Chapter 2
Laser Diode Beam Propagation Basics

Abstract Laser diode beam propagation characteristics, the collimating and focusing behaviors and the $M^2$ factor are discussed using equations and graphs. Thin lens equation modified to be applicable for laser beams is introduced. An example about collimating and focusing a laser diode beam is presented. Raytracing technique is briefly discussed.

Keywords Thin lens equation · Propagation · Collimating · Focusing · Beam spot · Gaussian beam · Geometric rays

To understand laser diode beam propagation characteristics, some basic knowledge about laser beam propagation theory is necessary.

2.1 Basic Mode Gaussian Beam

Most laser beams have a circular shape cross section with a Gaussian intensity profile. Such beams are basic TE mode Gaussian beams. The characteristics of a Gaussian beam are described by a set of three equations [1].

$$w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}$$  \hspace{1cm} (2.1)

$$R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$  \hspace{1cm} (2.2)

$$I(r,z) = I_0(z)e^{-2r^2/w(z)^2}$$  \hspace{1cm} (2.3)

where $z$ is the distance from the waist of the laser beam, $\lambda$ is the wavelength, $w_0$ is the $1/e^2$ intensity beam waist radius, $w(z)$ is the $1/e^2$ intensity beam radius at $z$, $I_0(z)$ is the intensity at the waist.
$R(z)$ is the beam wavefront curvature radius at $z$, $I_0(z)$ is the beam peak intensity in a cross section plane at $z$, $r$ is the radial coordinate in a cross section plane at $z$, and $I(r,z)$ is the beam intensity radial distribution in a cross section plane at $z$. Figure 2.1 shows the propagation of two Gaussian laser beams with $w_0 = 1.0$ and 0.5 mm, respectively, both beams have $\lambda = 0.633$ μm.

For a laser beam the Rayleigh range $z_R$ is defined as that at $z = z_R$ the beam radius is $w(z_R) = \sqrt{2}w_0$. From Eq. 2.1 we can find that.

$$z_R = \frac{\pi w_0^2}{\lambda} \quad \text{(2.4)}$$

$z_R$ is proportional to $w_0^2$. $z_R$ for the two laser beams are marked out in Fig. 2.1. From Eq. 2.1 we can also find that at far field, $z$ is large, term $\lambda z / \pi w_0^2 = z / z_R \approx 1$, the $1/e^2$ intensity beam far field half divergent angle $\theta$ can be found by

$$\theta = \frac{w(z)}{z} = \frac{\lambda}{\pi w_0} = \frac{w_0}{z_R} \quad \text{(2.5)}$$
The percentage laser energy encircled inside the $1/e^2$ intensity radius can be calculated by:

$$\frac{\int_0^{w(z)} e^{-2r^2/w(z)^2} r \, dr}{\int_0^{\infty} e^{-2r^2/w(z)^2} r \, dr} = 86.4\%$$

(2.6)

where $r$ is the radial variable. Similarly, the percentage laser energy encircled inside the half magnitude radius can be calculated by
\[
\frac{\int_0^{0.59w(z)} e^{-2r^2/w(z)^2} r\,dr}{\int_0^\infty e^{-2r^2/w(z)^2} r\,dr} = 69.2 \% \tag{2.7}
\]

The characteristics of basic transverse mode Gaussian beams have been studied in depth. Many literatures studying this subject have been published. The most cited one is the book written by Siegman [1].

### 2.2 $M^2$ Factor Approximation

The beams of some solid state lasers and laser diodes are not exact basic mode Gaussian beams, they may contain higher order Gaussian modes. It is difficult to find the mode structure details in these beams, since the unavoidable measurement errors often lead to inconclusive results. But all these non-basic Gaussian mode beams will have far field divergences larger than the far field divergences of basic mode Gaussian beams with the same beam waist radii. A practical way of handling such laser beams is to neglect the mode structure details, assume the beams still have Gaussian intensity distributions and introduce a “$M^2$ factor” to the beams. Eqs. 2.1 and 2.2 are modified to the forms [2]

\[
w(z) = w_0 \left[ 1 + \left( \frac{M^2 \lambda z}{\pi w_0^2} \right)^2 \right]^{1/2} \tag{2.8}
\]
The beam Rayleigh range and the far field divergence become, respectively

\[
R(z) = z \left[ 1 + \left( \frac{\pi w_0^2}{M^2 \lambda z} \right)^2 \right] \tag{2.9}
\]

\[
z_R = \frac{\pi w_0^2}{M^2 \lambda} \tag{2.10}
\]

\[
\theta = \frac{w(z)}{z} = \frac{M^2 \lambda}{\pi w_0} = \frac{w_0}{z_R} \tag{2.11}
\]

