

# Comparison of Alternative Equivalent Circuits of Induction Motor with Real Machine Data

J. Bradna, J. Bauer, S. Fligl and V. Hlinovsky

**Abstract** The algorithms based on separated control of the motor flux and torque is used in order to gain the maximum performance from the induction machine. To push the efficiency and dynamics limits of the IM to its limits mostly FOC or DTC control strategies are used. Both are based on the knowledge of the hardly measurable variable-machine flux. To obtain the information about inner machine flux models based on the machine equivalent circuit are mostly used. Therefore the accuracy of the equivalent circuits has direct influence on the accuracy of the machine control. To reduce the complexity of the mathematical model the resistances and inductances are concentrated to one component and three phase winding is assumed to be symmetrical. In order to design control strategy for the induction motor, system equations and equivalent circuit must be established at first. This paper examines and compares some of the issues of adequate machine modeling and attempts to provide a firmer basis for selection of an appropriate model and to confirm or disprove the equivalence of different approaches. The results of the IM model run up are then compared to the results obtained from the measurements on the real machine and the equivalency is discussed..

**Keywords** Induction motor · Equivalent circuit · IM model

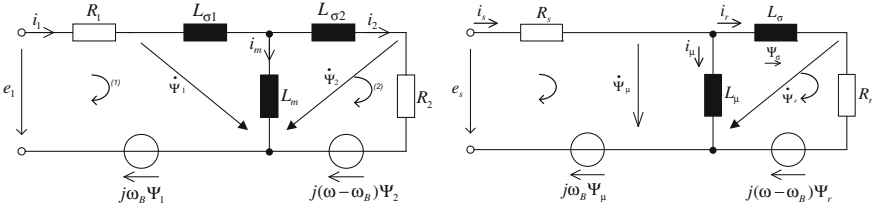
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**Fig. 1** Equivalent T-circuit of an IM  $\Gamma$ -circuit of the IM

## 1 Introduction

Induction machines (IM) are widely used as electro-mechanics changers in variable speed drives [1]. For describing the electromagnetic processes in induction machines different equivalent circuits can be used. To reduce the complexity of the mathematical model, resistances and inductances are represented as concentrated components and the 3-phase winding system is assumed to be symmetrical [2]. These equivalent circuits are also used as a starting point for the design of a drive controller [1]. Moreover for the function of a regulation algorithm of the drive or model of a machine, designed on the basis of the machine's equivalent circuit, the parameters of the equivalent circuit must be known with sufficient accuracy.

In IM drive controllers the representations based on an equivalent T-formed circuit are used, because they serve as a solid basis for an advanced motor control. However, other authors tend to use gamma models instead [3–5].

## 2 Equivalent Circuit in T-form

In the equivalent IM circuit in T-form (Fig. 1) can be established two kirchhoff loops and (1), (2) for the derivation of the fluxes can be written

$$\dot{\underline{\Psi}}_1(t) - \underline{e}_1 + R_1 \dot{i}_1(t) + \omega_B j \underline{\Psi}_1(t) = 0 \quad (1)$$

$$-\dot{\underline{\Psi}}_2(t) + R_2 \dot{i}_2(t) + j \underline{\Psi}_2(t)(\omega - \omega_B) = 0, \quad (2)$$

where:  $\underline{\Psi}_1$ —stator flux complex space vector,  $\underline{\Psi}_2$ —rotor flux complex space vector,  $\underline{e}$ —supplying voltage complex space vector,  $\omega_B$ —angular speed of the reference frame.

Then (1) and (2) can be rewritten in a standard vector form (3)

$$\dot{\underline{\Psi}}(t) = A_T \underline{\Psi}(t) + \underline{g}(t) \quad (3)$$

where  $\mathbf{A}_T$  is the state space matrix (4) and  $\mathbf{g}$  respects the influence of the supplying voltage.

$$\mathbf{A}_T = \begin{pmatrix} -\frac{R_1(L_m+L_{\sigma 2})}{c} & \omega_B & \frac{L_m R_1}{c} & 0 \\ -\omega_B & -\frac{R_1(L_m+L_{\sigma 2})}{c} & 0 & \frac{L_m R_1}{c} \\ \frac{L_m R_2}{c} & 0 & -\frac{R_2(L_m+L_{\sigma 1})}{c} & \omega_B - \omega \\ 0 & \frac{L_m R_2}{c} & \omega - \omega_B & -\frac{R_2(L_m+L_{\sigma 1})}{c} \end{pmatrix} \mathbf{g} = \begin{pmatrix} e_{1a}(t) \\ e_{1b}(t) \\ 0 \\ 0 \end{pmatrix} \quad (4)$$

where  $c = L_{\sigma 1} + L_{\sigma 2} + L_m L_{\sigma 1} + L_m L_{\sigma 2}$

### 3 Equivalent Circuit in $\Gamma$ -form

The  $\Gamma$ -equivalent circuit is second equivalent circuit used in electric drives. The advantage of this circuit is a simplification consisting in fusion of the rotor and stator inductances into one inductance on the rotor side without loss of information. The two loops for the stator resp. rotor flux (5) and (6) can be established too

