Optimal Guidance and Control for Space Robot Operation

Takuro Kobayashi and Shinichi Tsuda

Abstract This paper deals with a control of space robot for capturing moving targets. It would be desirable to use the space robot to repair the failed satellite and to remove space debris since the work load to do these tasks by astronauts will be extremely heavy. Extensive studies have been done for the control of space robot. Unfortunately these studies have not incorporated the orbital motion which is essential for space robot. Coplanar motion between space robot and target is discussed in this study. Suboptimal control, which uses piecewise optimized feedback gain by optimal tracking control method, is applied to chase the target. Also Hill's equation was applied to the relative orbital equations of motions. Based on the above formulation dynamical simulation was conducted to demonstrate the validity of our approach.

Keywords Hill's equation · Motion control · Optimal control · Space robot

1 Introduction

Recently space robots have been extensively used for the space activities like the International Space Station. Moreover Japanese government announced future space programs which will utilize the robot technology like for Moon exploration.

A lot of studies have also been made for more advanced space robots like a robot satellite, in which the robot will be operated in an autonomous manner. These robots are expected to play an important role, like a space debris capture and retrieval. However these studies have not given any consideration about the effect of orbital motion, which generates the relative motion between the space robot and moving target.

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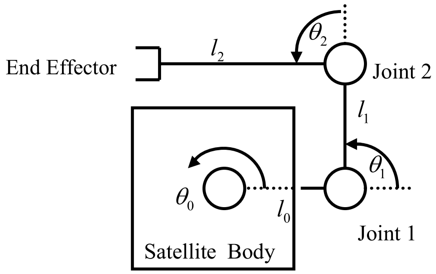


Fig. 1 Model of space robot with end effector

Based on the above consideration the control of space robot, like a satellite robot, is discussed, in which its hand, an end effector of the space robot, tracks the moving target [1]. Firstly the kinematics of space robot is formulated using the generalized Jacobian [2]. And then the control method to correct the position error between the end effector and the target with feedback is defined. Dynamical equation was also derived to obtain the relation between joint variables and applied torque. In this development a linearized approximation, in which the centrifugal and Coriolis terms were neglected, was done by assuming the small deviation of joint variables and their velocity. The tracking control is formulated by applying optimal control theory, LQR [3]. Piecewise optimization for time-varying state-space equation is applied in this paper. This is a practical solution for suboptimal control and as shown in the simulation result it looks working well. Hill's equation was introduced to deal with the relative motion between the space robot and the target, which are assumed to be on a circular orbit without loss of generality.

2 Model of Space Robot

Figure 1 shows the model of space robot with an end effector.

Two dimensional motion is assumed here, therefore, θ_0 gives the satellite attitude angle and two joint angles are defined to express the robot arm posture.

The relation between the position of the end effector and joint variables is given by the following general equation:

$$\mathbf{r} = \mathbf{f}(\mathbf{q_M}) \tag{1}$$

where $\mathbf{r}$ and $\mathbf{q_M}$ are the end effector position andjoint variable vectors, respectively. The velocity relation between end effector and the joint variables is as following:

$$\dot{\mathbf{r}} = \mathbf{J_M}\dot{\mathbf{q}}_\mathbf{M} \tag{2}$$

Since the base of the space robot is not fixed, the satellite attitude is also changed by the arm operation. In this respect the generalized Jacobian was proposed to analytically deal with this issue. Momentum and angular momentum conservations give us the relations between end effector and joint variable velocity as below:

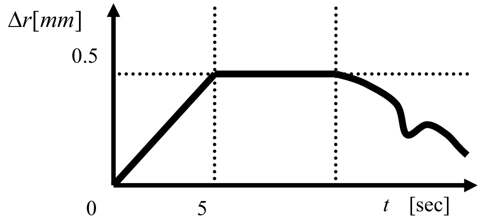


Fig. 2 Profile of Δr

$$\dot{\mathbf{r}} = \mathbf{J}^{*}\dot{\mathbf{q}}_{\mathbf{M}} \tag{3}$$

$$\mathbf{J}^{*} = \hat{\mathbf{J}}_{\mathbf{M}} - \hat{\mathbf{J}}_{\mathbf{S}}\hat{\mathbf{I}}_{\mathbf{S}}^{-1}\hat{\mathbf{I}}_{\mathbf{M}} \tag{4}$$

$$\hat{\mathbf{I}}_{\mathbf{S}}\dot{\mathbf{q}}_{\mathbf{S}} + \hat{\mathbf{I}}_{\mathbf{M}}\dot{\mathbf{q}}_{\mathbf{M}} = 0 \tag{5}$$

where $\hat{\mathbf{J}}_{\mathbf{M}}$: Jacobian matrix of the robot arm, $\hat{\mathbf{J}}_{\mathbf{S}}$: Jacobian matrix of the satellite, $\hat{\mathbf{I}}_{\mathbf{M}}$: robot arm inertia tensor and $\hat{\mathbf{I}}_{\mathbf{S}}$: satellite inertia tensor.

