Simulation plays a major role in computer aided design of integrated circuits (ICs). Mathematical models describe the dynamical processes and interactions of electrical devices. Verification of a circuit’s behavior by means of solving these model equations in time and frequency domain is a mandatory task in the design process. The structures’ sizes are decreasing, the packing density increases and so do the driving frequencies. This requires to use refined models and to take into account secondary, parasitic effects. The very high dimensional problems that emerge in this way may be solvable with the help of computer algebra in an unreasonable amount of time only. Clearly, this conflicts with the short time-to-market demands in industry.

Model order reduction (MOR) presents a way out of this dilemma. Redundancies are resolved, less relevant quantities are replaced by the most significant ones. In this way, the problem’s complexity is reduced, keeping the main characteristics. Solving lower dimensional problems one can get statements on the circuit’s performance more quickly.

This book surveys the state of the art in the challenging research field of MOR for integrated circuits (ICs), and also addresses future research directions in this area. Special emphasis is put on aspects stemming from miniaturization to nanoscale. Contributions cover complexity reduction using e.g., balanced truncation, Padé-approximation/Krylov-techniques, or POD approaches. For semiconductor applications, a focus is on generalizing current techniques to differential algebraic equations, on including design parameters, on preserving stability and other physical properties as well as model structures, and on including nonlinearities by means of piecewise linearizations along solution trajectories (TPWL) and interpolation techniques for nonlinear parts. Furthermore, the influence of interconnect and power grids on the physical properties of the device is considered, and also top-down system design approaches in which detailed block descriptions are combined with behavioral models. MOR faces the same requests as those appearing in Response Surface Modeling (RSM) techniques for robust design, with emphasis on parameterization, parameter screening, and nonlinearity. Further topics consider MOR and the combination of approaches from optimization and
statistics, and the inclusion of Partial Differential Equation (PDE) models with emphasis on MOR for the resulting partial differential algebraic systems.

It is a proven fact that semiconductor industries see MOR as a key tool for simulations of nano-technology designs, with high demand of well educated experts in the field. This implies that timely education is needed. The current number of books in this area is very limited, so that this volume helps to fill a gap in providing the state of the art material, and to stimulate further research in the area of MOR for ICs. Furthermore, the methods which currently are being developed have also relevance in other application areas such as mechanical multibody systems, mechatronics, micro- and nano-technology, chemical and biological processes, etc.

Furthermore, this book reflects and documents the vivid interaction between three active research projects in this area, namely the EU-Marie Curie Action ToK project O-MOORE-NICE\(^1\) (members in Belgium, The Netherlands and Germany), the EU-Marie Curie Action RTN-project COMSON\(^2\) (members in The Netherlands, Italy, Germany, and Romania), and the German federal project *System Reduction for Nanoscale IC Design (SyreNe)*.\(^3\)

The material collected in this book reflects the contributions to the workshop “Model Reduction in Circuit Simulation”, held at University of Hamburg, October 30–31, 2008. The workshop was jointly organized by the abovementioned projects SyreNe and O-Moore-Nice!, supported by the Federal Ministry of Education and Research (BMBF) of Germany and the EU FP6 programme “Marie Curie Industry Host Fellowships”, respectively.

Part I of the book consists of overview papers of tutorial value corresponding to the invited presentations. Chapter 1 by Wilhelmus H. A. Schilders describes the need for novel MOR techniques in the electronics industry and thus provides a good motivation for the the technical papers to follow. In Chap. 2, Roland Freund derives the mathematical equations describing linear RCL circuits (which are the main tool for modeling interconnect and other parasitic effects in IC design) and presents Páde and Páde-type approximation methods respecting the structure of typical mathematical RCL models. A different approach to MOR is taken by Tatjana Stykel in Chap. 3 where the application of balancing-related methods from systems theory to circuit equations is surveyed. Special emphasis is put there on the exploitation of the topological structure implied by the electrical network. In Chap. 4, Sanda Lefteriu and Athanasios C. Antoulas pave the way to macro-modeling using measured data rather than starting from model equations. The main principle employed here is (tangential) interpolation using the Loewner matrix framework.

