Chapter 2
Mathematics and Mathematics Education: Beginning a Dialogue in an Atmosphere of Increasing Estrangement

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Abstract In 2009, Norma Presmeg wrote a piece for a special issue of the ZDM on interdisciplinarity. Presmeg's paper presented her view of the general spirit of and possibilities for mathematics education research. This prompted a dialogue on the state of mathematics education by Ted Eisenberg and Michael Fried, published in the same issue of ZDM. This paper gives an account of that dialogue and the symposium in honor of Ted that arose out of it; in doing so, it also further elaborates on the themes that motivated this book.

Keywords Human sciences vs exact sciences · Mathematical content · Mathematicians · Mathematics education researchers · Mathematics education · Values

My Dialogue with Ted

When Dani Berend broached the idea of a conference in honor of Ted Eisenberg, it was immediately clear to me what the subject of the conference should be. It should concern the relationship between mathematics and mathematics education as disciplines. The thin mathematical backgrounds of many researchers in mathematics education, and worse, their apparent lack of interest in mathematics, had become one of Ted and my constant conversation topics. Ted often lamented to me how out of place he felt in a field more and more dominated by sociology, psychology, politics, anthropology, and philosophy, and less and less by mathematics. His feelings were understandable. For almost his entire academic career, Ted sat in a mathematics department, and, besides his own mathematics education research, he taught regular courses in the mathematics department, working hard to introduce students to calculus and linear algebra. Teaching mathematics, and, therefore, knowing mathematics, has always been for him at the center of mathematics education, and he has always maintained it should be. Not only is a mathematics department

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M.N. Fried, T. Dreyfus (eds.), *Mathematics & Mathematics Education: Searching for Common Ground*, Advances in Mathematics Education,
DOI 10.1007/978-94-007-7473-5_2,
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the right place for the study of mathematics education, in his view, but also a solid mathematics background was requisite for fruitful mathematics education *research*.

For my part, when we spoke about these things, I often took the position of the devil's advocate and defended the usefulness of non-mathematical mathematics education research. But I was not always consistent in that role, not wholly the devil's advocate. On the one hand, I could not help often commiserating with Ted about the limited mathematical understanding of many researchers who consider mathematics their specialty and about research that tries to solve the problems of the world rather than those of mathematics teaching and learning. On the other hand, I could deny some genuine leanings towards the other side, an inevitable outcome, perhaps, of my training as a historian of mathematics, where one learns to see mathematical thought as contingent and embedded in culture.

In any case, these conversations came to head in 2008 when Ted was asked to review a paper by Norma Presmeg entitled "Mathematics Education Research Embracing Arts and Sciences" (Presmeg 2009). In this position paper, Norma argued that since the purview of mathematics education research includes more than mathematical content *per se*—that it concerns how students think about mathematics, how mathematics becomes part of students' inner and outer lives, how it is integrated into students' sociocultural world, for example—it is necessarily a multidisciplinary affair. And the introduction of multidisciplinary considerations brings with it also the introduction of different kinds of methodologies. Research in the field, for this reason, takes on a character often closer to the human sciences than the exact sciences, even though the focus of the field is still mathematics.

Indeed, Norma never discounted the importance of mathematical content in her piece: she was explicit about that. "The subject matter," she said, "of research mathematicians is the content of mathematics, and without this content there would be no mathematics education" (p. 132). On the other hand, it was central to her argument that mathematics education research is not mathematics, not even applied mathematics! She recalled Millroy's 1992 study of carpenters in Cape Town which pointed to distinctions between the mathematics implicit in students' out-of-school culture and the explicit mathematics in their in-school culture. But bridging these two cultures requires the kind of ethnographic approach typical of anthropological research. Students' mathematical understanding in these cultural contexts requires understanding the language conditioned by such cultures, the systems of signs one uses to construct and communicate ideas, including mathematical ideas, and this brings one to the semiotic research which has become prevalent in mathematics education research. It becomes apparent by such examples how one can be led in a very natural way into extra-mathematical disciplines.

For Norma, these borrowings from other disciplines were not only a necessary widening of the field, but also a refreshing and welcome one. For Ted, ethnographic research, semiotics, and so on were dragging the field too far afield. Yet, Ted also understood that Norma's paper was an accurate picture of the state of the art, and while for her that meant finally "coming home" (p. 134), as she put it, for Ted it meant alienation. The situation was particularly painful for Ted because of the enormous respect he has for Norma, unquestionably and rightfully a leading figure in mathematics education. It was in that uneasy spirit Ted wrote his review of Norma's paper. The gist of his review was that while the paper went against his most basic beliefs about the nature of mathematics education and direction it should be going, he saw that according to criteria nearly universally accepted in the field he could hardly criticize the paper, let alone reject it.