By definition \( M^2 \geq 1 \). For \( M^2 = 1 \), Eqs. 2.8–2.11 reduce to Eqs. 2.1, 2.2, 2.4 and 2.5, the beam reduces to a basic mode Gaussian beam. Figures 2.4 and 2.5 plot Eqs. 2.8 and 2.9 for two beams with \( M^2 = 1 \) and 1.2, respectively, both beams have \( w_0 = 1.0 \) mm and \( \lambda = 0.633 \) mm. Most collimated single TE mode laser diode beams have a \( M^2 \) of 1.1 ~ 1.2. \( M^2 \) factor has been widely used now to describe various quasi Gaussian laser beams. Some laser developers even use \( M^2 \) factor to describe multi TE mode laser beams. To this author’s opinion, this is not an appropriate use of \( M^2 \) factor.

### 2.3 Collimation or Focusing a Laser Diode Beam: Thin lens Equation

Thin lens equation was originally derived as a simple analytical model to describe how a lens manipulates geometric rays.\(^1\) Thin lens equation is an approximated model, but accurate enough in most applications, and is therefore widely used. With some modifications, thin lens equation can be used to describe how a laser beam propagates through a lens. In this book we use thin lens equation as the main mathematical model.

The widely used thin lens equation for geometric rays emitted by an object point has the form

\[
\frac{i}{f} = \frac{o}{o - f} \tag{2.12}
\]

\(^1\) Almost any optics text book discusses thin lens equations and Snell’s law.
Fig. 2.4 *Thick curves are* $w(z)$ versus $z$ for two laser beams with $w_0 = 1.0$ mm, and $M^2 = 1$ and 1.2, respectively.

Fig. 2.5 *Thick curves are* $R(z)$ versus $z$ for of two laser beams with $w_0 = 1.0$ mm, and $M^2 = 1$ and 1.2, respectively.
where \( o \) is the object distance measured from the object point to the lens principal plane, the lens focuses the rays from the object point and produces an image of the object point, \( i \) is the image distance measured from the image point to the lens principal plane, and \( f \) is the focal length of the lens. The lens magnification ratio \( m \) is given by

\[
m = \frac{i}{o}
\]  

Equation 2.12 shows that for \( o = f_+ \), \( i \to \infty \) and \( m \to \infty \), the rays are collimated, where \( f_+ \) means a value slightly larger than \( f \). For \( o = f_- \), \( i \to -\infty \) and \( m \to -\infty \), the rays are also collimated, where \( f_- \) means a value slightly smaller than \( f \). For \( o \to \infty \), \( i \to f \) and \( m \to 0 \), the rays are focused. It’s noted that the actual smallest possible focused spot radius is the diffraction limited radius \( 1.22 \lambda d / d \), where \( \lambda \) is the light wavelength and \( d \) is the ray bundle diameter.

Equation 2.12 was first modified to be applicable to a basic mode Gaussian beam without considering the \( M^2 \) factor [3] and later expanded to include the \( M^2 \) factor [4]. The later form of thin lens equation looks like that

\[
i = \frac{o (\frac{o}{f} - 1) + (\frac{z_R}{M^2 f})^2}{(\frac{f}{o} - 1)^2 + (\frac{z_R}{M^2 f})^2}
\]  

(2.14)

where \( o \) is the object distance measured from the waist of the laser beam incident on the lens to the principal plane of the lens, \( i \) is the image distance measured from the waist of the laser beam output from the lens to the principal plane of the lens, and \( z_R \) is the incoming laser beam Rayleigh range defined in Eq. 2.4 (if use the \( z_R \) defined in Eq. 2.10, the \( M^2 \) factor will be used twice). \( z_R/(M^2 f) \) is an important parameter in Eq. 2.14. For \( z_R/(M^2 f) \to 0 \), Eq. 2.14 reduces to Eq. 2.12, that means such a laser beam can be considered as geometric rays emitted by an object point. For \( z_R/(M^2 f) \to \infty \), Eq. 2.14 leads to \( i = f \), the laser beam is focused. Section 2.4 will discuss the collimation and focusing of laser beams in more details.