$$\dot{\Psi}_\mu(t) - e_s(t) + R_s i_s(t) + \omega_B j \Psi_\mu(t) = 0 \quad (5)$$

$$-\dot{\Psi}_r(t) + R_r i_r(t) + j \Psi_r(t)(\omega - \omega_B) = 0 \quad (6)$$

where:  $i_s$ —stator current complex space vector,  $i_r$ —rotor current complex space vector,  $L_\sigma$ —leakage inductance,  $L_\mu$ —magnetizing inductance. When we use similar labeling to that for a T-circuit (5), (6) can be rewritten into standard vector form (7) again.

$$\underline{\Psi}(t) = \mathbf{A}_\Gamma \Psi(t) + \mathbf{b}(t) \quad (7)$$

where  $\mathbf{A}_\Gamma$  is the system state space matrix.

$$\mathbf{A}_\Gamma = \begin{pmatrix} -\frac{R_s}{L_\mu} - \frac{R_s}{L_\sigma} & \omega_B & \frac{R_2}{L_\sigma} & 0 \\ -\omega_B & -\frac{R_s}{L_\mu} - \frac{R_s}{L_\sigma} & 0 & \frac{R_3}{L_\sigma} \\ \frac{R_r}{L_\sigma} & 0 & -\frac{R_r}{L_\sigma} & \omega_B - \omega \\ 0 & \frac{R_r}{L_\sigma} & \omega - \omega_B & -\frac{R_r}{L_\sigma} \end{pmatrix}; \mathbf{b} = \begin{pmatrix} e_{sa}(t) \\ e_{sb}(t) \\ 0 \\ 0 \end{pmatrix} \quad (8)$$

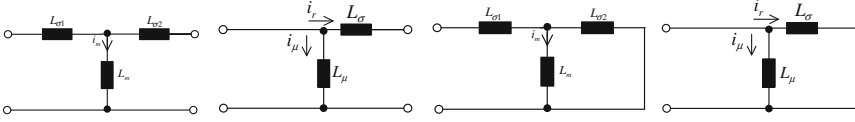


Fig. 2 Equivalency of the impedances

## 4 Recalculation Formulas Between T and G Circuit

We must express the recalculation relations between the variables in the circuits in order to compare them. Let us to follow the most widespread approach in text books—impedance equivalency and time constant comparison [2–3].

Firstly, let us suppose the input voltage and the stator resistance to be the same for both circuits. Secondly we can substitute the fluxes under the assumption that in a similar way as the fluxes the rotor currents have a certain recounting as follows:

$$e_s = e_1, R_s = R_1, i_s = i_1, i_r = \frac{L_m i_2}{L_m + L_{\sigma 1}} \quad (9)$$

Next we assume that inductance circuits have the same behaviour when the right-hand sides of the circuits are opened or short-circuited. From the principle of the superposition it then follows that for the opened right-hand side the same current will flow through the primary circuit (left-hand side) (10). The same behaviour shall be guaranteed in the case of the short-circuit on the right-hand side (11) Fig. 2.

For the impedance can be then written

$$Z_T = \omega j (L_m + L_{\sigma 1}), Z_{\Gamma} = \omega j L_{\mu} \Rightarrow L_{\mu} = L_m + L_{\sigma 1} \quad (10)$$

$$\begin{aligned} Z_T &= \omega j \left( L_{\sigma 1} + \frac{L_m L_{\sigma 2}}{L_m + L_{\sigma 2}} \right), Z_{\Gamma} = \frac{\omega j L_{\mu} L_{\sigma}}{L_{\mu} + L_{\sigma}} \Rightarrow L_{\sigma} \\ &= \left( \frac{L_{\sigma 1}}{L_m} + 1 \right) L_{\sigma 1} + \left( \frac{L_{\sigma 1}}{L_m} + 1 \right)^2 L_{\sigma 2} \end{aligned} \quad (11)$$

Finally, the very last assumption is the equality of the rotor and stator time constants:  $T_1 = T_s$  and  $T_2 = T_r$

The stator time constants can be expressed as (12). Because of (9) the equivalency is obvious.

$$T_1 = \frac{L_m + L_{\sigma 1}}{R_1}, T_s = \frac{L_{\mu}}{R_s} \quad (12)$$

In the case of the rotor time constant the situation is depicted in Fig 3. This can be described by (13).

