# A Unified Approach to Data Analysis and Modeling of the Appearance of Materials for Computer Graphics and Multidimensional Reflectometry

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**Abstract** Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional , computer vision and computer graphics. In this paper, we outline a unified perception-based approach to modeling of the appearance of materials for computer graphics and reflectometry. We discuss the differences and the common points of data analysis and modeling for BRDFs in both physical and in virtual application domains. We outline a mathematical framework that captures important problems in both types of application domains, and allows for application and performance comparisons of statistical and machine learning methods. For comparisons between methods, we use criteria that are relevant to both statistics and machine learning, as well as to both virtual and physical application domains. Additionally, we propose a class of multiple testing procedures to test a hypothesis that a material has diffuse reflection in a generalized sense. We treat a general case where the number of hypotheses can potentially grow with the number of measurements. Our approach leads to tests that are more powerful than the generic multiple testing procedures.

**Keywords** BRDF · Computer graphics · Data analysis · Light reflection · Machine learning · Metrology · Perception · Realistic image representation · Reflectometry · Statistics of manifolds

## 1 Introduction

Characterizing the appearance of real-world surfaces is a fundamental problem in multidimensional reflectometry, computer vision and computer graphics. For many applications, appearance is sufficiently well characterized by the bidirectional reflectance distribution function (BRDF).

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In the case of a fixed wavelength, BRDF describes reflected light as a four-dimensional function of incoming and outgoing light directions. In a special case of rotational symmetry, isotropic BRDFs are are used. Isotropic BRDFs are functions of only three angles. On the other hand, for modelling or describing complicated visual effects such as goniochromism or irradiance, an extra dimension accounting for the wave length has to be added.

In computer graphics and computer vision, usually either physically inspired analytic reflectance models [1–3], or parametric reflectance models chosen via qualitative criteria [4–7] are used to model BRDFs. These BRDF models are only crude approximations of the reflectance of real materials. Moreover, analytic reflectance models are limited to describing only special subclasses of materials.

In multidimensional reflectometry, an alternative approach is usually taken. One directly measures values of the BRDF for different combinations of the incoming and outgoing angles and then fits the measured data to a selected analytic model using optimization techniques. There are several shortcomings to this approach as well.

An alternative approach to fitting parametric models is in constructing more realistic BRDFs on the basis of actual BRDF measurements. This approach bridges the gap between computer graphics and industrial reflectometry. For example, [8] and [9] modelled reflectance of materials in nature as a linear combination of a small set of basis functions derived from analyzing a large number of densely sampled BRDFs of different materials.

There were numerous efforts to use modern machine learning techniques to construct data-driven BRDF models. Brady et al. [10] proposed a method to generate new analytical BRDFs using a heuristic distance-based search procedure called Genetic Programming. In [11], an active learning algorithm using discrete perceptional data was developed and applied to learning parameters of BRDF models such as the Ashikhmin—Shirley model [12].

In computer graphics, it is important that BRDF models should be processed in real-time. Computer-modelled materials have to remind real materials qualitatively, but the quantitative accuracy was not considered as important. The picture in reflectometry and metrology was almost the opposite: there was typically no need in real-time processing of BRDFs, but quantitative accuracy was always the paramount. In view of this, some of the breakthrough results from computer vision and animation would not fit applications in reflectometry and in many industries.

Another difference with virtual reality models is that in computer graphics measurement uncertainties are essentially never present. This is not the case in metrology, reflectometry and in any real-world based industry [13]. Since measurement errors can greatly influence shape and properties of BRDF manifolds, there is a clear need to develop new methods for handling BRDFs with measurement uncertainties.

In this paper, we treat BRDF measurements as samples of points from a high-dimensional and highly non-linear non-convex manifold. We argue that any realistic statistical analysis of BRDF measurements, or any parameter or manifold learning procedure applied to BRDF measurements has to account both for

nonlinear structure of the data as well as for an ill-behaved noise. Standard statistical and machine learning methods can not be safely directly applied to BRDF data. Our study of parameters for generalized Lambertian models in Sects. 5 and 6 clarifies certain pitfalls in analysis of BRDF data, and helps to understand and develop more refined estimates for generalized Lambertian models in Sect. 7.

We introduce and apply in Sect. 6 the notion of Pitman closeness to compare different estimators and parameter learning methods that could be applied to BRDF models. To the best of our knowledge, [14] and [15] were the first works where the Pitman closeness criterion was introduced to either fields of computer graphics as well as metrology. This criterion for comparison of estimators appeals to the actually observable precision of estimators and is assumption-free and loss function-independent, and thus seems to be especially appropriate for applications in metrology, as well as for comparative studies of parameter learning procedures derived for different types of loss functions.

