, A. Aldroubi, Shift-orthogonal wavelet bases using splines. IEEE Signal Process. Lett. **3**(3), 85–88 (1996)
93. K.F. Üstüner, L.A. Ferrari, Discrete splines and spline filters. IEEE Trans. Circuits Syst. II: Analog Digital Signal Process. **39**(7), 417–422 (1992)
94. J. Zak, Finite translations in solid-state physics. Phys. Rev. Lett. **19**(24), 1385–1387 (1967)
95. Yu.S. Zav'yalov, B.I. Kvasov, V.L. Miroshnigenko, *Methods of Splinefunctions* (Nauka, Moscow, 1980) (In Russian)
96. V. Zheludev, in *Spline harmonic analysis and wavelet bases* ed. by W. Gautcshi, Proc. Symp. Appl. Math.
97. V. Zheludev, Local spline approximation on a uniform mesh. U.S.S.R. Comput. Math. Math. Phys. **27**(5), 8–19 (1987)
98. V. Zheludev, Local smoothing splines with a regularizing parameter. Comput. Math. Math Phys. **31**, 11–25 (1991)
99. V. Zheludev, An operational calculus connected with periodic splines. Soviet. Math. Dokl. **42**(1), 162–167 (1991)
100. V. Zheludev, Wavelets based on periodic splines. Russian Acad. Sci. Dokl. Math. **49**(2), 216–222 (1994)
101. V. Zheludev, in *Periodic splines, harmonic analysis, and wavelets* ed. by Y.Y. Zeevi, R. Coifman, Signal and Image Representation in Combined Spaces, Wavelet Anal. Appl. vol. 7 (Academic Press, San Diego, CA, 1998) p. 477–509
102. V. Zheludev, Wavelet analysis in spaces of slowly growing splines via integral representation. Real Anal. Exchange **24**(1), 229–261 (1998/1999)
103. V. Zheludev, Interpolatory subdivision schemes with infinite masks originated from splines. Adv. Comput. Math. **25**(4), 475–506 (2006)
104. V. Zheludev, A. Averbuch, Computation of interpolatory splines via triadic subdivision. Adv. Comput. Math. **32**(1), 63–72 (2010)
105. V. Zheludev, V.N. Malozemov, A.B. Pevnyi, Filter banks and frames in the discrete periodic case ed. by N. N. Uraltseva in *Proceedings of the St. Petersburg Mathematical Society, Vol. XIV*, of Amer. Math. Soc. Transl., Ser. 2, vol. 228 (2009) pp. 1–11

Springer

http://www.springer.com/978-94-017-8925-7

Spline and Spline Wavelet Methods with Applications to Signal and Image Processing
Volume I: Periodic Splines
Averbuch, A.Z.; Neittaanmäki, P.; Zheludev, V.A.
2014, XXV, 496 p. 185 illus., 128 illus. in color., Hardcover
ISBN: 978-94-017-8925-7