David Makinson and the Extension of Classical Logic

Sven Ove Hansson and Peter Gärdenfors

Abstract There are two major ways to deal with the limitations of classical logic. It can be replaced by systems representing alternative accounts of the laws of thought (non-classical logic), or it can be supplemented with non-inferential mechanisms. David Makinson has a leading role as proponent of the latter approach in the form of the *inferential-preferential* method in which classical logic is combined with representations of preference or choice. This has turned out to be a highly efficient and versatile method. Its applications in non-monotonic logic and belief revision are used as examples.

Keywords David Makinson · Classical logic · Inferential-preferential method · Nonmonotonic logic · Belief revision · Input/output logic

If we wish to understand the significance of a major researcher's contributions to science we need to see them in a larger context. Logic took big steps forward in the twentieth century, and David Makinson has been a major contributor to its progress. In order to appreciate his role in the development of the discipline, let us begin with a bird's-eye view of how logic has evolved.

1 A Tradition from Antiquity

The logic that we inherited from antiquity had its focus on single steps of reasoning. Its major achievement was the Aristotelian theory of syllogisms, a fairly precise account of steps of thought that take us with certainty from two premises

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to a conclusion. The argumentation steps represented in the standard syllogisms are those in which the notions "some", "all", and "no" have a pivotal role. These types of argument steps are relatively rare in actual human reasoning, but they are particularly well suited for precise modelling. It was an admirable intellectual achievement to identify them as a starting-point and a model for a rigorous account of reasoning. But in the two millennia that followed, progress towards more generality was remarkably slow. When Kant in 1787 described logic as closed and perfected ("geschlossen und vollendet", AA 3:7), it might have been more adequate to describe it as stuck in a position from which it was difficult to move forward.

As a theory of human rational thought and ratiocination, Aristotle's theory of syllogisms has three major limitations. First, it only treats small steps of argumentation one at a time, rather than complete inferences or lines of argumentation. Therefore it cannot answer questions such as whether a certain conclusion follows from a given set of premises (Hodges 2009). Secondly, it only covers arguments expressible in terms of a small selection of terms, namely "all", "some", and "no". Although Aristotle provided (in *Topics*) logical principles for other terms such as betterness, these attempts did not reach the high level of precision achieved in the syllogisms, and they were not integrated with (or easily integrable into) the theory of syllogisms. Thirdly, the theory of syllogisms only covers (deductive) inferences that can be made with full certainty, and it does not have much to say about inductive and other fallible inferences.

The second of these limitations was attacked in several ways through the centuries, most importantly through the parallel development of propositional logic. The logic of propositions reached a high level of sophistication and formal rigour in George Boole's *The Laws of Thought* (1854)—but it was still unconnected to syllogistic logic and also still subject to the other two limitations.

The first limitation, that to single steps of argumentation, was overcome with the quantifiers that Frege introduced in his *Begriffsschrift* (1879). With these new tools it was possible to develop more comprehensive systems that allowed a general investigation of what can and cannot be inferred from a set of premises. These developments opened the way for mathematically more advanced investigations of logic. Consequently, in the twentieth century logic has attracted some of the most brilliant mathematical minds, but unfortunately also repelled some able but less mathematically minded philosophers.

Although Frege's system was revolutionary in terms of its expressive power, it was still traditional in the sense of retaining the major laws of thought that had been almost universally accepted since antiquity: bivalence, the laws of identity, non-contradiction and the excluded middle, and the elimination of double negations. As could be expected, however, the introduction of more comprehensive logical systems opened up the possibility of constructing precisely defined alternative logics representing divergent specifications of the laws of thought. This gave rise to an abundance of non-classical logics including intuitionistic, many-valued, paraconsistent, and quantum logics.[1]

The new logic developed by Frege and his immediate successors was still subject to the other two limitations, namely the restriction to a small selection of pivotal terms (the logical constants) and the exclusive focus on inferences that can be made with full certainty. In the last century these limitations have to a considerable extent been overcome through a large number of extensions of the logical apparatus that cover new subject areas: inductive logic, semantically based modal logic, the logic of conditionals, epistemic and doxastic logic, deontic logic, preference logic, action logic, belief revision, ... It is in this development that David Makinson has been one of the most influential contributors.

2 The Inferential-Preferential Method

Many logicians have combined the two developments just mentioned, namely the extension of logic to new subject-matter and the construction of non-classical alternatives to classical logic. It has often been believed that in order to extend logic to subject-matter such as inductive and non-monotonic reasoning, classical logic has to be replaced by something else. But there is also another approach to the extension of logic to new areas—an approach that David Makinson has (gently) advocated more consistently and efficiently than anyone else: Instead of giving up classical logic, we can combine it with non-inferential mechanisms, in particular mechanisms of preference and choice. The combination of inferential and preferential mechanisms has turned out to be surprisingly powerful and versatile in a wide range of applications.