Norma's paper was intended for a special issue on "interdisciplinarity in mathematics education" for ZDM—The International Journal on Mathematics Education. The editor of the special issue, Bharath Sriraman, saw in Ted's review an opportunity for some interesting counterpoint and suggested that Ted write up his criticism as a reaction to Norma's paper. He suggested, moreover, that Ted and I do it jointly. Perhaps because I had played the devil's advocate a bit too often, Ted responded that he thought I was more in Norma's camp than his. So Bharath, who does not give up easily, suggested that Ted and I write a dialogue on the issues Ted raised in his critique. When Ted finally told me about Bharath's proposal, my first inclination was to say that all this was so much at the center of Ted's concerns he should really take up the project himself. But, since I always have trouble uttering a simple "no," I said I would think about it.

Thus it stood until March 2008, when I went to Rome to attend the centenary of the ICMI, the International Commission on Mathematics Instruction, the oldest and most prominent international organization dedicated to mathematics education. As is well known, the ICMI was established at the Fourth International Congress of Mathematicians held in Rome in 1908. The ICMI still belongs to the IMU, the International Mathematics Union, and its connection with the greater mathematical world has deep roots. Felix Klein, for example, was the first president of the ICMI, and he was followed by other eminent mathematicians holding the presidency or other high posts in the commission, figures such as Jacques Hadamard, Marshall Stone, Sergei Sobolev, Saunders Mac Lane, Hans Freudenthal, and Hyman Bass. Yet, at this celebration of the first hundred years of the ICMI, years in which the commission survived tensions from nationalistic fervor and the violence of two world wars, and years of triumph in which it saw great changes in mathematics and mathematics education, the founding of international mathematics education journals and large scale international mathematics education conferences—it was at this happy occasion that some mathematics educators saw fit to ask for divorce.

The one that particularly stands out in my mind is Mamokgethi Setati. Setati did not want merely to broaden the scope of the field; she sought to reestablish its entire agenda—and in a way that left little room for mathematical content. For her, mathematics education should focus all of its energies on confronting the problems of the developing world, "the eradication of poverty, empowerment of women and gender equality" (Menghini et al. 2008, p. 182), no less. It was in this context that she also called for a reexamination of ICMI's relationship with the IMU (p. 184), a euphemistic way of saying, "End the marriage." To me, it was immediately clear that her position was untenable. As I wrote in my review of the proceedings of the ICMI centenary (Menghini et al. 2008):

> Following the implications of Setati's position, it seems difficult to avoid two equally dubious conclusions: (1) mathematics is not at the heart of mathematics education or at least

> must be subordinated to more general social issues, or, at the other extreme, (2) mathematics has a privileged position in dealing with global social problems such as poverty and gender inequality. (Fried 2009, p. 524)

As human beings, of course, we must be concerned with social justice. But this is not the question: the question is whether we must be concerned with social justice *as* mathematics educators and, more, whether social justice should trump all else relevant to mathematics education. I do not think Setati was a lone voice, though I hardly believe her view as to where mathematics education should be going reflected a consensus at the Rome meeting. That said, it was evident, whether one liked it or not, the field *had* broadened far beyond teaching and learning mathematical content: I could not help feeling we had reached a watershed and a real possibility that mathematical content might be swept away altogether. It was then and there I decided I had thought about it enough: I would tell Ted I am ready to work on the dialogue.

Although Norma's paper was the pretense, Ted and I wanted our piece as much as possible to be like the conversations he and I had so often. I think it was Ted's idea that the paper should take the form of an exchange of letters. He wrote about his vision and his discontent, and I responded. Although it gave to me, in effect, the last word, still it allowed us both to write a more or less connected account of our take on the state of mathematics education research. The format also allowed a certain informality appropriate to airing views rather than presenting findings. But that should not detract from the seriousness of the exchange. Where mathematics education ought to be going and what mathematics education research ought to be are not empirical questions that *findings* could ever settle. These are matters that require continual sober discussion. In fact, the place of empirical research in mathematics education was one issue we raised in our letters. There were many issues we put on the table.

Mathematical content in mathematics education and mathematics education research was only one of these issues; however, then and in our own off-the-page conversations it was a focal one. In Ted's way of thinking about it, it could be discussed in terms of university geography, that is, where on campus should a unit on mathematics education be located? Should it be where the exact sciences and engineering faculties are or where the humanities and social sciences faculties are? Even taken so literally, where one sees oneself is unavoidably a question of identity or self-definition, and ours *was* a discussion about self-definition. Certainly Norma's paper was about how mathematics education should be defined—an art? a science? both together? Setati's view too was surely a statement of self-definition, albeit one I could not swallow.