Equation 2.14 has some interesting characteristics that are different from Eq. 2.13. One characteristics is the maximum and minimum focusing distance that can be found by differentiating Eq. 2.14 and let \( \Delta i / \Delta o = 0 \), we obtain

\[
o = f \pm z_R / M^2
\]  

(2.15)

Plug \( o = f + z_R / M^2 \) into Eq. 2.14 we can find the maximum focusing distance as

\[
 i_{\text{max}} = f \left( \frac{z_R}{M^2 f} + 1 \right)
\]  

(2.16)

Plug \( o = f - z_R / M^2 \) into Eq. 2.14 we can find the minimum focusing distance as
$i_\text{min} = f \frac{2z_R}{M^2f} - 1$  \hspace{1cm} (2.17)

$z_R/(M^2f)$ again play an important role here. For $z_R/(M^2f) \gg 1$, Eqs. 2.16 and 2.17 lead to $i_{\text{max}} = i_{\text{min}} = f$, that is a focusing situation. For $z_R/(M^2f) \ll 1$, Eqs. 2.16 and 2.17 lead to $i_{\text{max}} \to \infty$ and $i_{\text{min}} \to -\infty$, the beam is collimated similar to collimated geometric rays emitted by an object point.

For a typical laser diode beam, $z_R$ is several microns, assuming this laser diode beam is collimated by a lens with a focal length of several millimeters, we have $z_R/(M^2f) \approx 0.001$, Eq. 2.16 reduces to $i_{\text{max}} \approx f^2 M^2/2 z_R \approx 500f \approx 1$ m, Eq. 2.17 reduces to $i_{\text{min}} \approx -f^2 M^2/2 z_R \approx -500f \approx -1$ m, the negative value of $i_{\text{min}}$ indicates that the laser beam outgoing from the lens has an imaginary waist at the left hand side of the collimation lens.

The waist of a collimated laser diode beam is a few millimeters, the $z_R$ of the collimated beam is several meters. When this collimated laser diode beam is focused by a lens with a focal length of tens of millimeters, we have $z_R/(M^2f) \approx 100$, Eqs. 2.16 and 2.17 give $i_{\text{max}} \approx 1.01f$ and $i_{\text{min}} \approx 0.99f$, respectively, the focused spot of the beam can shift around the lens focal point in the range of $\approx 10$ $\mu$m.

Equation 2.14 is plotted in Fig. 2.6 by dash curves with $z_R/(M^2f)$ being a parameter, Eq. 2.12 is also plotted in Fig. 2.6 by the solid curve for comparison. Figure 2.6 shows that for $olf = 1$, $ilf = 1$ for any $z_R/(M^2f)$ values. The maximum and minimum focusing distances $i_{\text{max}}$ and $i_{\text{min}}$ are marked by the black dots and squares on each curve. For smaller $z_R/(M^2f)$, $i$ changes faster as $o$ changes and the value of $i_{\text{max}}$ and $|i_{\text{min}}|$ are larger.

The magnification of a laser beam propagating through a lens is defined by the ratio of $w'/w_0$, where $w'_0$ is the waist radius of the beam output from the lens. $w'/w_0$ can be found by modifying Eq. 2.13 as [4].

$$m = \frac{w'_0}{w_0} = \frac{1}{\left[\left(\frac{o}{f} - 1\right)^2 + \left(\frac{z_R}{M^2f}\right)^2\right]^{0.5}}$$ \hspace{1cm} (2.18)

$w'/w_0 \approx 1$ indicates the beam is collimated. $w'/w_0 \ll 1$ indicates the beam is focused. $w'/w_0 = 1$ means the beam is relayed. From Eq. 2.18 we can see that $w'/w_0 = 1$ can appear for various combinations of $olf$ and $z_R/(M^2f)$. It can be seen that for $z_R/(M^2f) \to 0$, Eq. 2.18 reduces to Eq. 2.13.