This method was used by Lewis (1973) in his pioneering work on the logic of counterfactual conditionals. In collaboration with others Makinson introduced it into belief revision theory. In a joint paper he and Carlos Alchourrón used it to define safe contraction (Alchourrón and Makinson 1985). The central problem in belief contraction is to find out which sentences we should remove from a logically closed set K in order to obtain another logically set $K \div p$ such that $p \notin K \div p \subseteq K$. In safe contraction a sentence q is discarded in this contraction if and only if it satisfies both an inferential and a preferential criterion. The inferential criterion is that q is an element of some subset X of K that implies p but has no proper subset that implies p. The preferential criterion is that q has a bottom position among the elements of X according to a hierarchy $<$ that regulates the contraction (i.e. there is no $r \in X$ with $r < q$). In partial meet contraction, the recipe for belief contraction that Makinson developed together with Alchourrón and Gärdenfors (Alchourrón et al. 1985), the

[1] On the connections between intuitionistic and classical logic, see Gödel (1986, esp. pp. 286–295 and 300–303) and Humberstone and Makinson (2011). On the connections between relevance logic and classical logic, see Friedman and Meyer (1992) and Makinson's chapter on relevance logic in this book, "Relevance Logic as a Conservative Extension of Classical Logic".

selection mechanism is instead applied to maximal subsets of K not implying p, and the contraction outcome is obtained as the meet of the selected such subsets.

Makinson's book *Bridges from Classical to Nonmonotonic Logic* (2005) is highly recommended not only as an unsurpassed presentation of non-monotonic logic but also as an introduction to the inferential-preferential methodology. In this book he constructs non-monotonic logics using what he calls three "natural bridges between classical consequence and the principal kinds of nonmonotonic logic to be found in the literature" (p. ix). One of these bridges is the use of additional background assumptions in the form of a set of propositions that are used along with the current premises in the derivation of consequences. A problem with this method is that even if the set of premises is consistent, it may yield an inconsistent set of outcomes. This can be solved by using only a selection of the background assumptions (presumably selected because they are more highly preferred than the others). In the following section we will present this methodology in greater detail, as applied to non-monotonic reasoning.

The second method is to restrict the set of Boolean valuations on the language that are used to semantically determine the relation of consequence. In other words, some of the possible assignments of truth values to atomic sentences are not used in evaluating logical consequence. This, of course, again requires some mechanism of choice. This is the basic framework that was used for instance in the well-known system presented by Kraus et al. (1990).

The third method is similar to the first in that it also adds background assumptions to be used together with the present premises to derive consequences. However, in this case the background assumptions are rules rather than propositions. By a rule is meant a pair $\langle x, y\rangle$ of propositions. A set A of sentences is closed under the rule $\langle x, y\rangle$ if and only if: if $x \in A$ then $y \in A$. The set of non-monotonic consequences of A can then be characterized by being closed both under classical consequence and under a set of rules. Just as in the first method, in order to avoid the derivation of inconsistences from a consistent set of premises a mechanism of choice is needed, in this case one that makes a choice among the rules. This is the idea behind the default rule systems developed by Reiter (1980), Brewka (1991), and others.

The reader is referred to Makinson's book for a systematic discussion of the intuitive and formal connections between insertions of preferential principles at different places in the inferential system. Other such connections have been obtained in belief revision theory, see for instance Gärdenfors and Makinson (1988), Fuhrmann (1997a, b), Grove (1988), Rott (2001), and Hansson (2008, 2013). The inferential-preferential method has also been used to show how consistent normative recommendations can be obtained from a system of norms containing potentially conflicting elements (Hansson and Makinson 1997).

3 Non-monotonic Reasoning as Classical Logic with Defeasible Premises

The first of the three above-mentioned uses of the inferential-preferential method is arguably the simplest one from a conceptual point of view. Its simplicity makes it particularly well suited for elucidating the philosophical background of that method. Let us therefore have a closer look at this particular bridge from classical to non-monotonic logic.

In classical logic, the focus is on a syntactic relation $P \vdash q$ between a set P of premises and a conclusion q. The validity of an argument that is representable by the relation $\vdash$ is assumed to depend only on the logical structure of q and the sentences in P, and consequently to be independent of their meaning, their truth, and the context. However, actual human reasoning is not based exclusively on premises that are taken as given. It is also influenced by the stronger or weaker expectations that we have about facts of the world. Such expectations can be expressed by sentences that are regarded as plausible enough to be used as a basis for inference so long as they do not give rise to inconsistency. Gärdenfors and Makinson (1994) took this extension of classical logic as a basis for analysing non-monotonic reasoning. The key idea for using expectations can be put informally as follows:

> P non-monotonically entails q (denoted $P \mathrel{|\!\sim} q$) if and only if q follows logically from P together with as many as possible of the set E of our expectations as are compatible with P.

Note that the expectations are represented in the same way as the premises, namely as a set of beliefs. Contrary to many other accounts of non-monotonic reasoning, this approach does not need a special formal representation for default beliefs.

The expectations in E are not premises that have to be accepted. Instead they are defeasible in the sense that if the premises P are in conflict with some of these expectations, then we do not use them when determining whether q follows from P. In order to make this inference recipe precise, the meaning of "as many as possible" must be explicated.