Mathematics and Mathematics Education: Difference and Confluence

Dani Berend's idea to have a conference in honor of Ted ultimately became, therefore, a symposium concerning the very identity of mathematics education as a field, specifically, its identity as it relates to mathematics as a field. But for this reason, it could not a question about mathematics education alone, for since it is asked *with respect* to the discipline of mathematics, whatever identity mathematics education crystalizes for itself will also leave a mark on the identity of mathematics as a discipline: the two really are wed. Indeed, having originally entitled this symposium "Is there still room for mathematics in mathematics education?" it quickly became apparent that this could not be asked without its complement, "Is there still room for mathematics education in mathematics?" Thus the symposium was renamed, "searching for common ground." As a title, it expressed, first of all, Ted's profound belief that mathematics education and mathematics do have a common ground that must be taken seriously and never ignored. But it also suggested we have to some extent lost hold of that common ground and must search together to regain it.

Naturally, with this in the background, we should want to concentrate on commonalities; yet, this cannot be done without, at the same time, being cognizant of how the mathematics education community is set off from the mathematics community. Failing to discern the separateness these communities only invites claims that mathematics education is populated by poor mathematicians and mathematics by poor educational thinkers. (And here, I should emphasize that by the mathematics education community I mean chiefly the academic community of mathematics educators rather than teachers in the field.)

Besides the more obvious differences between the two communities—that mathematics education researchers do not prove theorems as a matter of course and that mathematicians do not consider theories of learning and thinking, for example—I should mention two ways in which these communities at least appear move in different directions, particularly, ways in which mathematics education as an academic field of research turns towards the social sciences for its sense of identity.

The first has to do with how mathematics educators approach their questions, their methodology. The methodological approaches mathematics education must apply to understand aspects of teaching and learning and often questions of curriculum do truly have more in common with social sciences than with the exact sciences or engineering, the theory of "didactic engineering," notwithstanding. This is evident in part by the sheer variety of methodological approaches in the field, that is, by the lack of a single paradigm for doing research. In fact, it might be argued that this methodological eclecticism is one reason why the question of identity is so much more prevalent in mathematics education and the social sciences than it is in the exact sciences (perhaps with the exception of biology, but for different reasons). But more importantly, like the social sciences, the methodological concerns of mathematics education research share a history, rooted in Comte and Weber, of aiming to be "value-free," an "objective" science like physics, and, yet, ever falling short of that ambition. We want a *science* of learning and teaching, but we cannot escape, in its baldest form, our own commitments as to what we think students *ought* to learn and how they *ought* to learn it and how we *ought* to teach them; our most basic questions always lead us to questions of values.

The second has to do with our aims as educators. For the separateness of mathematics education research from the discipline of mathematics can also be felt even where both are focused on education. This is so because, being typically associated with a university mathematics department, mathematicians are placed in the position of training new mathematicians or scientists. Mathematics education, by contrast, concerns the whole gamut of learning and teaching mathematics, including university level mathematics, but typically concentrating on learning and teaching school mathematics. The one must ask what constitutes a mathematically *trained* person, while the other, a mathematically *educated* person. These are not, to be sure, mutually exclusive categories; however, what it means to be mathematically trained and what it means to be mathematically educated are also not identical, and the difference is not just one of degree. To be fully mathematically trained is to be a mathematician; but one can be highly mathematically educated without being a mathematician; and, conversely, there are competent mathematicians who are surprisingly uninformed about the history of mathematics and matters connected to its philosophical foundations. A trained mathematician must produce mathematics; one who is mathematically educated must feel at home with mathematics, appreciate its power, and know it as a part of one's culture. What is crucial for the latter is not always crucial for the former, and the contrary. And from this it follows also that researchers in mathematics education must take into account considerations that are at least broader than those mathematicians *qua* educators must take into account and some ways are qualitatively different.

But the picture is hardly black and white with regards to either of these differences; the differences are real, certainly, but they are not such that we can sit contently each in our own separate bailiwicks. Consider the second. To feel at home with mathematics and appreciate its power one must *engage* in mathematics at some level. To become mathematically educated one must understand something about mathematics from within, and that means having a foot in mathematically training. Even to understand *understanding*, as researchers in mathematics education should want to do (as pursued, for example, in Sierpinska 1994), one must engage in mathematics. For example, if one learns that the derivative of a function at a point P is the slope of the tangent to the graph of the function at P, one will have a certain level of understanding of the derivative; if one stops there, however, one could easily believe one understands the derivative *tout court*. Learning a little more, one is faced with a new idea, the "gradient," which is still called the "derivative"; one might convince oneself that it is similar to the old idea since the gradient can be related to the tangent plane of a surface. But then one goes further and learns another idea, the "Jacobian matrix," and, again, this is called a "derivative." One's notion of the derivative as a "slope" no longer suffices to understand the "derivative"; one needs a more general idea of a linear operator approximating the function at a point, which, in odd way, brings one back to the slope of the tangent. The point is without having *experienced* such levels of meaning and circulation of ideas, a student's understanding of mathematical concepts, as well as an educator's understanding of understanding are bound to be rather one dimensional.