From (4) and (5) we obtain the following relationships:

$$\mathbf{\Delta q_M} = \mathbf{J}^{*-1}\mathbf{\Delta r} \tag{6}$$

$$\mathbf{\Delta q_S} = -\hat{\mathbf{I}}_{\mathbf{S}}^{-1}(\hat{\mathbf{I}}_{\mathbf{M}}\mathbf{\Delta q_M}) \tag{7}$$

Thus when we define $\mathbf{\Delta r}$, we can obtain $\mathbf{\Delta q_M}$ and $\mathbf{\Delta q_S}$.

$\mathbf{\Delta r}$ will be given at a small interval. This idea is shown in Fig. 2 as an example, in which

$$\mathbf{\Delta r} = \frac{\mathbf{r_t} - \mathbf{r_d}}{|\mathbf{r_t} - \mathbf{r_d}|}\Delta r \tag{8}$$

$$\Delta r = \begin{cases} 0.1 \cdot t & 0 \le t \le 5 \\ 0.5 & 5 < t \\ |\mathbf{r_t} - \mathbf{r_d}| & |\mathbf{r_t} - \mathbf{r_d}| < 0.5 \end{cases} \tag{9}$$

in order to eliminate the error of the end effector (6) was modified as follows:

$$\mathbf{\Delta q_M} = \mathbf{J}^{*-1}\left[\Delta\mathbf{r} + \lambda(\mathbf{r_d} - \mathbf{r})\right] \tag{10}$$

where λ, $\mathbf{r_d}$ and $\mathbf{r}$ are feedback gain, goal position and current position, respectively.

3 Dynamics of Space Robot

The dynamics of the robot is generally expressed in the following form:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} \tag{11}$$

where $\mathbf{M}(\mathbf{q})$: inertia matrix, $\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}})$: matrix derived from centrifugal and Corioli forces, and $\boldsymbol{\tau}$: applied torque. The linearization was made in the vicinity of the current posture of the robot and the second term in left hand side of the above equation was neglected by assuming both the small angle deviation and very slow joint velocity. This will be shown in the simulation result. Thus we obtain a linearized dynamical equation as bellow:

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q})^{-1}\boldsymbol{\tau} \tag{12}$$

The specific expression of this equation is given in Appendix 1.

4 Optimal Tracking Control

Linear optimal control, LQR, is applied to the control of the space robot to track the target. Generally state space representation for optimal tracking control is as follows. The state equation and observation equation are given by the vector equations as below:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned} \tag{13}$$

and when a desired trajectory $\mathbf{q_d}(t)$ is defined in the time interval between t_0 and t_f, the tracking error is given by the following equation:

$$\mathbf{e}(t) = \mathbf{q_d}(t) - \mathbf{q}(t) \tag{14}$$

where $\mathbf{q_d}(t) \equiv \mathbf{q_d}(n) = \mathbf{q_d}(n-1) + [\boldsymbol{\Delta}\mathbf{q_S}(t)\ \ \boldsymbol{\Delta}\mathbf{q_M}(t)]^T$ for the n-th step. And the performance index is defined in the quadratic formula:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left(\mathbf{e^T Q e} + \mathbf{u^T R u}\right) dt \tag{15}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are a positive semi-definite symmetric and a positive definite symmetric matrices, respectively.