Expectations are suppressed in classical logic, but in practical reasoning they are not. Everyday arguments are full of hidden premises that need to be made explicit in order to make the argument logically valid. In each separate case, it may be possible to add the hidden assumptions to make the derivation performable with classical methods. Expectations are normally shared among speakers, and unless they are contradicted, they serve as a common background for arguments.

To give a concrete example of how this mechanism leads to non-monotonic inferences, suppose that among the expectations are "Swedes are protestants", "Swedes with Italian parents are catholics" and "Protestants are not catholics". Now if "Anton is a Swede" is given as a premise, then one can conclude "Anton is a protestant" using the first expectation. If the premise "Anton has Italian parents" is added, then one can also conclude "Anton is a catholic". This conclusion is inconsistent with the first, given the expectations, so in order to maintain consistency the expectation "Swedes are protestants" is given up and one concludes only "Anton is a catholic".

The reasoning pattern is non-monotonic since a previous conclusion is lost when more premises are added.

In order to make precise "as many as possible of the expectations", Gärdenfors and Makinson (1994) studied two selection mechanisms. The first is based on selection functions of precisely the same type as those that are used in models of belief revision. The second is based on an ordering of "defeasibility" of the expectations that is similar to the entrenchment orderings for belief revision used by Gärdenfors and Makinson (1988). They also formulated a set of postulates for non-monotonic reasoning that is essentially equivalent to the postulates used by Kraus et al. (1990). Gärdenfors and Makinson (1994) could then prove completeness results: (1) The selection mechanism based on selection functions is characterized by a basic set of postulates only involving one expectation. (2) The selection mechanism based on defeasibility orderings is characterized by the full set of postulates also involving several expectations. They also showed that the selection mechanisms based on orderings form a subclass of the preferential models studied by Shoham (1988), Kraus et al. (1990).

On the basis of the similarity between those preferential models based on defeasibility/entrenchment ordering that have been developed for belief revision and those that have been developed for non-monotonic reasoning, the two authors established a method for translating postulates for belief revision into postulates for non-monotonic reasoning, and vice versa (Makinson and Gärdenfors 1990; Gärdenfors and Makinson 1994). The key idea for the translation from belief revision to non-monotonic logic is that a statement of the form $q \in K * p$ is seen as a non-monotonic inference from p to q given the set K of sentences that is then interpreted as the set of background expectations ($K = E$). In this way the statement $q \in K * p$ in belief revision is translated into the statement $p \mid\sim_K q$ in non-monotonic logic (or into $p \mid\sim_K q$, if one wants to emphasize the role of the background beliefs). Conversely, a statement of the form $p \mid\sim q$ for non-monotonic logic is translated into a statement of the form $q \in K * p$ for belief revision, where K is introduced as a fixed belief set. In our view, this example shows that the "bridges" that Makinson has been instrumental in building with the inferential-preferential method can be used not only to connect classical and non-classical logic with each other but also to connect different areas of logical analysis with each other.

4 Logic as an Auxiliary Tool

In the examples we have described above, logical inference is the driving force, but it is supplemented with some non-logical mechanism such as a choice process in order to obtain the desired properties. However, it is not self-evident that the logical component should have the dominant role in such a combination. The opposite relation is also possible, as exemplified by the input/output systems introduced by Makinson and van der Torre (2000).

An input/output system is a mechanism that takes propositions as inputs and delivers propositions as outputs. In most interpretations of this versatile framework the inputs are conditions, whereas the outputs may for instance be norms, preferences, goals, intentions or ideals. The central component of an input/output system is some relation that assigns output propositions to input propositions. In principle, logical inference need not be involved. However, logic can be helpful in at least three ways. First, it can prepare the inputs, for instance by seeing to it that an input consisting of two inputs x and y are treated in the same way as one consisting of the single input $x\&y$. Secondly, it can unpack the outputs or specify their interpretation, for instance by closing them under logical consequence. Thirdly, it can co-ordinate inputs and outputs, for instance by making sure that outputs are reusable as inputs.

Makinson and van der Torre state quite explicitly that in their systems, logic has only an auxiliary role. In a model like this, logic is not the central transformation mechanism. Instead it should be "seen in another role, as 'secretarial assistant' to some other, perhaps non-logical, transformation engine" (p. 384). Furthermore:

> The process as a whole is one of 'logically assisted transformation', and is an inference only when the central transformation is so. This is the general perspective underlying the present paper. It is one of 'logic at work' rather than 'logic in isolation'; we are not studying some kind of non-classical logic, but a way of using the classical one (2000, p. 384).

Logicians tend to think of logic as the leading principle of (rational) human thought. Practical experience as well as results from cognitive science give us reasons to assign a much less dominant role to logic (see e.g. Gärdenfors 1994; Benthem 2008). The concept of "logically assisted" mechanisms is, in our view, well worth investigating as a means to obtain more realistic models of the role of logic in mental processes.

5 Conclusion

As we mentioned above, two major types of solutions have been developed to overcome the limitations of classical logic: replacing it with something else and combining it with non-inferential mechanisms. David Makinson has been a most influential proponent of the second method. Developing the inferential-preferential model, he has shown how it can be applied in a powerful way to expand classical reasoning to a wide range of applications.

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