# Contents

1 Introduction .......................... 1
   1.1 Fracture Phenomena in Nature and Engineering .......... 1
   1.2 Fracture Mechanics ........................ 5
   1.3 Computational Methods for Cracks .................... 8
   1.4 Basic Literature on Fracture Mechanics ............... 10

2 Classification of Fracture Processes .................. 13
   2.1 Macroscopic Manifestations of Fracture ............... 13
   2.2 Microscopic Appearances of Fracture ................. 17
   2.3 Classification of Fracture Processes ................. 18
   Reference ..................................... 20

3 Basics of Fracture Mechanics ....................... 21
   3.1 Model Assumptions .......................... 21
   3.2 Linear-Elastic Fracture Mechanics ............... 23
      3.2.1 Two-Dimensional Crack Problems .............. 23
      3.2.2 Eigenfunctions of the Crack Problem ......... 31
      3.2.3 Three-Dimensional Crack Problems ........... 35
      3.2.4 Stress Intensity Factors: K-Concept ......... 38
      3.2.5 Energy Balance During Crack Propagation .... 42
      3.2.6 The J-Integral .......................... 51
      3.2.7 Cracks in Anisotropic Elastic Bodies ......... 54
      3.2.8 Interface Cracks .......................... 58
      3.2.9 Cracks in Plates and Shells ................. 62
      3.2.10 Fracture Mechanical Weight Functions ....... 65
      3.2.11 Thermal and Electric Fields ................. 76
   3.3 Elastic-Plastic Fracture Mechanics ............... 80
      3.3.1 Introduction .................................. 80
      3.3.2 Small Plastic Zones at the Crack ............. 81
      3.3.3 The Dugdale Model .......................... 86
      3.3.4 Crack Tip Opening Displacement (CTOD) ........ 87
      3.3.5 Failure Assessment Diagram (FAD) .............. 89
      3.3.6 Crack Tip Fields ............................ 91
5 FE-Techniques for Crack Analysis in Linear-Elastic Structures.......................... 193
5.1 Interpreting the Numerical Solution at the Crack Tip......................... 193
5.2 Special Finite Elements at the Crack Tip..................... 197
  5.2.1 Development of Crack Tip Elements......................... 197
  5.2.2 Modified Isoparametric Displacement Elements...... 198
  5.2.3 Computing Intensity Factors from Quarter-Point
  Elements........................................... 207
5.3 Hybrid Crack Tip Elements........................................... 212
  5.3.1 Development of Hybrid Crack Tip Elements.............. 212
  5.3.2 2D Crack Tip Elements Based on Mixed
  Hybrid Model......................................... 214
  5.3.3 3D Crack Tip Elements Based on Hybrid
  Stress Model......................................... 218
5.4 Method of Global Energy Release Rate..................................... 223
  5.4.1 Realization Within FEA........................................ 223
  5.4.2 Method of Virtual Crack Extension.............................. 224
5.5 Method of Crack Closure Integral........................................ 227
  5.5.1 Basic Equations of Local Energy Method................. 227
  5.5.2 Numerical Implementation in FEA 2D...................... 229
  5.5.3 Numerical Implementation in FEA 3D...................... 233
  5.5.4 Consideration of Crack Face, Volume
  and Thermal Loading...................................... 239
5.6 FE-Computation of J-Contour Integrals..................................... 240
5.7 FE-Calculation of Fracture Mechanics Weight Functions............. 243
  5.7.1 Determination by Point Forces................................. 243
  5.7.2 Determination of Parametric Influence Functions...... 245
  5.7.3 Derivation from Displacement Fields.......................... 247
  5.7.4 Application of the J-VCE-Technique......................... 249
  5.7.5 Calculation by Means of the Bueckner-Singularity...... 250
5.8 Examples.......................................................................... 251
  5.8.1 Tension Sheet with Internal Crack.............................. 251
  5.8.2 Semi-Elliptical Surface Crack Under Tension............ 255
References........................................................................... 258

