

# Contents

<b>1</b>	<b>Introduction to Nonlinear Vibrations and Dynamics</b> . . . . .	1
1.1	Usual Sources of Nonlinearity in Mechanical and Other Engineering . . . . .	1
1.1.1	Introduction . . . . .	1
1.1.2	Geometrical Nonlinearities . . . . .	1
1.1.3	Physical Nonlinearities . . . . .	2
1.1.4	Structural or Designed Nonlinearities . . . . .	3
1.1.5	Constraints . . . . .	4
1.1.6	Nonlinearity of Friction . . . . .	5
1.2	Formulation of Equations . . . . .	7
1.2.1	Introduction . . . . .	7
1.2.2	Principle of Virtual Work . . . . .	7
1.2.3	d'Alembert's Principle . . . . .	11
1.2.4	Lagrange's Equations of Motion . . . . .	12
1.2.5	Newton's Method . . . . .	16
1.3	Applied Examples . . . . .	18
	References . . . . .	48
<b>2</b>	<b>Perturbation and Variational Methods</b> . . . . .	49
2.1	Introduction . . . . .	49
2.2	The Basic Ideas of Perturbation Analysis . . . . .	50
2.2.1	Variation of Free Constants and Systems in Standard Form . . . . .	51
2.2.2	Standard Averaging as an Almost Identical Transformation . . . . .	52
2.2.3	Method of Multiple Scales . . . . .	55
2.2.4	Direct Separation of Motions . . . . .	57
2.2.5	Relationship Between These Methods . . . . .	58
2.2.6	Application . . . . .	59
2.2.7	Introduction . . . . .	61
2.2.8	The Method of Multiple Scales . . . . .	61

- 2.3 Parameterized Perturbation Method . . . . . 69
  - 2.3.1 Introduction . . . . . 69
  - 2.3.2 Application . . . . . 70
- 2.4 Singular Perturbation Method . . . . . 71
  - 2.4.1 Introduction . . . . . 71
  - 2.4.2 Application . . . . . 72
- 2.5 Homotopy Perturbation Method and Its Modifications . . . . . 74
  - 2.5.1 A Brief Introduction to the Homotopy  
Perturbation Method . . . . . 74
  - 2.5.2 Application . . . . . 78
- 2.6 Variational Iteration Method . . . . . 90
  - 2.6.1 Introduction . . . . . 90
  - 2.6.2 Application . . . . . 92
- 2.7 He’s Variational Approach . . . . . 96
  - 2.7.1 Basic Idea . . . . . 96
  - 2.7.2 Application . . . . . 98
- 2.8 Couple Variational Method . . . . . 105
  - 2.8.1 Introduction . . . . . 105
  - 2.8.2 Application . . . . . 105
- 2.9 Energy Balance Method . . . . . 108
  - 2.9.1 Introduction . . . . . 108
  - 2.9.2 Application . . . . . 110
- 2.10 Coupled Method of Homotopy Perturbation  
and Variational Method . . . . . 115
  - 2.10.1 Introduction . . . . . 115
  - 2.10.2 Application . . . . . 116
- References . . . . . 129
  
- 3 Considerable Analytical Methods . . . . . 133**
  - 3.1 Harmonic Balance Method . . . . . 133
    - 3.1.1 Introduction . . . . . 133
    - 3.1.2 Governing Equation of Motion and Formulation . . . . . 134
    - 3.1.3 First-Order Analytical Approximation . . . . . 136
    - 3.1.4 Second-Order Analytical Approximation . . . . . 137
    - 3.1.5 Third-Order Analytical Approximation . . . . . 138
    - 3.1.6 Approximate Results and Discussion . . . . . 139
  - 3.2 He’s Parameter Expansion Method . . . . . 141
    - 3.2.1 Introduction . . . . . 141
    - 3.2.2 Modified Lindstedt–Poincaré Method . . . . . 142
    - 3.2.3 Bookkeeping Parameter Method . . . . . 142
    - 3.2.4 Application . . . . . 142
    - 3.2.5 Governing Equation . . . . . 144
    - 3.2.6 HPEM for Solving Problem . . . . . 145

3.3	Differential Transformation Method . . . . .	146
3.3.1	Introduction . . . . .	146
3.3.2	Differential Transformation Method . . . . .	147
3.3.3	Padé Approximations. . . . .	149
3.3.4	Application . . . . .	150
3.4	Adomian’s Decomposition Method . . . . .	154
3.4.1	Basic Idea of Adomian’s Decomposition Method . . . . .	154
3.4.2	Application . . . . .	156
3.5	He’s Amplitude–Frequency Formulation. . . . .	161
3.5.1	Introduction . . . . .	161
3.5.2	Applications . . . . .	162
3.5.3	Problems . . . . .	167
	References . . . . .	181
<b>4</b>	<b>Introduction of Considerable Oscillatory Systems . . . . .</b>	<b>185</b>
4.1	Duffing’s Oscillation Systems . . . . .	185
4.1.1	Introduction to Duffing’s Oscillation . . . . .	185
4.1.2	The Forced Duffing Oscillator . . . . .	192
4.1.3	Universalization and Superposition in Duffing’s Oscillator. . . . .	198
4.2	The Van der Pol Oscillator Systems . . . . .	203
4.2.1	The Unforced Van der Pol Oscillator . . . . .	203
4.2.2	The Forced Van der Pol Oscillator . . . . .	208
4.2.3	Two Coupled Limit Cycle Oscillators . . . . .	214
4.3	Mathieu’s Equation . . . . .	220
4.3.1	Introduction . . . . .	220
4.3.2	Effect of Damping . . . . .	229
4.3.3	Effect of Nonlinearity . . . . .	230
4.4	Ince’s Equation . . . . .	233
4.4.1	Introduction . . . . .	233
4.4.2	Coexistence . . . . .	234
4.4.3	Ince’s Equation. . . . .	236
4.4.4	Designing a System with a Finite Number of Tongues. . . . .	239
4.4.5	Application . . . . .	240
	References . . . . .	247
<b>5</b>	<b>Applied Problems in Dynamical Systems. . . . .</b>	<b>249</b>
5.1	Problem 5.1. Displacement of the Human Eardrum . . . . .	249
5.1.1	Introduction . . . . .	249
5.1.2	Variational Iteration Method. . . . .	249
5.1.3	Perturbation Method . . . . .	250