In Part II, contributed papers are collected. Chapters 5 and 7 provide different MOR methods and techniques to serve parameterized problems. In Chap. 5,

\(^1\) [www.tu-chemnitz.de/mathematik/industrie_technik/projekte/omoorenice/](http://www.tu-chemnitz.de/mathematik/industrie_technik/projekte/omoorenice/).

\(^2\) [www.comson.org](http://www.comson.org) for details.

\(^3\) See [www.syrene.org](http://www.syrene.org) for details.
surrogate models (or RSMs) based on table models and neural network models are built that are valid for a range of parameters. These are applied to solve multi-input, multi-output (MIMO) problems. In Chap. 7, parameterized response functions are using expansions into bivariate rational basis functions. The coefficients are determined by optimizing a special cost function.

The Chaps. 6, 13 and 16 consider techniques to overcome problems associated with having massive numbers of inputs and outputs (or terminals, ports). In Chap. 6, recycling of Krylov sub spaces is applied to sequentially solve problems with changing right-hand sides occurring in an approach based on superposition of single input systems. Several recycling variants are studied for the Generalized Conjugate Residual method as basic linear solver. In Chap. 13, the sources are approximated by a finite expansion in basis functions. This allows for replacing current sources by a reduced number of controlled current sources. The network with the replaced sources remains passive and reciprocal. In Chap. 16, the situation where the large number of ports originates from validation problems that include many parasitics is considered. Extended SVD-based MOR (ESVDMOR) uses SVD on the input and output super matrices of the moments of the transfer function. A variant uses truncated SVD. Both approaches lead a system with virtual input and output operators and reduced number of terminals. For both approaches, stability, passivity, and reciprocity can be preserved under reasonable assumptions.

The Chaps. 8 and 9 address two different problems with circuits involving inductors. In Chap. 8, the transfer function at a node that is a candidate to be eliminated within the TICER method is first checked for the contribution coming from an attached inductor element. When passivity is preserved, RC couplings replace the first order inductor effect. After this the TICER step can be completed. In Chap. 9, the phase shift effect in a periodic steady state solution of an oscillator is determined by defining appropriate perturbation vectors covering the effects from other parts of the circuit. This allows to efficiently solve a balun circuit. The simulation time is further reduced by applying MOR (balanced truncation) to the RC models for the transmission lines coupling the oscillators.

In Chap. 10, POD-based model order reduction for semiconductors in electrical networks is investigated, where the semiconductor is modeled by the drift diffusion equations. The greedy approach is used to construct POD models which are valid over certain parameter ranges (frequencies). Numerical investigations for the reduction of a 4-diode rectifier network indicate that POD surrogate models depend on the position of the semiconductor in the network.

Chapter 11 deals with periodic linear systems as they arise in harmonic balance. It extends the notion of causal and non causal reachability and observability Gramians to discrete sequences of periodic descriptor systems (for instance arising after linearising around a periodic steady state solution). Periodic projected Lyapunov equations can be defined as well as (non-)causal Hankel singular values. This framework allows to extend MOR by balanced truncation to this class of problems.
In Chap. 12, synthesis of reduced models is considered: how to formulate those as netlist using RLC components and to reduce the amount of controlled sources. Foster synthesis only applies to partial fraction expansion of the transfer function. It is shown how MOR techniques that preserve the main structure of the equations and the input-output connectivity, formulated for problems in impedance form (current in, voltage out), can also be applied to problems in admittance form (voltage in, current out). This allows studying a second method, unstamping via RLCSYN. After synthesizing the reduced impedance model, this can be converted to an admittance model.


In Chap. 15, symbolic reduction and simplification of expressions in nonlinear circuit equations is combined with reduction of blocks by numerical MOR techniques. The method relies on a hierarchical structure of the circuit.

In Chap. 17, MOR techniques like POD and Trajectory Piece-Wise Linear (TPWL) for nonlinear problems are considered, together with variants (like adapted POD, DEIM). The effect of the chosen variant on the quality of the approximations is considered as well as the efficiency and the robustness with respect to variations of the input sources.

In Chap. 18 an approach to nonlinear balancing and MOR is considered. For linear systems the notion of sliding interval balancing (SIB) is introduced, and it is shown that subsystems conserve stability. The truncated SIB realization bounds the optimal approximation in an input-output sense. Nonlinear balancing is approached by applying SIB on the linear variational system.

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