Contents

Part I Foundations

1 Dual Tableau for Classical First-Order Logic ........................................ 3
  1.1 Introduction .................................................................................. 3
  1.2 Classical First-Order Logic with Identity .................................... 4
  1.3 Rasiowa–Sikorski Proof System for Classical First-Order Logic with Identity ...................................................... 5
  1.4 Tableau System for Classical First-Order Logic with Identity ...... 12
  1.5 Quasi Proof Trees .................................................................... 14
  1.6 Duality ....................................................................................... 17
  1.7 Transformation of Proofs ............................................................ 19
  1.8 Discussion of Various Rules for Identity .................................... 19
  1.9 Dual Tableaux and Hilbert-Style Systems .................................. 22
  1.10 Dual Tableaux and Gentzen-Style Systems .................................. 24
  1.11 Dual Tableaux and Dual Resolution ........................................... 27

2 Dual Tableaux for Logics of Classical Algebras of Binary Relations ... 33
  2.1 Introduction .................................................................................. 33
  2.2 Algebras of Binary Relations ...................................................... 34
  2.3 Logics of Binary Relations ............................................................ 36
  2.4 Relational Dual Tableaux .............................................................. 38
  2.5 A Basic Relational Logic ............................................................... 39
  2.6 A Method of Proving Soundness and Completeness of Relational Dual Tableaux .................................................. 43
  2.7 Relational Logic with Relations 1 and 1’ ..................................... 45
  2.8 Discussion of Various Rules for Relation 1’ ................................ 50
  2.9 Full Relation Algebras and Relational Logics .............................. 54
  2.10 An Example of a Relational Dual Tableau Proof ......................... 55
  2.11 Relational Entailment ................................................................. 61
  2.12 Decision Procedures for Some Relational Logics ....................... 62
3 Theories of Point Relations and Relational Model Checking 69
  3.1 Introduction .......................................................... 69
  3.2 Relational Logics with Point Relations Introduced with Axioms .. 70
  3.3 Relational Logics with Point Relations Introduced with Definitions .................................................... 72
  3.4 Model Checking in Relational Logics ............................ 75
  3.5 Verification of Satisfaction in Relational Logics ............ 80

Part II Reasoning in Logics of Non-classical Algebras of Relations

4 Dual Tableaux for Peirce Algebras ......................................... 85
  4.1 Introduction ........................................................... 85
  4.2 Peirce Algebras ....................................................... 86
  4.3 Peirce Logic ............................................................ 87
  4.4 Dual Tableau for Peirce Logic ........................................ 88
  4.5 Entailment, Model Checking, and Satisfaction in Peirce Logic ... 93
  4.6 Peirce Algebras and Terminological Languages ................. 99

5 Dual Tableaux for Fork Algebras ...........................................105
  5.1 Introduction ........................................................... 105
  5.2 Fork Algebras .......................................................... 106
  5.3 Fork Logic .............................................................. 108
  5.4 Dual Tableau for Fork Logic ........................................ 110
  5.5 Relational Interpretation of First-Order Theories ............ 116

6 Dual Tableaux for Relational Databases ...................................121
  6.1 Introduction ........................................................... 121
  6.2 The Calculus of Typed Relations ....................................122
  6.3 A Logic of Typed Relations ..........................................125
  6.4 Dual Tableau for the Logic of Typed Relations ............... 127
  6.5 Relational Representation of Database Dependencies ........ 132
  6.6 Dual Tableau for Database Dependencies ....................... 135

Part III Relational Reasoning in Traditional Non-classical Logics

7 Dual Tableaux for Classical Modal Logics .................................143
  7.1 Introduction ........................................................... 143
  7.2 Classical Propositional Logic ........................................144
  7.3 Propositional Modal Logics ..........................................144
  7.4 Relational Formalization of Modal Logics ........................146
  7.5 Dual Tableaux for Standard Modal Logics ........................151
  7.6 Entailment in Modal Logics .........................................153
  7.7 Model Checking in Modal Logics ..................................156
  7.8 Verification of Satisfaction in Modal Logics ................. 157
8 Dual Tableaux for Some Logics Based on Intuitionism ..........161
  8.1 Introduction ..........................................................161
  8.2 Relational Formalization of Intuitionistic Logic ............162
  8.3 Relational Formalization of Minimal Intuitionistic Logic ....167
  8.4 Relational Formalization of Some Intermediate Logics ........171
  8.5 Relational Formalization of a Logic for Hardware Verification ..174

9 Dual Tableaux for Relevant Logics .....................................177
  9.1 Introduction ...........................................................177
  9.2 Relevant Logics .......................................................178
  9.3 Translation of Relevant Logics into Relational Logics ........179
  9.4 Relational Dual Tableau for Logic RLV .........................183
  9.5 Relational Dual Tableaux for Axiomatic Extensions of Logic RLV ..........................................................189