We use the generalized Lambertian model parameter estimators from Sect. 7 to build statistical tests to test a hypothesis whether any particular material is diffuse, even if in a weak sense, or not. Testing validity of BRDF models is important for computer graphics, even though rarely done in a rigorous way, with [16] being a notable exception dealing with several types of tests for parametric models. Surprisingly, hypothesis testing for BRDF data is rarely studied in metrology and reflectometry as well. Recent works [17] and [18] deals with hypothesis testing for diffuse reflection standards. In this paper, we treat a more general case of generalized Lambertian BRDFs, which demands simultaneous testing for a set of stochastically ordered hypotheses, where the number of those hypothesis is the number of measured BRDF layers and so can potentially grow with the number of measurements available. We build a class of tests for this complicated set of hypotheses, and show that our approach leads to tests that are more powerful than the generic multiple testing procedures often recommended by default in the literature.

## 2 Unified Approach

In our research, we advocate the use of a universal approach to data analysis and material appearance modeling, based on those goals that are common both for computer graphics as well as for industrial applications. Accurate simulations are important for virtual applications as well, because that would allow to use computer graphics algorithms for testing and development of complicated real-life technologies, avoiding the difficulties of running expensive physical experiments. Our approach can be seen as complementing the methodology proposed in [19].

An important first step of our approach is to formulate the problems within a rigorous mathematical framework, necessarily including criteria for comparisons between simulations, predictions and actual measurements. An important advantage here is that for a mathematically formalized problem, there is a large toolbox of

methods from applied mathematics, statistics and machine learning that can be applied to this problem. For example, when a criterion for comparisons of results is not specified, the whole decision-theoretic framework is not even applicable.

Methods from different areas often rely on different types of model assumptions. In those situations where it is desirable to compare a range of methods, we advocate the use of assumption-free and loss function-independent criteria. A possible example is the closeness of estimators.

Following [19], we also split our framework into 3 specialized parts:

1. Light reflection models (via BRDFs) and their validation.
2. Light transport simulation.
3. Perception-based studies tailored for specific applications.

Notice that a proper treatment of Stage 3 can greatly reduce computational expense of the global illumination algorithms. Algorithms are substantially accelerated if one develops a perception-based metric evaluating perceptual importance of scene features, because in this case all the numerous unimportant features can be singled out and the amount of operations to process the unimportant features can be greatly reduced.

In the framework developed in [19], only physically based error metrics are used at Stage 1 and Stage 2 (see also [16, 20]). In our framework, we start applying perception-based metrics already at the Stage 1. In [21], we started developing perception-based metrics for the space of BRDFs. We believe that this approach makes a difference when studying sampling of BRDF manifolds and efficient ways to measure the light reflection.

The main goal of Stage 3 in the approach of [19] is to create photorealistic synthetic images which are perceptionally indistinguishable from real scenes. Our goal at Stage 3 is formulated in a more flexible way. Suppose that our study involves materials (or physical scenes) $\mathscr{M}_1, \mathscr{M}_2, \ldots, \mathscr{M}_m$, and we are interested to evaluate the function $\mathfrak{F}$ on $m$ arguments, $\mathfrak{F}(\mathscr{M}_1, \mathscr{M}_2, \ldots, \mathscr{M}_m)$. A typical example here is the customer preference function, that returns the number $i$ of the material $\mathscr{M}_\mathrm{i}$ that appears the most attractive. Then our goal at Stage 3 is in simulating the scenes $\widehat{\mathscr{M}_1}, \ldots, \widehat{\mathscr{M}_m}$ such that

$$\mathfrak{F}(\mathscr{M}_1, \mathscr{M}_2, \ldots, \mathscr{M}_m) = \mathfrak{F}(\widehat{\mathscr{M}_1}, \ldots, \widehat{\mathscr{M}_m}).$$

For the customer preference example, our simulated images are of satisfactory quality if the customer's choice can be guessed correctly by looking at the simulated images alone.

These specialized tasks are much less strict than building fully photorealistic images. Mathematically, we substantially reduce dimensionality of the problem, and relax our demands on precision of light transport studies. It can be expected that the algorithms would run faster due to the smaller and less demanding problem.

## 3 Main Definition

The bidirectional reflectance distribution function (BRDF), $f_r(\omega_i, \omega_r)$ is a four-dimensional function that defines how light is reflected at an opaque surface. The function takes a negative incoming light direction, $\omega_i$, and outgoing direction, $\omega_r$, both defined with respect to the surface normal **n**, and returns the ratio of reflected radiance exiting along $\omega_r$ to the irradiance incident on the surface from direction $\omega_i$. Each direction $\omega$ is itself parameterized by azimuth angle $\phi$ and zenith angle $\theta$, therefore the BRDF as a whole is 4-dimensional. The BRDF has units $sr^{-1}$, with steradians (sr) being a unit of solid angle.