The solution x^0 of the above optimal control problem is resolved as the TPBVP (Two Point Boundary Value Problem) expressed by the following differential equation:

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \\ &\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}} \\ -\mathbf{C}^{\mathbf{T}}\mathbf{Q}\mathbf{C} & -\mathbf{A}^{\mathbf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{C}^{\mathbf{T}}\mathbf{Q} \end{bmatrix} \mathbf{q_d} \end{aligned} \tag{16}$$

where $\mathbf{p}(t)$ satisfies the relation:

$$\mathbf{u^0}(t) = -\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{p}(t) \tag{17}$$

Let us assume that $\mathbf{p}(t)$ is as below:

$$\mathbf{p}(t) = \mathbf{K}(t)\mathbf{x}(t) + \mathbf{p_1}(t) \tag{18}$$

where $\mathbf{K}(t)$ is a symmetric and positive definite matrix with the same dimension as the vector $\mathbf{x}(t)$. Replacing $\mathbf{p}(t)$ by $\mathbf{p_1}(t)$, we obtain the following state equation

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{F} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}} \\ -\mathbf{G} & -\mathbf{F}^{\mathbf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{p}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{C}^{\mathbf{T}}\mathbf{Q} \end{bmatrix} \mathbf{q}_{\mathbf{d}} \tag{19}$$

$$\mathbf{F} = \mathbf{A} - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{K} \tag{20}$$

$$-\mathbf{G} = \mathbf{K}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{K} - \mathbf{K}\mathbf{A} - \mathbf{A}^{\mathbf{T}}\mathbf{K} - \mathbf{C}^{\mathbf{T}}\mathbf{Q}\mathbf{C} - \dot{\mathbf{K}} \tag{21}$$

Let us substitute the followings into (19) and (21):

$$\mathbf{G} = \mathbf{0} \quad \text{and} \quad \dot{\mathbf{K}} = \mathbf{0}$$

Then we have the state equation and algebraic Riccati equation as below.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{p}}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{F} & -\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}} \\ -\mathbf{G} & -\mathbf{F}^{\mathbf{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{p}_1 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{C}^{\mathbf{T}}\mathbf{Q} \end{bmatrix} \mathbf{q}_{\mathbf{d}} \tag{22}$$

$$\mathbf{K}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{K} - \mathbf{K}\mathbf{A} - \mathbf{A}^{\mathbf{T}}\mathbf{K} - \mathbf{C}^{\mathbf{T}}\mathbf{Q}\mathbf{C} = \mathbf{0} \tag{23}$$

We apply a feedback gain **K**, which is given by the steady state solution, to our tracking control.

5 Relative Motion by Orbital Motion

The relative motion is described by Hill's equation with the reference coordinate system of the target. X axis is directed toward the target movement and Y axis is normal to the orbital plane. Z axis is reversely directed toward the earth center.

Then we obtain the following equations:

$$\begin{aligned} \ddot{x} &= -g_t \frac{x}{r_t} - 2\omega\dot{z} - \dot{\omega}z + \omega^2 x + A_{\mathbf{x}} \\ \ddot{y} &= -g_t \frac{y}{r_t} + A_{\mathbf{y}} \\ \ddot{z} &= 2g_t \frac{z}{r_t} - 2\omega\dot{x} - \dot{\omega}x + \omega^2 z + A_{\mathbf{z}} \end{aligned} \tag{24}$$

where g_t is gravity acceleration, and **A** is external force, for example, by thrusters. As assumed in the previous section the target is on a circular orbit and external forces are not acting, then $\dot{\omega} = 0$ and $\omega = \sqrt{g_t/r_t}$.

$$\begin{aligned} \ddot{x} + 2\omega\dot{z} &= 0 \\ \ddot{y} + \omega^2 y &= 0 \\ \ddot{z} - 2\omega\dot{x} - 3\omega^2 z &= 0 \end{aligned} \tag{25}$$

The solution of these equations is shown in Appendix 2. In order to avoid the collision between the satellite and the target, the satellite robot arm will be approaching from the line of the target movement. And the coordinate system is converted to the space robot reference frame with X axis directed toward the reverse of the target movement and Y axis directed toward the reverse of the center of the earth.

Table 1 Satellite and robot characteristics

	Body	Link 1	Link 2
Mass [kg]	1500	30	50
Link length [m]	1.5	1.5	2.5
Moment of inertia [kg m^2]	1000	22.5	26
Initial angle [deg]	−90	90	90

Table 2 Target orbital properties

	Target
Center of rotation: X direction [m]	4.8
Center of rotation: Y direction [m]	1
Rotational radius [m]	1.5
Altitude [km]	1000
Angular velocity [deg/sec]	1
Relative velocity: X direction [m/sec]	−0.001
Relative velocity: Y direction [m/sec]	−0.0001

6 Simulation Results

The simulation, in which the end effector is tracking the target during 90 seconds, was conducted.

We made the following assumptions for simulation study:

(1) The target is in reachable zone by the end effector of the robot during the period, and as noted above,
(2) Only coplanar motions are allowed for the robot and target.