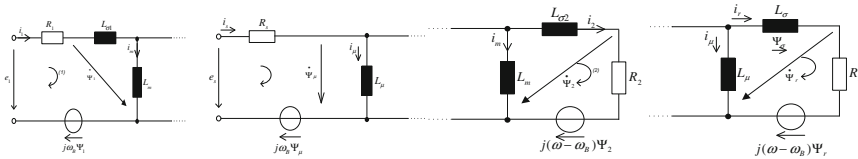


Fig. 3 Equivalency of the time constants

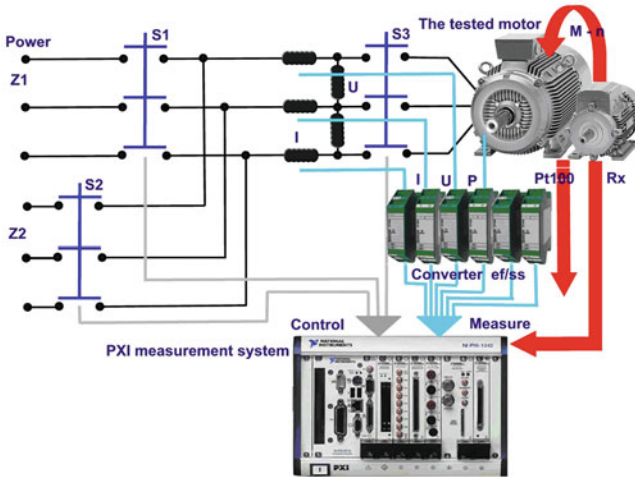


Fig. 4 System “OMEGA 2011”

$$T_2 = \frac{L_m + L_{\sigma 2}}{R_2}, T_r = \frac{L_\mu + L_\sigma}{R_r} \Rightarrow R_r = \frac{R_2(L_m + L_{\sigma 1})^2}{L_m^2} \quad (13)$$

### 5 Induction Motor Parameter Test Bed

Measuring system is based on components from National Instruments—PXI device. Whole system is managed by interactive programme “Omega 2011”, which was created in program Lab Windows CVI in language “C++”. Whole system is placed in chassis with eight positions, which protects system from disturbance and outer damage. The efficient power supply and fast data bus PCI are placed in the base unit for direct processing measured data in the memory of control computer (Fig. 4).

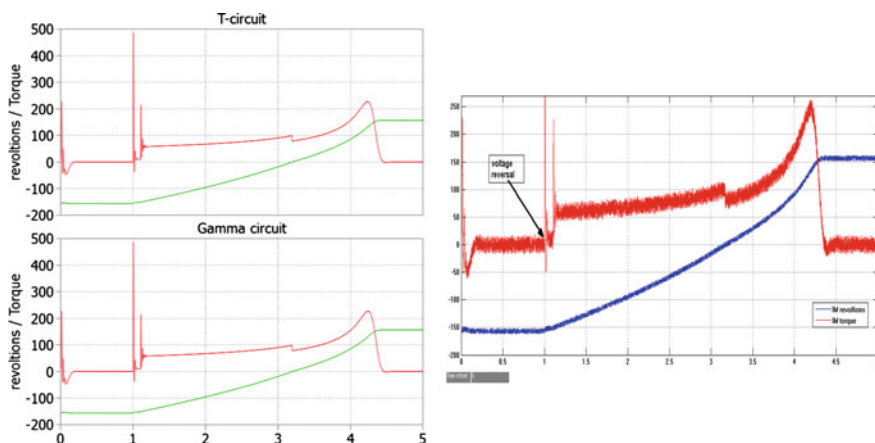
Nominal parameters of the IM are summarized in (Table 1), the parameters for the T-equivalent circuit calculated from the measurements are in (Table 2). These parameters were recalculated according to (10–13) to obtain the parameters for the

**Table 1** Nominal values of the IM

Nominal values of the IM-R623 + 4 + Db + H02			
$P_n$ (kW)	5,5	$I_{1n}$ (A)	11,8
$f_n$ (Hz)	50	$n_n$ ( $\text{min}^{-1}$ )	1430
$U_{1n}$ (V)	380	$J$ ( $\text{kg m}^2$ )	0,14

**Table 2** Equivalent circuit parameters

T-Circuit		$\Gamma$ -Circuit	
$R_1$ ( $\Omega$ )	3,300	$R_S$ ( $\Omega$ )	3,300
$L_{\sigma 1}$ (H)	0,026	$R_R$ ( $\Omega$ )	2,690
$R_2$ ( $\Omega$ )	2,300	$L_{\sigma}$ (H)	0,052
$L_{\sigma 2}$ (H)	0,200	$L_{\mu}$ (H)	0,3499
$L_m$ (H)	0,3235		

**Fig. 5** Simulation results—IM reversal measured reversal of IM

$\Gamma$ -circuit. Based on these parameters were established two models in Matlab/Simulink.

The Fig. 5 (left) show the simulation of reversal of the IM from negative nominal speed to positive. Figure 5 (right) shows measured results by system Omega 2011.

## 6 Conclusions

Simulation results show that both equivalent circuits are equal. Also the recalculation equations for transformation from T-circuit to  $\Gamma$ -circuit are correct. This is proof for the full equivalence of both circuits. However, size adjustment for rotor state variables must be undertaken if those are to be exchanged directly between

the Gamma and the T model running in parallel. Also measured and simulated reversal of the IM show same behavior of IM and same variable scales.

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