5.1.4	Homotopy Perturbation Method . . . . .	252
5.1.5	Numerical Solution . . . . .	253
5.2	Problem 5.2. Slides Motion Along a Bending Wire . . . . .	254
5.2.1	Introduction . . . . .	254
5.2.2	Energy Balance Method. . . . .	255
5.2.3	Variational Iteration Method. . . . .	256
5.2.4	Parameter Lindstedt–Poincaré Method. . . . .	257
5.3	Problem 5.3. Movement of a Mass Along a Circle . . . . .	260
5.3.1	Introduction . . . . .	260
5.3.2	Energy Balance Method. . . . .	261
5.3.3	Variational Iteration Method. . . . .	262
5.3.4	Parameter Lindstedt–Poincaré Method. . . . .	263
5.4	Problem 5.4. Rolling a Cylinder on a Cylindrical Surface . . . . .	265
5.4.1	Introduction . . . . .	265
5.4.2	Energy Balance Method Results . . . . .	266
5.4.3	Variational Iteration Method Results . . . . .	267
5.4.4	Parameter Lindstedt–Poincaré Method Results . . . . .	267
5.5	Problem 5.5. Movement of Rigid Rods on a Circular Surface. . . . .	268
5.5.1	Introduction . . . . .	268
5.5.2	Energy Balance Method. . . . .	269
5.5.3	Variational Iteration Method. . . . .	270
5.5.4	Parametrized Perturbation Method. . . . .	272
5.6	Problem 5.6. Application of Two Degrees of Freedom Viscously Damped. . . . .	275
5.6.1	Introduction . . . . .	275
5.6.2	Application of the Homotopy Perturbation Method . . . . .	276
5.7	Problem 5.7. Application of Viscous Damping with a Nonlinear Spring . . . . .	282
5.7.1	Introduction . . . . .	282
5.7.2	Application of Homotopy Perturbation Method. . . . .	283
5.7.3	Underdamped System $\left(\zeta^2 < 1 \text{ or } \frac{c}{2m} < \sqrt{\frac{k}{m}}\right)$ . . . . .	285
5.7.4	Overdamped System $\left(\zeta^2 > 1 \text{ or } \frac{c}{2m} > \sqrt{\frac{k}{m}}\right)$ . . . . .	288
5.7.5	Critically Damped System $\left(\zeta^2 = 1 \text{ or } \frac{c}{2m} = \sqrt{\frac{k}{m}}\right)$ . . . . .	292
5.7.6	Discussion and Conclusion. . . . .	294
5.8	Problem 5.8. Application of Cubic Nonlinearity . . . . .	297
5.8.1	Introduction . . . . .	297
5.8.2	First Assumption. . . . .	299
5.8.3	Second Assumption. . . . .	303
5.9	Problem 5.9. Van der Pol Oscillator . . . . .	305
5.9.1	Introduction . . . . .	305
5.9.2	The Application of PM in the Van der Pol Oscillator . . . . .	306
5.9.3	Homotopy Perturbation Method . . . . .	307

- 5.9.4 Application of VIM in the Van der Pol Oscillator. . . . . 308
- 5.9.5 Application of ADM in the Van der Pol Oscillator . . . . . 309
- 5.10 Problem 5.10. Application of a Slender, Elastic Cantilever Beam . . . . . 313
  - 5.10.1 Introduction . . . . . 313
  - 5.10.2 Solution Using the First Case of the Homotopy Perturbation Method . . . . . 315
  - 5.10.3 Solution Using Second Case of the Homotopy Perturbation Method . . . . . 320
- 5.11 Problem 5.11. Dynamic Behavior of a Flexible Beam Attached to a Rotating Rigid Hub . . . . . 322
  - 5.11.1 Introduction . . . . . 322
  - 5.11.2 Application of the Homotopy Perturbation Method . . . . . 323
  - 5.11.3 Application of the Energy Balance Method . . . . . 325
  - 5.11.4 Results. . . . . 326
- 5.12 Problem 5.12. The Motion of a Ring Sliding Freely on a Rotating Wire . . . . . 326
  - 5.12.1 Introduction . . . . . 326
  - 5.12.2 Application of HPEM . . . . . 328
- 5.13 Problem 5.13. Application of a Rotating Rigid Frame Under Force . . . . . 330
  - 5.13.1 Introduction of Case 1 . . . . . 330
  - 5.13.2 Application of HPEM . . . . . 330
  - 5.13.3 Introduction of Case 2 . . . . . 332
  - 5.13.4 Solution of Case 2 Using Frequency Formulation . . . . . 333
- 5.14 Problem 5.14. Application of a Nonlinear Oscillator in Automobile Design . . . . . 335
  - 5.14.1 Introduction . . . . . 335
  - 5.14.2 Solution Using the Amplitude Frequency Formulation . . . . . 336



<http://www.springer.com/978-94-007-6774-4>

Dynamics and Vibrations  
Progress in Nonlinear Analysis  
Kachapi, S.H.H.; Ganji, D.D.  
2014, XVII, 338 p., Hardcover  
ISBN: 978-94-007-6774-4