Equation 2.18 is plotted in Fig. 2.7 with $z_R/(M^2f)$ as a parameter. We can see from Fig. 2.7 that for any $z_R/(M^2f)$ values, $m$ peaks at $olf = 1$. For $olf = 1$, $m$ is larger for smaller $z_R/(M^2f)$, since this is a collimation situation, the waist size of a collimated beam is much larger than the waist size of the incident beam. For $z_R/(M^2f) > 1$, the value of $m$ does not change much as the value of $olf$ changes,
since this is a focusing situation, the waist size of the focused beam or the focused spot does not change much.

Fig. 2.6  $i/f$ versus $o/f$ curves. The dash curves are for several laser beams with $z_R/(M^2f)$ being a parameter. The solid curves are for geometric rays emitted by an object point for comparison. $i_{\text{max}}$ and $i_{\text{min}}$ are marked by black dots and squares on each curve, respectively

Fig. 2.7  $m$ versus $o/f$ curve with $z_R/(M^2f)$ as a parameter
Fig. 2.8 Illustration of the collimating characteristics of a laser diode beam. The black dots denote the focal points of the lens.
2.4 Collimation or Focusing a Laser Diode Beam: Graphical explanations

In Sect. 2.3, we discussed the mathematical model describing the collimation or focusing of a laser beam. In this section we draw several graphs to provide a more clear view about the collimating or focusing of a laser diode beam. We start from the collimating situation.

Figure 2.8a shows a laser beam with its waist located at the focal plane of the lens, the beam is collimated after propagating through the lens, the waist of the collimated beam is located at the other focal plane of the lens. Figure 2.8b shows the waist of the input laser beam moves away from the focal plane of the lens by a small distance, the waist of the beam propagated through the lens also moves away from the lens focal plane, this beam is not well collimated. Figure 2.8c shows the waist of the input laser beam moves away from the lens to a location with $o = f + z_R/M^2$, then the waist of the beam propagated through the lens reaches the maximum focusing distance $i_{\text{max}}$. Figure 2.8d shows the waist of the input laser beam moves further away from the lens, the waist of the beam propagated through the lens starts moving back towards the lens. We note that the waist size of the beam output from the lens changes in Fig. 2.8a–d, and that the waist positions of the beam output from the lens are the same in Fig. 2.8b and d, but the waist sizes are different.

When the waist of the input laser beam moves from the collimated position shown in Fig. 2.8a towards the lens by a small distance, the beam propagated through the lens is still divergent with the imaginary beam waist appearing at the left hand side of the lens, as shown in Fig. 2.8e. Figure 2.8f shows the waist of the input laser beam moves closer to the lens to a location with $o = f - z_R/M^2$, then
Fig. 2.9 Illustration of the focusing characteristics of a laser diode beam. The black dots denote the focal points of the lens
the imaginary waist of the beam propagated through the lens reaches the minimum focusing distance \( i_{\text{min}} \). Figure 2.8g shows the waist of the input laser beam moves further closer to the lens, the imaginary waist of the beam propagated through the lens starts moving back towards the lens. Figure 2.8h shows the laser diode beam waist is located at \((olf - 1)^2 = 0.5\) with \( z_R^2/(M^2f)^2 = 0.5 \). According to Eqs. 2.18 and 2.14, we have \( i = o \) and \( w_0 = w_0 \), respectively, the beam is relayed.

In Fig. 2.8a–h, the value of \( z_R/M^2f \) is visibly smaller than 1, the situation can be categorized as collimation situation. It’s noted that the drawings in Fig. 2.8 are only for illustration purpose, they don’t have the exact proportions.