The BRDF was first defined by Nicodemus in [22]. The defining equation is:

$$f_r(\omega_i, \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}. \quad (1)$$

where $L$ is radiance, or power per unit solid-angle-in-the-direction-of-a-ray per unit projected-area-perpendicular-to-the-ray, $E$ is irradiance, or power per unit surface area, and $\theta_i$ is the angle between $\omega_i$ and the surface normal, **n**. The index $i$ indicates incident light, whereas the index $r$ indicates reflected light.

In the basic definition it is assumed that the wavelength $\lambda$ is fixed and is the same for both the incoming and the reflected light. In order to model complicated visual effects such as iridescence, luminescence and structural coloration, or to model materials such as pearls, crystals or minerals, as well as to analyze the related data, it is necessary to have an extended, wavelength-dependent definition of BRDFs. Fortunately, formally this new definition is relatively straightforward and is obtained by rewriting Eq. (1) for $f_r(\lambda_i, \omega_i, \lambda_r, \omega_r)$, where $\lambda_i$ and $\lambda_r$ are the wavelengths of the incoming and the reflected light respectively.

## 4 Important Models of Diffuse Reflection

Lambertian model [4] represents reflection of perfectly diffuse surfaces by a constant BRDF. Because of its simplicity, Lambertian model is extensively used as one of the building blocks for models in computer graphics. Most of the recent studies of light reflection by means of advanced machine learning methods still rely on the Lambertian model. Examples include color studies [23, 24], analytic inference [25], perception studies [26], and face detection [27].

It was believed for a long time that the so-called standard diffuse reflection materials exhibit Lambertian reflectance, but recent studies with actual BRDF measurements convincingly reject this hypothesis [17, 18, 28].

We refer to [14, 15] for a brief discussion of the Oren"-Nayar "directed–diffuse" microfacet model [1], a sophisticated model by [29], and the Lommel"-Seeliger model [30] of the lunar and Martian reflection.

## 5 Statistical Analysis of BRDF Models

In this section, we treat parameter estimation for BRDF models of standard diffuse reference materials. These materials are supposed to have ideal diffuse reflection with constant BRDFs. Graphically, for each incoming angle $\theta_i, \varphi_i$, the resulting BRDF $f_r(\omega_i, \omega_r)$ is a (subset of) two-dimensional upper hemisphere. The radius $\rho$ of this hemisphere is the parameter that we aim to estimate in this paper.

As we mentioned before, the Lambertian model has been shown to be inaccurate even for those materials that were designed to be as close to perfectly diffuse as possible. Therefore, parameter estimates determined for the Lambertian model can hardly be used in practice. However, there are two methodological reasons that make these estimators worth a separate study.

First, BRDF measurements represent a sample of points from a high-dimensional and highly non-linear non-convex manifold. Moreover, these measurements are collected via a nontrivial process, possibly involving random or systematic measurement errors of digital or geometric nature. These two observations suggest that any realistic statistical analysis of BRDF measurements has to account both for nonlinear structure of the data as well as for a very ill-behaved noise and heavy-tailed noise. Any type of statistical inference is more complicated in these conditions, see, e.g., [31]. Standard statistical methods typically assume nice situations like i.i.d. normal errors, and can not be safely directly applied to BRDF data. The same applies to statistical analysis of image data in general [32]. Our study of parameters for Lambertian models clarifies certain pitfalls in analysis of BRDF data, and helps to understand and develop more refined estimates for more realistic BRDF models that will be studied in subsequent papers.

Second, we would use the Lambertian model parameter estimators to build statistical tests to test a hypothesis whether any particular material is perfect diffuse or not. This will be studied in a separate paper.

Suppose we have measurements of a BRDF available for the *set of incoming angles*

$$\Omega_{inc} = \{\omega_i^{(p)}\}_{p=1}^{P_{inc}} = \{(\theta_i^{(p)}, \varphi_i^{(p)})\}_{p=1}^{P_{inc}}. \tag{2}$$

Here $P_{inc} \geq 1$ is the total number of incoming angles where the measurements were taken. Say that for an incoming angle $\{\omega_i^{(p)}\}$ we have measurements available for angles from the *set of reflection angles*

$$\Omega_{refl} = \bigcup_{p=1}^{P_{inc}} \Omega_{refl}(p), \tag{3}$$

where

$$\Omega_{refl}(p) = \{\omega_r^{(q)}\}_{q=1}^{P_{refl}(p)} = \{(\theta_r^{(q)}, \varphi_r^{(q)})\}_{q=1}^{P_{refl}(p)},$$

where $\{P_{refl}(p)\}_{p=1}^{P_{inc}}$ are (possibly different) numbers of measurements taken for corresponding incoming angles.