Parameters used in the simulation are shown Tables 1 and 2.

And the following parameters are assumed in the simulation:

$$\lambda = 0.05$$

and

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Figure 3 illustrates the initial geometrical relation between the robot satellite and moving target.

Figures 4 and 5 show the end effector has reached the target with an error less than 0.001 m at 45.6 second. The maximum velocity of the joints was smaller than 0.05 rad/sec which is compatible with other space robot arm.

Figure 6 illustrates the detailed relative distance between end effector and target from 45 second to 90 second. This chart shows good tracking performance.

Fig. 3 Initial geometry between target and satellite

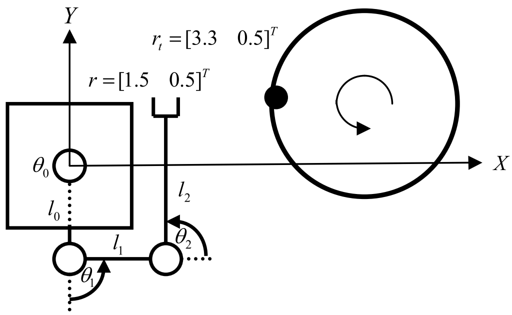


Fig. 4 Positions of target and robot at 0, 30, 60, 90 seconds

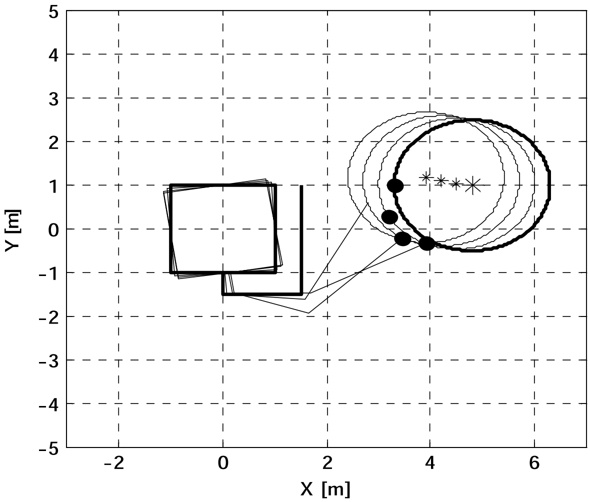
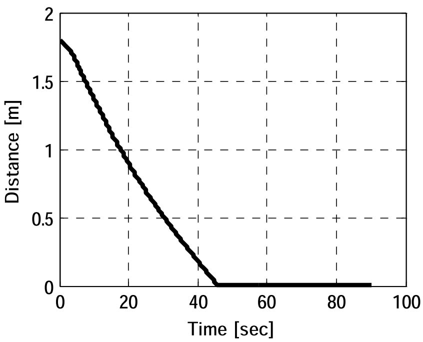


Fig. 5 Relative distance between end effector and target



The histories of joint angles and their velocity, which were obtained from the kinematic equation (10), are given in Figs. 7 and 8. These are trajectories followed by tracking control.

Fig. 6 Relative distance between end effector and target

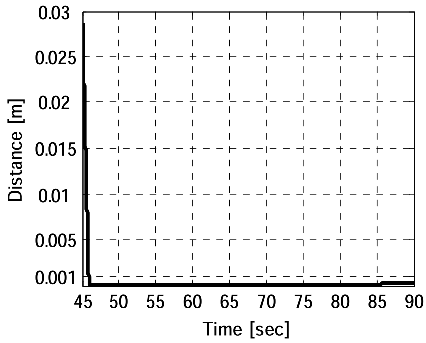


Fig. 7 History of joint angles by kinematics

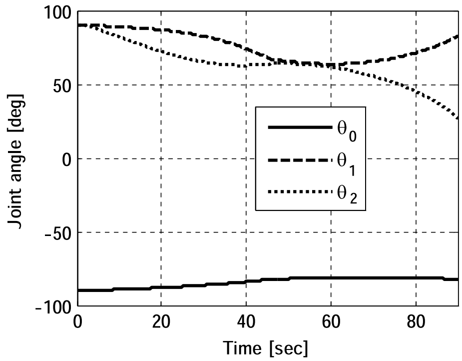
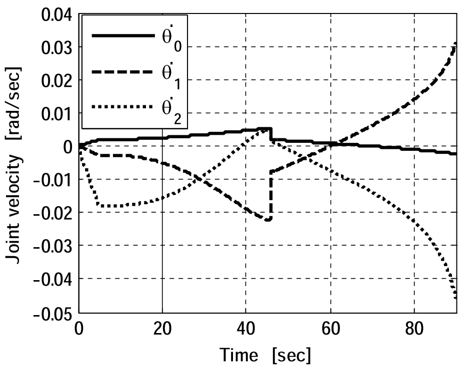


Fig. 8 History of joint angle velocity by kinematics



Figures 9 and 10 illustrate the result of optimal tracking control, in which good agreement with the desired trajectories given by Figs. 9 and 10 is observed. And small angular velocities of joints are preserved.