The focusing characteristics of a laser diode beam is illustrated in Fig. 2.9. We start from the situation shown in Fig. 2.9a, where the waist of the input laser beam is located at the focal plane of the lens, the beam propagated through the lens is focused with its waist located at the other focal plane of the lens. Figure 2.9b shows the waist of the input laser beam moves away from the lens to a location with \( o = f + z_R/M^2 \), the waist of the beam propagated through the lens reaches the maximum focusing distance \( i_{\text{max}} \). Figure 2.9c shows the waist of the input laser beam moves further away from the lens, the waist of the beam propagated through the lens starts moving back towards the lens. Figure 2.9d shows the input laser beam with its imaginary waist at the right hand side of the lens as shown by the dot curves (that means if there is no lens, the laser beam waist will reach there), the waist of the beam propagated through the lens also moves away from the lens focal point towards the lens. Figure 2.9e shows the imaginary waist of the input laser beam moves toward the lens from the right hand side to a position with \( o = f - z_R/M^2 = -(z_R/M^2 - f) \), the waist of the beam propagated through the lens reaches the minimum focusing distance \( i_{\text{min}} \), the negative sign of \( o \) indicates that the waist of the input laser beam is at the right hand side of the lens. Figure 2.9f shows the waist of the input laser beam moves further towards the lens, the waist of the beam propagated through the lens starts moving away from the lens back towards the lens focal point.

We note that in Fig. 2.9a–f, \( z_R/(M^2f) > 1 \), the situation can be categorized as focusing situation. The drawings in Fig. 2.9 are only for illustration purpose, they don’t have the exact proportions.

### 2.5 An Example of Collimating and Focusing a Laser Diode Beam

A numerical example can help to better understand the collimation and focusing characteristics of laser diode beams. We consider a laser diode beam with \( 1/e^2 \) intensity beam waist radius of 0.5 and 1.5 \( \mu \text{m} \) in the fast and slow axis direction, respectively, and a wavelength of 0.67 \( \mu \text{m} \). Here we assume \( M^2 = 1 \) and the beam astigmatism \( A = 0 \) for simplicity. This laser diode beam is collimated by a lens with a focal length of \( f_1 = 10 \text{ mm} \) and then focused by another lens with a focal length \( f_2 = 100 \text{ mm} \), as shown in Fig. 2.10.
From Eq. 2.10 we find the Rayleigh ranges in the fast and slow axis directions for this laser diode beam to be $z_{RF} = 1.17 \, \mu m$ and $z_{RS} = 10.55 \, \mu m$, respectively. The $i$ versus $o$ curve, and the $w_{0F'}$ and $w_{0S'}$ versus $o$ curves described by Eqs. 2.14 and 2.18 for lens 1 are plotted in Fig. 2.11a and b, respectively, where $w_{0F'}$ and $w_{0S'}$ are the waist radius in the fast and slow axis direction, respectively, of the beam propagated through the lens. It’s noted that the middle points at the horizontal and vertical axes in Fig. 2.11a and the middle point at the horizontal axis of Fig. 2.11b are the value of $f_1$. It can be seen that in the fast axis direction, the maximum focusing distance is about 43 m, in the slow axis direction the maximum focusing distance is about 4.7 m. In a collimation situation shown in Fig. 2.11a, $w_{0F'}$ and $w_{0S'}$ can be calculated by Eq. 2.18 to be $w_{0F'} = m \times 0.5 \, \mu m \approx 4.27 \, \mu m$ and $w_{0S'} = 1.42 \, \mu m$. 

**Fig. 2.10** A laser diode beam is collimated by a lens and focused by another lens. The *solid curves* and *dash curves* are for the beams in the fast and slow axis directions, respectively.

**Fig. 2.11** A laser diode beam is collimated by a lens with 10 mm focal length. *a* $i$ versus $o$ curve. *b* $w_{0F''}$ and $w_{0S''}$ versus $o$ curves. In both *a* and *b* the *solid and dash curves* are for the fast and slow axis directions, respectively.
When $o' = f$, the beam is not well collimated, $w_{0F}'$ and $w_{0S}'$ are smaller than 4.27 and 1.42 mm, respectively, as shown in Fig. 2.11b. After being collimated, the laser diode beam has larger waist radius and smaller divergence in the fast axis direction than in the slow axis. From Eq. 2.10 we can find the Rayleigh ranges for this collimated laser diode beam to be $z_{RF}' = 85.3$ m and $z_{RS}' = 9.5$ m in the fast and slow axis directions, respectively. The collimated laser diode beam is then focused by lens 2 with a focal length of $f_2 = 100$ mm.