Overall, we have the set of random observations

$$\{f(\theta_i^{(p)}, \varphi_i^{(p)}, \theta_r^{(q)}, \varphi_r^{(q)}) \mid (\theta_i^{(p)}, \varphi_i^{(p)}) \in \Omega_{inc}, (\theta_r^{(q)}, \varphi_r^{(q)}) \in \Omega_{refl}(p)\}. \quad (4)$$

Our aim is to infer properties of the BRDF function (1) from the set of observations (4). In general, the connection between the true BRDF and its measurements is described via a stochastic transformation $T$, i.e.

$$f(\omega_i, \omega_r) = T(f_r(\omega_i, \omega_r)), \quad (5)$$

where

$$T : \mathscr{M} \times \mathscr{P} \times \mathscr{F}_4 \to F_4, \quad (6)$$

with $\mathscr{M} = (M, \mathfrak{A}, \mu)$ is an (unknown) measurable space, $\mathscr{P} = (\Pi, \mathfrak{P}, \mathbb{P})$ is an unknown probability space, $\mathscr{F}_4$ is the space of all Helmholtz-invariant energy preserving 4-dimensional BRDFs, and $F_4$ is the set of all functions of 4 arguments on the 3-dimensional unit sphere $S^3$ in $\mathbb{R}^4$.

Equations (5) and (6) mean that there could be errors of both stochastic or non-stochastic origin. In this setting, the problem of inferring the BRDF can be seen as a statistical inverse problem. However, contrary to much literature on this subject, we do not assume linearity of the transformation $T$, we do not assume that $T$ is purely stochastic, and we do not assume an additive model with zero-mean parametric errors, as these assumptions do not seem realistic for BRDF measurements.

Of course, this setup is intractable in full generality, but for special cases such as inference for Lambertian model, we would be able to obtain quite general solutions (see also [21]).

It is also easily observable (see, e.g., [28]) that for all materials their sub-BRDFs, consisting of measurements for different incoming angles, look substantially different (no matter if we believe in the underlying Lambertian model or not). This suggests that different sub-BRDFs of the same material still have different parameter values, and this in turn calls for applying statistical procedures separately for different sub-BRDFs and for combining the results via model selection, multitesting and related techniques.

# 6 Means, Medians and Robust Estimators

## 6.1 Basic Properties of Distributions in BRDF Data

In our choice of estimators for parameters in BRDF models, we have to take into account specific properties of BRDF data. It is important to notice that, due to the complicated structure of measurement devices, outliers are possible in the data. Additionally, due to technical difficulties in measuring peak values of BRDFs (see [33, 34]), we have to count on the fact that certain (even though small) parts of the data contain observations with big errors. This also leads us to conclusion that, even for simplest additive error models, we cannot blindly assume that random errors are identically distributed throughout the whole manifold. Additionally, missing data are possible and even inevitable for certain angles. Measurement angles are often non-uniformly distributed. In view of the above arguments, a useful estimator for any BRDF model has to exhibit certain robustness against outliers and dependent or mixed errors.

An estimator has to be universal enough in the sense that it has to be applicable to BRDF samples without requiring extra regularity in the data set, such as uniformly distributed design points, pre-specified large number of measurements, or absence of missing values. This observation suggests that simpler estimators are more practical for BRDF data than complicated (even if possibly asymptotically optimal) estimators, as the later class of estimators has to rely on rather strict regularity assumptions about the underlying model.

## 6.2 Pitman Closeness of Estimators

Let $\Omega$ be a probability space and let $\widehat{\theta}_1 : \Omega \to \mathbb{R}$ and $\widehat{\theta}_2 : \Omega \to \mathbb{R}$ be estimators of a parameter $\theta \in \mathbb{R}$. Then the *Pitman relative closeness* of these two estimators at the point $\theta$ is defined as

$$\mathscr{P}(\widehat{\theta}_1, \widehat{\theta}_2; \theta) = \mathbb{P}(|\widehat{\theta}_1 - \theta| < |\widehat{\theta}_2 - \theta|). \tag{7}$$

The estimator $\widehat{\theta}_1$ is *Pitman closer* to $\theta$ than $\widehat{\theta}_2$, if

$$\mathscr{P}(\widehat{\theta}_1, \widehat{\theta}_2; \theta) > 1/2.$$

While this criterion for comparison of estimators is much less known as, say, unbiasedness or asymptotic variance, it appeals to the actually observable precision of estimators, and thus seems of much interest for applications in metrology.

The closeness criterion appeals to the actually observed precision of estimators and is assumption-free and loss function-independent, and thus seems to be especially appropriate for comparative studies of parameter learning procedures derived

for different types of loss functions. As a drawback, the Pitman closeness has some nontrivial properties such as non-transitivity [35], which leads to counterintuitive results in several examples [36]. On the other hand, these nontrivial properties help to clarify some classic statistical paradoxes such as the Stein paradox [37, 38].

We refer to [39] for an extensive discussion of the relative closeness of estimators and other related notions and their properties. Besides unbiasedness, asymptotic variance and relative closeness, there are many other criteria for comparing quality of statistical estimators. At least 7 of them can be found in [40].