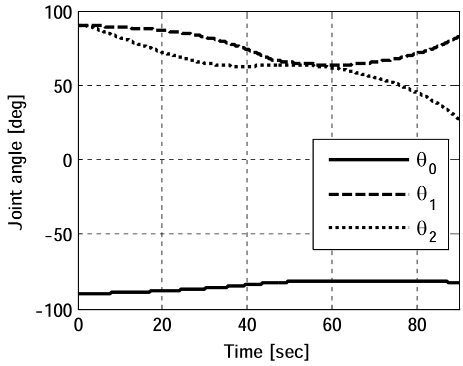


Fig. 9 Joint angle history by optimal tracking control

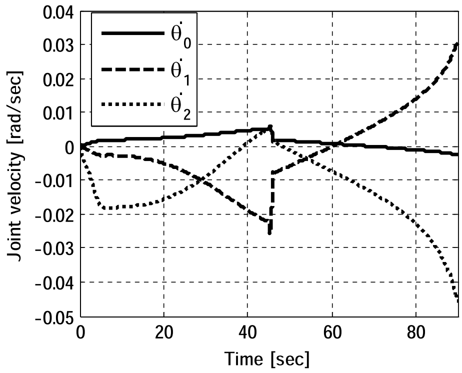


Fig. 10 Joint angle velocities by optimal tracking control

7 Conclusions

The space robot tracking control to capture a target was discussed. In general for orbiting targets like the space debris we have to consider two motions, rotation about their center of mass and orbital motion, at the same time. Especially targets like space debris and failed satellites are noncooperative for capturing them by space robot so that tracking control is inevitable.

Kinematics and dynamics are formulated, including orbital motion which has not been discussed yet. And the optimal tracking control method was applied by using piecewise optimized feedback gains.

Simulation result shows satisfactory performance of the control system by our approach.

Appendix 1: State Space Equation of Joint Variables for Space Robot

$$\frac{d}{dt}\begin{bmatrix}\theta_0\\ \dot{\theta}_0\\ \theta_1\\ \dot{\theta}_1\\ \theta_2\\ \dot{\theta}_2\end{bmatrix}=\begin{bmatrix}0&1&0&0&0&0\\0&0&0&0&0&0\\0&0&0&1&0&0\\0&0&0&0&0&0\\0&0&0&0&0&1\\0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}\theta_0\\ \dot{\theta}_0\\ \theta_1\\ \dot{\theta}_1\\ \theta_2\\ \dot{\theta}_2\end{bmatrix}+\begin{bmatrix}0&0&0\\ M_{11}(\mathbf{q})^{-1}&M_{12}(\mathbf{q})^{-1}&M_{13}(\mathbf{q})^{-1}\\0&0&0\\ M_{21}(\mathbf{q})^{-1}&M_{22}(\mathbf{q})^{-1}&M_{23}(\mathbf{q})^{-1}\\0&0&0\\ M_{31}(\mathbf{q})^{-1}&M_{32}(\mathbf{q})^{-1}&M_{33}(\mathbf{q})^{-1}\end{bmatrix}\begin{bmatrix}\tau_0\\ \tau_1\\ \tau_2\end{bmatrix}$$

Appendix 2: Solutions of Hill's Equations

$$x=x_0-2\frac{\dot{z}_0}{\omega}(1-\cos\omega t)+\left(4\frac{\dot{x}_0}{\omega}+6z_0\right)\sin\omega t-(6\omega z_0+3\dot{x}_0)t$$

$$y=\frac{\dot{y}_0}{\omega}\sin\omega t+y_0\cos\omega t$$

$$z=4z_0+2\frac{\dot{x}_0}{\omega}-\left(2\frac{\dot{x}_0}{\omega}+3z_0\right)\cos\omega t+\dot{z}_0\cos\omega t$$

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