The $i'$ versus $o'$ curve, and the $w_{0F}''$ and $w_{0S}''$ versus $o'$ curves are plotted in Fig. 2.12a and b, respectively, where $w_{0F}''$ and $w_{0S}''$ are the waist radius of the focused beam in the fast and slow axis direction, respectively. As we can see in Fig. 2.12a that the maximum and minimum focusing distances in the slow axis direction is about 100.53 and 99.47 mm, respectively. The maximum and minimum focusing distances in the fast axis direction is about 100.06 and 99.94 mm, respectively. The waist radius of the beam focused by lens 2 can be calculated using Eq. 2.18 as $w_{0F}'' = m \times 4.27$ mm $\approx 0.018$ mm and $w_{0S}'' = m \times 1.42$ mm $\approx 0.072$ mm. It’s noted that the middle points at the horizontal and vertical axes in Fig. 2.12a and the middle point at the horizontal axis of Fig. 2.12b are the value of $f_2$.

From Fig. 2.12a we can see one interesting phenomenon; if $o' \neq f_2$, the waists of the focused beam in the fast and slow axis can appear at different positions, although the laser diode considered in this case has no astigmatism. We call this phenomenon “ghost astigmatism”. For the setup shown in Fig. 2.10, the maximum ghost astigmatism $A_{max}$ is about 0.52 mm that appears at $o' = f_2 + z_{RS}' = 9.6$ m, as shown in Fig. 2.12a. From Fig. 2.12b we can see another interesting phenomenon;
when \( o' = f_2 \), the beam is best focused with the focused spot sizes being the largest in both fast and slow axis directions. The best focused beam has the largest focused spot, that sounds weird, but it is the characteristics of Gaussian laser beams. So, if we want a smallest possible focused spot, we can defocus the beam. For example, when the beam in the slow axis direction is focused at the maximum focusing distance with \( o' = f_2 + z_{RS} = 9.6 \text{ m} \), the focused spot radius is 0.018 mm, that is four times smaller than \( w_{0S''} = 0.072 \text{ mm} \).

2.6 Raytracing Technique

Thin lens equation is an approximation that is only appropriate to simulate the propagation of a laser beam through a thin lens. In some applications, more accurate step by step raytracing is required.

When a ray propagates through an air/glass interface, the ray will be refracted according to Snell law

\[
n \sin \theta = n' \sin \theta'
\]

(2.19)

where \( n \) and \( n' \) are the air and glass refractive indices, respectively, \( \theta \) and \( \theta' \) are the ray incident and exit angles with respect to the local normal of the lens surface, respectively. In a design case, the laser beam parameters, the lens surface profile, and the distance between the laser beam waist and the lens surface vertex are known, we can determine the position of the point on the lens surface where the 1/e² intensity contour of the incident laser beam contacts the lens surface, as shown in Fig. 2.13, that means \( z, w(z) \) and \( R(z) \) can be found. A conceived incident ray perpendicularly passing through the wavefront at this contacting point can be determined, as shown in Fig. 2.13. Applying Snell law to this ray, we can determine a conceived ray refracted by the lens surface. The conceived refracted ray has an angle \( \theta'' \) between it and the local normal of the lens surface, as shown in Fig. 2.13. Since the equation describes this local normal can be found using the lens surface profile, \( \theta \) can be found too and \( \theta' \) can be calculated by Eq. 2.19. The angle \( \phi \) between the conceived refracted ray and the optical axis can be found. Then we can find the radius of the wavefront perpendicular to the refracted ray by

\[
R'(z) = \frac{w'(z)}{\tan \phi}
\]

(2.20)

where \( w'(z) = w(z) \) is the refracted beam size at this point, as shown in Fig. 2.13. Now we find \( w'(z) \) and \( R'(z) \), we can calculate the waist radius \( w'_0 \) of the refracted beam and the distance \( z' \) along the optical axis from the contacting point to the waist of the refracted beam by modifying Eqs. 2.1 and 2.2 to

2 Almost any optics text books discusses Snell law
This raytracing technique is more time consuming, but provides accurate results.

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