We apply the notion of Pitman closeness to compare different estimators that could be used in BRDF models. Based on this and other criteria, we show that, in the context of the BRDF model parameter estimation and parameter learning, procedures based on either median or trimmed mean are safer to use and are often more accurate than procedures based on sample means.

## *6.3 Mean and Median*

We refer to [14, 15] for definitions of the sample mean, the sample median and the trimmed (truncated) mean. Sample mean is known to be an asymptotically efficient estimator, as well as a uniformly minimum-variance unbiased estimator, for the expected value of the random variable. However, it is important to note that these nice properties are guaranteed only for sufficiently "nice" distributions (see [41] or [42]), while sometimes even marginal deviations from these smooth models seriously spoil performance of the sample mean estimator. In view of the above discussion of properties of BRDF data, we conclude that it is not advisable to apply the sample mean directly as an estimator of the Lambertian radius.

In this and in the next subsection, we present some results of an extensive Monte Carlo experiment comparing relative closeness of different types of basic nonparametric estimators. Each of the graphs contains values of relative closeness obtained for samples of all sizes ranging from 1 to 1000 observations. We performed 1,000,000 comparisons for each sample size. We refer to [14] and [15] for more examples and details.

However, if we are dealing with a heavy-tailed distribution, the picture changes. Suppose we are presented with a Cauchy distribution, and our goal is to estimate the mode (the mean does not exist in this case). Then Fig. 1 shows that the relative closeness of the mean tends to 0 when compared with the median.

Mean surprisingly loses its efficiency even in rather smooth toy situations. Suppose that a sample from i.i.d. standard normal distribution is contaminated with 5 % of i.i.d. normals with mean 0 and variance 10. The result is shown on Fig. 2. Mean's closeness compared to median drops to 0.3. Even more surprisingly, if we start with a sample of i.i.d. normals with mean 0 and variance 100 and contaminate this sample with just 5 % of i.i.d. normals with mean 0 and small variance 1, the drop in mean's closeness compared to median is even worse. Figure 3 shows that the relative closeness of mean drops to 0.1.

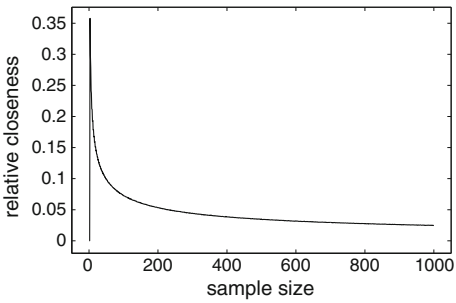


**Fig. 1** Median grossly outperforms mean for heavy-tailed distributions

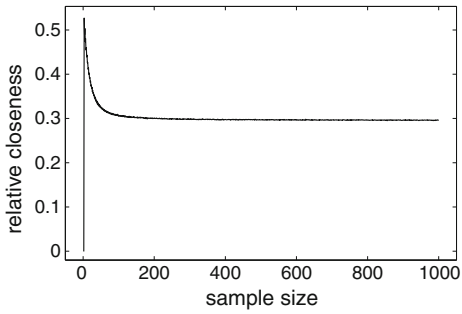


**Fig. 2** Median can outperform mean for mixtures of normal distributions

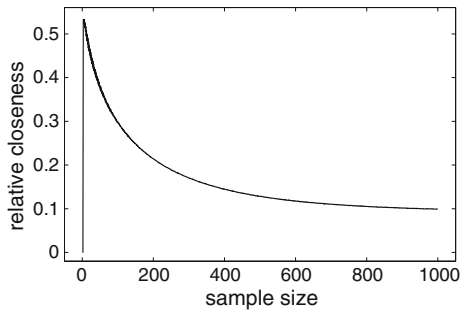


**Fig. 3** Median can outperform mean for mixtures of normal distributions with small errors

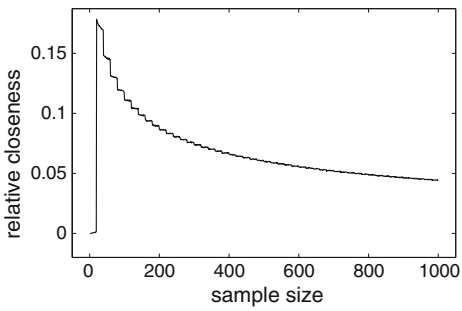


**Fig. 4** Trimmed mean totally dominates mean for cauchy distributions

### 6.4 Truncated Mean and Mean

If our data are generated by sufficiently nice distribution such as, say, a normal distribution, then the sample mean is an efficient estimator. In those cases, it can be rigorously proven that Mean is better than Trimmed Mean in the sense of both Pitman closeness, as well as asymptotic relative efficiency.