On the other side of the equation, well-trained mathematicians who lack a broader view of their subject—its history, its place in society, its philosophy—may still be able to do what they have to do very well, but they face the danger of being something like excellent technicians only. It is thus not by accident that in the very best mathematicians there tends to be a confluence of training and education: knowing history or philosophy of mathematics or the social implications of mathematics may not allow them to solve more problems or prove more theorems, but it makes them more worthy of the name "mathematician." I am certain that Felix Klein had this in mind when he urged teachers to learn history. As he wrote in his *Elementary Mathematics from an Advanced Standpoint*:

> ...I shall draw attention, more than is usually done... to the *historical development of the science*, to the accomplishments of its great pioneers. I hope, by discussions of this sort, to further, as I like to say, your general *mathematical culture*: alongside of knowledge of details, as these are supplied by the special lectures, there should be a grasp of subject-matter and of historical relationship [emphases in the original]. (Klein 1908/1939, II, p. 2)

As for values, while it is true, as I described above, that mathematics education, like all education, is value-laden, it has been one of the leitmotifs of modern philosophy and history of science that even the most exact sciences themselves are not exactly *Wertfrei*! One begins to see this by considering how a certain question or idea or approach in mathematics is deemed interesting or important or beautiful. It is not just because it is correct, or even clever. One might say it is because it is useful. But what makes something useful? This has its own set of values attached to it. And the priority of utility as a measure of importance is itself a matter of values: one recalls Hardy's pride in never having done anything useful in mathematics!

The social background of values is also evident in mathematics and science: a mathematician or scientist's winning a prize or obtaining a speedy promotion depends on whether the community of mathematicians or scientists values the person's work—and that evaluation is not so much a determination as a collective judgment. Aesthetics plays an important part here too for the work is likely to be judged by the number of beautiful results it contains. Of course it may be that beauty only *appears* to be related to values, that it is actually a completely determined thing itself. However, if agreement is any measure of that, it worth recalling that when David Wells asked readers of the *Mathematical Intelligencer* to judge theorems according to their beauty, he had to conclude finally that "...the idea that mathematicians largely agree in their aesthetic judgments is at best grossly oversimplified" (Wells 1990, p. 40).

And there can be real clashes of values in mathematics. Such a clash was described in Siobhan Roberts' biography of H.S.M. Coxeter (Roberts 2006). Coxeter, being a classical geometer, represented a position favoring a visual and intuitive approach to geometry. Standing opposed to Coxeter—this was chiefly during the 1940s and 50s—was the more fashionable Bourbaki, who, according Pierre Cartier, considered that "...[geometry] was based on pure logical reasoning, as little visual insight as possible. Visual insight [in the view of Bourbaki] was considered a concession to human weakness" (quoted in Roberts 2006, p. 122) (a statement of values if ever there was one!).

What is interesting for us is that this difference of values within the mathematics community was played out in discussions about mathematics education. Recall it was in the context of debating reforms in French mathematics education (at Royaumont in 1959) that Dieudonné, one of the founding members of Bourbaki, cried out, famously, "Down with Euclid! Death to Triangles!" Coxeter, for his part, participating in activities and producing publications "... went on a crusade to bring his passion for the visual and intuitive methods to any and all willing spectators," as Roberts puts it (Roberts 2006, p. 163). Thus we see that far from a value-free existence, the mathematical world has its own biases and preferences and these bring it directly into regions of common ground with mathematics education. More precisely, it was as questions about mathematics education that these differences in mathematical values—those of the Bourbaki camp towards the formal, non-visual and those of the Coxeter camp towards the intuitive, visual approach—found a natural means of expression.

As a last word, I should say that when Ted and I wrote our dialogue, it was clear to both of us that this was only one round of a greater dialogue. We had no intention giving a final statement about any of the issues we raised. It was only a beginning. The question is where it should continue, where should its locus be? The remark above in connection to the Royaumont conference suggests, perhaps, this may be the role of mathematics education itself, even if, as I have already argued, mathematics has a stake in the dialogue as well. Indeed, the fact that our dialogue was published by a leading journal for mathematics education may not have been an anomaly, a departure from the main issues of mathematics education research, but an indication of a new issue of emerging importance in the field itself.

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http://www.springer.com/978-94-007-7472-8

Mathematics & Mathematics Education: Searching for Common Ground

Fried, M.N.; Dreyfus, T. (Eds.)

2014, XI, 402 p. 53 illus., Hardcover
ISBN: 978-94-007-7472-8