6 Numerical Calculation of Generalized Energy
Balance Integrals......................................................................... 263
6.1 Generalized Energy Balance Integrals..................................... 263
6.2 Extension to General Loading Cases...................................... 267
  6.2.1 Preconditions for Path-Independence.......................... 267
  6.2.2 Crack Face, Volume and Thermal Loading.................... 268
6.3 Three-Dimensional Variants................................................. 270
  6.3.1 The 3D-Disk Integral............................................. 271
  6.3.2 Virtual Crack Propagation in 3D............................... 273
6.4 Numerical Calculation as Equivalent Domain Integral
6.4.1 Transformation into an Equivalent Domain Integral 2D
6.4.2 Transformation into an Equivalent Domain Integral 3D
6.4.3 Numerical Implementation
6.5 Consideration of Dynamic Processes
6.6 Extension to Inhomogeneous Structures
6.7 Treatment of Mixed-Mode-Crack Problems
6.7.1 Separation into Crack Opening Modes I and II
6.7.2 Interaction-Integral-Technique
6.8 Calculation of $T$-Stresses
6.9 Examples
6.9.1 Internal Crack Under Crack Face Loading
6.9.2 Edge Crack Under Thermal Shock
6.9.3 Dynamically Loaded Internal Crack
6.9.4 Crack in a Functionally Graded Material
6.10 Concluding Assessment of Methods

References

7 FE-Techniques for Crack Analysis in Elastic-Plastic Structures
7.1 Elastic-Plastic Crack Tip Elements
7.2 Determination of Crack Tip Opening Displacements
7.3 Calculation of the $J$-Integral and its Meaning
7.3.1 Elastic-Plastic Extensions of $J$
7.3.2 Application to Stationary Cracks
7.3.3 Application to Moving Cracks
7.4 Examples
7.4.1 Compact-Tension Specimen
7.4.2 Tensile Plate with Surface Crack

References

8 Numerical Simulation of Crack Propagation
8.1 Nodal Release Technique
8.2 Techniques of Element Modification
8.2.1 Element Splitting
8.2.2 Element Elimination Technique
8.2.3 Adapting Element Stiffness
8.3 Moving Crack Tip Elements
8.4 Adaptive Remeshing Strategies
8.4.1 Error-Controlled Adaptive Meshing
8.4.2 Simulation of Crack Propagation
8.5 Cohesive Zone Models ........................................... 338
  8.5.1 Physical Background ..................................... 338
  8.5.2 Numerical Realization ................................... 343
8.6 Damage Mechanical Models .................................. 345
8.7 Examples of Fatigue Crack Propagation .................... 347
  8.7.1 Shear Force Bending Specimen .......................... 347
  8.7.2 ICE-Wheel Failure ....................................... 349
8.8 Examples of Ductile Crack Propagation ..................... 351
  8.8.1 Cohesive Zone Model for CT-Specimen ................. 351
  8.8.2 Damage Mechanics for SENB-Specimen ................. 354
References .......................................................... 358

9 Practical Applications ............................................. 361
  9.1 Fatigue Crack Growth in a Railway Wheel ................. 361
    9.1.1 Material Data of Austempered Ductile Iron ADI ... 361
    9.1.2 Finite Element Calculation of the Wheel ............ 362
    9.1.3 Specification of Crack Postulates .................... 365
    9.1.4 Fracture Mechanical Analysis ......................... 366
  9.2 Brittle Fracture Assessment of a Container Under Impact Loading ........................................... 373
    9.2.1 FE-Model of the Drop Test ............................ 373
    9.2.2 Fracture Mechanical Results of the Simulation ...... 376
    9.2.3 Application of Submodel Technique .................. 376
  9.3 Ductile Fracture of a Weldment in a Gas Pipeline ....... 377
    9.3.1 Introduction ........................................... 377
    9.3.2 Fracture Mechanics Assessment Concept FAD ......... 378
    9.3.3 Large Scale Test of a Piping with Pre-cracked Weldments ........................................... 382
    9.3.4 FE-Analysis of Large Scale Piping Test ............ 386
References .......................................................... 389

Appendix: Fundamentals of Strength of Materials ............... 391

Index .............................................................. 443