The picture can be reversed when our data are allowed to contain outliers or when the data can be, at least partially, generated by a heavy-tailed distribution (which is the case when large values of measurement errors are possible, as is the case for BRDF measurements of specular peaks). We give here a toy example with a Cauchy distribution. Figure 4 illustrates the relative efficiency of mean compared to the trimmed mean with 10 % of the extremes in data being discarded. The unusual shape of the relative closeness curve has no explanation at the moment.

Here the mean is an inconsistent estimator of the median of the distribution, while the truncated mean is not only a consistent estimator of the median, but, with a proper choice of the truncation point, is capable of outperforming the sample median in estimating the median [43]! One needs to drop out about 76 % of the data, though. In fact, even more efficient estimators exist [44], but they require to drop out almost all of the data, and we would not advise to use them for estimation in BRDF models or for any work with moderate sample sizes.

## 7 Parameter Estimation for Generalized Lambertian Models

For each $\omega_i^{(p)}$ from the set of incoming angles $\Omega_{inc}$, let $\rho^{(p)}$ denote the Lambertian radius of the BRDF's layer

$$\{f(\theta_i^{(p)}, \varphi_i^{(p)}, \theta_r^{(q)}, \varphi_r^{(q)}) \,|\, (\theta_r^{(q)}, \varphi_r^{(q)}) \in \Omega_{refl}(p)\}, \tag{8}$$

where $\Omega_{refl}(p)$ is defined by (3). Thus, we are estimating the $P_{inc}$-dimensional parameter vector

$$\{\rho^{(p)}\}_{p=1}^{P_{inc}}. \tag{9}$$

For $1 \le p \le P_{inc}$, let

$$\{f_{(i)}^{(p)}\}_{i=1}^{P_{refl}(p)} \tag{10}$$

be the non-decreasing sequence of order statistics of the subsample (8). Then the *sample median estimator* of the parameter vector (9) is defined as

$$\{\widehat{smed^{(p)}}\}_{p=1}^{P_{inc}}, \tag{11}$$

where

$$\widehat{smed^{(p)}}(f) = \begin{cases} f^{(p)}_{((P_{refl}(p)+1)/2)}, & P_{refl}(p) \text{ is odd;} \\ \frac{1}{2}(f^{(p)}_{(P_{refl}(p)/2)} + f^{(p)}_{(P_{refl}(p)/2+1)}), & P_{refl}(p) \text{ is even.} \end{cases}$$

Let $0 \leq \alpha < 1/2$ be a number, and let $[\cdot]$ denote the integer part of a real number. Then the *sample trimmed mean* estimator of the parameter vector (9) is defined as

$$\{\widehat{tm_{\alpha}^{(p)}}\}_{p=1}^{P_{inc}}, \tag{12}$$

where

$$\begin{aligned} \widehat{tm_{\alpha}^{(p)}}(f) = & \frac{1}{P_{refl}(p)(1-2\alpha)} \\ & \times \{([P_{refl}(p)\alpha] + 1 - P_{refl}(p)\alpha)(f^{(p)}_{([P_{refl}(p)\alpha]+1)} \\ & + f^{(p)}_{(P_{refl}(p)-[P_{refl}(p)\alpha])}) + \sum_{i=[P_{refl}(p)\alpha]+2}^{P_{refl}(p)-[P_{refl}(p)\alpha]-1} f^{(p)}_{(i)}\}. \end{aligned}$$

## 8 Hypothesis Testing for Generalized Diffuse Reflection Models

It is rather straightforward to build a test for checking whether any particular material is perfectly diffuse. Indeed, the corresponding null hypothesis can be tested via a $t$-statistic on the basis of the observed set of BRDF values. However, as we noted above, testing this hypothesis is not very informative as this null hypothesis will be rejected even for those materials that serve as diffuse reflectance standards.

Therefore, it makes more sense to test a hypothesis that a material has diffuse reflection in general, even though not perfectly diffuse with the same level of reflection for each incoming angle. This amounts to building a multiple testing procedure for testing the joint hypothesis $H_0 = \bigcap_{1 \leq p \leq P_{inc}} H_p$, where $H_p$ is the $p$-th null hypothesis stating that the $p$-th layer (8) is laying on a sphere.

As an application of the above estimators, we propose now a class of tests for the compound hypothesis $H_0$. Consider any sequence of test statistics $\{MT_p\}_{1 \leq p \leq P_{inc}}$, where $MT_p$ is used for testing the corresponding hypothesis $H_p$. For a given sample of points from the BRDF, let us apply the test based on $MT_p$ for testing the

hypothesis $H_p$ for all $p$. Denote the corresponding resulting $p$-values by $PV_1, \ldots, PV_{P_{inc}}$, and let $PV_{(1)} \leq \ldots \leq PV_{(P_{inc})}$ be the ordered set of these $p$-values. Then one could suggest to reject $H_0$ if $PV_{(p)} \leq p\alpha/P_{inc}$ for at least one $p$.

Under certain conditions, this multiple testing procedure is asymptotically consistent and more powerful than the procedure based on the Bonferroni principle applied to the same sequence of test statistics $\{MT_p\}_{1 \leq p \leq P_{inc}}$, which is often assumed to be the default way of testing several hypothesis simultaneously. Our procedure capitalizes on the physical fact that, as the incoming light angle grows, deviations from diffuse reflection can only grow as well. Therefore, in mathematical terms, the test statistics $\{MT_p\}_{1 \leq p \leq P_{inc}}$ would be highly positively correlated for any reasonable choice of these statistics. See [45] for details related to rigorous analysis of this type of multiple testing methods. A specific example of test statistics $\{MT_p\}_{1 \leq p \leq P_{inc}}$ was considered in [14] and [15].

Note that it is crucial to take into account the multiplicity of tests. Otherwise, irrespectively of what kind of test statistics we use, if the decisions about each of the basic hypothesis $H_0, \ldots, H_{P_{inc}}$ are made on the basis of the unadjusted marginal $p-$values, then the probability to reject some true null hypothesis will be too large and the test will not be reliable. Unfortunately, this mistake is commonly made in applications of multiple testing.

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## References

1. Oren M, Nayar SK (1995) Generalization of the lambertian model and implications for machine vision. Int J Comput Vis 14(3):227–251
2. Cook RL, Torrance KE (1981) A reflectance model for computer graphics. ACM Siggraph Comput Graph 15(3):307–316
3. He XD, Torrance KE, Sillion FX, Greenberg DP (1991) A comprehensive physical model for light reflection. In: ACM SIGGRAPH computer graphics, vol 25, no 4. 1. em plus 0.5em minus 0.4em. ACM, pp 175–186
4. Lambert J, Anding E, (1982) Lamberts Photometrie: (Photometria, sive De mensura et gradibus luminis, colorum et umbrae) (1760), ser. In: Engelmann W (ed) Ostwalds Klassiker der exakten Wissenschaften, vol 1–2. [Online]. Available http://books.google.de/books?id=Fq4RAAAAYAAJ
5. Phong BT (1975) Illumination for computer generated pictures. Commun ACM 18(6):311–317. [Online]. Available http://doi.acm.org/10.1145/360825.360839
6. Blinn JF (1977) Models of light reflection for computer synthesized pictures. In: Proceedings of the 4th annual conference on computer graphics and interactive techniques, ser. SIGGRAPH '77. ACM, New York, pp 192–198

7. Lafortune EP, Foo S-C, Torrance KE, Greenberg DP (1997) Non-linear approximation of reflectance functions. In: Proceedings of the 24th annual conference on computer graphics and interactive techniques, ACM Press/Addison-Wesley Publishing Co., pp 117–126
8. Matusik W (2003) A data-driven reflectance model. Ph.D. dissertation, Citeseer
9. Matusik W, Pfister H, Brand M, McMillan L (2003) A data-driven reflectance model. In ACM transactions on graphics (TOG), vol. 22, no. 3. 1. em plus 0.5em minus 0.4em. ACM, pp 759–769
10. Brady A, Lawrence J, Peers P, Weimer W (2014) Genbrdf: discovering new analytic brdfs with genetic programming. ACM Trans Graph 33(4):114:1–114:11. [Online]. Available http://doi.acm.org/10.1145/2601097.2601193
11. Brochu E, Freitas ND, Ghosh A (2008) Active preference learning with discrete choice data. In: Advances in neural information processing systems, pp 409–416
12. Ashikhmin M, Shirley P (2000) An anisotropic phong brdf model. J Graph Tools 5(2):25–32
13. Höpe A, Koo A, Forthmann C, Verdu F, Manoocheri F, Leloup F, Obein G, Wübbeler G, Ged G, Campos J et al (2014) xd-reflect-" multidimensional reflectometry for industry" a research project of the european metrology research program (emrp). In: 12th International conference on new developments and applications in optical radiometry (NEWRAD 2014)
14. Langovoy M, Wübbeler G, Elster C (2014) Statistical analysis of brdf data for computer graphics and metrology. In: Lecture notes in engineering and computer science: proceedings of the world congress on engineering and computer science 2014, WCECS 2014. 22–24 Oct 2014, San Francisco, USA, pp 785–790
15. Langovoy M (2015) Machine learning and statistical analysis for brdf data from computer graphics and multidimensional reflectometry. IAENG Int J Comput Sci 42(1):23–30
16. Subr K, Arvo J (2007) Statistical hypothesis testing for assessing monte carlo estimators: applications to image synthesis. In: 15th Pacific conference on computer graphics and applications, 2007. PG '07, pp. 106–115
17. Ferrero A, Rabal AM, Campos J, Pons A, Hernanz ML (2012) Spectral and geometrical variation of the bidirectional reflectance distribution function of diffuse reflectance standards. Appl Opt 51(36):8535–8540
18. Pinto CT, Ponzoni FJ, de Castro RM, Griffith DJ (2012) Spectral uniformity evaluation of reference surfaces for airborne and orbital sensors absolute calibration. Rev Bras Geofis 30
19. Greenberg DP, Torrance KE, Shirley P, Arvo J, Lafortune E, Ferwerda JA, Walter B, Trumbore B, Pattanaik S, Foo S-C (1997) A framework for realistic image synthesis. In: Proceedings of the 24th annual conference on computer graphics and interactive techniques, ACM Press/Addison-Wesley Publishing Co., pp 477–494
20. Arvo J, Torrance K, Smits B (1994) A framework for the analysis of error in global illumination algorithms. In: Proceedings of the 21st annual conference on computer graphics and interactive techniques, ACM, pp 75–84
21. Langovoy M, Wübbeler G, Elster C (2014) Novel metric for analysis, interpretation and visualization of BRDF data. Submitted
22. Nicodemus FE (1965) Directional reflectance and emissivity of an opaque surface. Appl Opt 4:767–775
23. Tieu K, Miller EG (2002) Unsupervised color constancy. In: Advances in neural information processing systems, pp 1303–1310
24. Rosenberg C, Ladsariya A, Minka T (2003) Bayesian color constancy with non-gaussian models. In: Advances in neural information processing systems
25. Wang S, Schwing A, Urtasun R (2014) Efficient inference of continuous markov random fields with polynomial potentials. In: Advances in neural information processing systems, pp 936–944
26. Freeman WT, Viola PA (1998) Bayesian model of surface perception. Adv Neural Inf Process Syst, vol 10
27. Li Y-M, Chen J, Qing L-Y, Yin B-C, Gao W (2004) Face detection under variable lighting based on resample by face relighting. In: Proceedings of 2004 international conference on machine learning and cybernetics, vol 6. IEEE, pp 3775–3780

28. Höpe A, Hauer K-O (2010) Three-dimensional appearance characterization of diffuse standard reflection materials. Metrologia 47(3):295. [Online]. Available http://stacks.iop.org/0026-1394/47/i=3/a=021
29. Simonot L (2009) Photometric model of diffuse surfaces described as a distribution of interfaced lambertian facets. Appl Opt 48(30):5793–5801
30. Fairbairn MB (2005) Planetary photometry: the Lommel-Seeliger law. J Roy Astron Soc Can 99:92
31. Meintanis S (1998) Moment-type estimation for positive stable laws with applications. IAENG Int J Appl Math 38:26–29
32. El Emary IM, Ramakrishnan S (2010) A critical review of statistical modeling of digital images. IAENG Int J Comput Sci 37(1):99–109
33. Ouarets S, Leroux T, Rougie B, Razet A, Obein G (2013) A high resolution set up devoted to the measurement of the bidirectional reflectance distribution function around the specular peak, at lne-cnam. In: 16th International Congress of Metrology. EDP Sciences, p 14008
34. Obein G, Ouarets S, Ged G (2014) Evaluation of the shape of the specular peak for high glossy surfaces. In: IS&T/SPIE electronic imaging, International Society for Optics and Photonics, p 901805
35. Rukhin AL (1996) On the pitman closeness criterion from the decision-theoretic point of view. Stat Decisions-Int J Stochast Methods Models 14(3):253–274
36. Robert CP, Hwang JG, Strawderman WE (1993) Is pitman closeness a reasonable criterion? J Am Stat Assoc 88(421):57–63
37. Sen PK, Kubokawa T, Saleh AME et al (1989) The stein paradox in the sense of the pitman measure of closeness. Ann Stat 17(3):1375–1386
38. Kourouklis S (1996) Improved estimation under pitman's measure of closeness. Ann Inst Stat Math 48(3):509–518
39. Keating JP, Mason RL, Sen PK (1993) Pitman's measure of closeness: a comparison of statistical estimators, vol 37. SIAM, Philadelphia
40. Savage CL (1972) The foundations of statistics. Dover Publications [Online]. Available http://books.google.de/books?id=UW5dAAAAIAAJ
41. Ibragimov IA, Khasminskii RZ (1981) Statistical estimation–asymptotic theory. Springer, New York
42. Borokov A (1999) Mathematical statistics. Taylor & Francis, [Online]. Available http://books.google.de/books?id=2CoUP7qRUxwC
43. Rothenberg TJ, Fisher FM, Tilanus CB (1964) A note on estimation from a cauchy sample. J Am Stat Assoc 59(306):460–463
44. Bloch D (1966) A note on the estimation of the location parameter of the cauchy distribution. J Am Stat Assoc 61(315):852–855
45. Sarkar SK (1998) Some probability inequalities for ordered mtp2 random variables: a proof of the simes conjecture. Ann Stat 26(2):494–504

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