10 Dual Tableaux for Many-Valued Logics ................................195
  10.1 Introduction ..........................................................195
  10.2 Finitely Many-Valued Logics ......................................196
  10.3 Relational Formalization of Finitely Many-Valued Logics ......199
  10.4 Dual Tableaux for Finitely Many-Valued Logics ...............204
  10.5 Three-Valued Logics ..................................................208

Part IV Relational Reasoning in Logics of Information and Data Analysis

11 Dual Tableaux for Information Logics of Plain Frames ..........217
  11.1 Introduction ..........................................................217
  11.2 Information Systems ...............................................218
  11.3 Information Logics NIL and IL ...................................223
  11.4 Relational Formalization of Logics NIL and IL ...............225
  11.5 Information Logic CI and Its Relational Formalization .......231

12 Dual Tableaux for Information Logics of Relative Frames .........237
  12.1 Introduction ..........................................................237
  12.2 Relative Frames .....................................................238
  12.3 Relational Formalizations of the Logics of Strong and Weak Relative Frames ..............................................240
  12.4 Relational Formalization of the Logic Rare-NIL ...............245
  12.5 Relational Formalization of the Logic Rare-CI ..................247
  12.6 Relational Formalization of the Logic of Strong Complementarity Frames ..................................................249

13 Dual Tableau for Formal Concept Analysis ..........................251
  13.1 Introduction ..........................................................251
  13.2 Basic Notions of Formal Concept Analysis ......................251
  13.3 Context Logic and Its Dual Tableau .............................253
  13.4 Entailment, Model Checking, and Satisfaction in Context Logic ..257
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>Dual Tableau for a Fuzzy Logic</td>
<td>263-266</td>
</tr>
<tr>
<td>14.1</td>
<td>Introduction</td>
<td>263</td>
</tr>
<tr>
<td>14.2</td>
<td>MTL-Algebras</td>
<td>264</td>
</tr>
<tr>
<td>14.3</td>
<td>The Logic MTL</td>
<td>264</td>
</tr>
<tr>
<td>14.4</td>
<td>Relational Formalization of Logic MTL</td>
<td>266</td>
</tr>
<tr>
<td>15</td>
<td>Dual Tableaux for Logics of Order of Magnitude Reasoning</td>
<td>277-280</td>
</tr>
<tr>
<td>15.1</td>
<td>Introduction</td>
<td>277</td>
</tr>
<tr>
<td>15.2</td>
<td>A Multimodal Logic of Order of Magnitude Reasoning</td>
<td>278</td>
</tr>
<tr>
<td>15.3</td>
<td>Dual Tableau for the Logic of Order of Magnitude Reasoning</td>
<td>280</td>
</tr>
<tr>
<td>Part V</td>
<td>Relational Reasoning about Time, Space, and Action</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Dual Tableaux for Temporal Logics</td>
<td>291-325</td>
</tr>
<tr>
<td>16.1</td>
<td>Introduction</td>
<td>291</td>
</tr>
<tr>
<td>16.2</td>
<td>Basic Temporal Logic</td>
<td>292</td>
</tr>
<tr>
<td>16.3</td>
<td>Semantic Restrictions on Basic Temporal Logic</td>
<td>294</td>
</tr>
<tr>
<td>16.4</td>
<td>Temporal Logics with Since and Until</td>
<td>300</td>
</tr>
<tr>
<td>16.5</td>
<td>Standard Temporal Logics with Nominals</td>
<td>306</td>
</tr>
<tr>
<td>16.6</td>
<td>Temporal Information Logics</td>
<td>311</td>
</tr>
<tr>
<td>17</td>
<td>Dual Tableaux for Interval Temporal Logics</td>
<td>315-354</td>
</tr>
<tr>
<td>17.1</td>
<td>Introduction</td>
<td>315</td>
</tr>
<tr>
<td>17.2</td>
<td>Halpern–Shoham Logic</td>
<td>316</td>
</tr>
<tr>
<td>17.3</td>
<td>Relational Logic for Halpern–Shoham Logic</td>
<td>317</td>
</tr>
<tr>
<td>17.4</td>
<td>Translation of Halpern–Shoham Logic into a Relational Logic</td>
<td>318</td>
</tr>
<tr>
<td>17.5</td>
<td>Dual Tableau for Halpern–Shoham Logic</td>
<td>320</td>
</tr>
<tr>
<td>17.6</td>
<td>Dual Tableaux for Other Interval Temporal Logics</td>
<td>325</td>
</tr>
<tr>
<td>18</td>
<td>Dual Tableaux for Spatial Reasoning</td>
<td>329-354</td>
</tr>
<tr>
<td>18.1</td>
<td>Introduction</td>
<td>329</td>
</tr>
<tr>
<td>18.2</td>
<td>Dual Tableaux for Spatial Theories Based on a Plain Contact Relation</td>
<td>330</td>
</tr>
<tr>
<td>18.3</td>
<td>Dual Tableaux for Spatial Theories Based on a Contact Relation on a Boolean Algebra</td>
<td>339</td>
</tr>
<tr>
<td>18.4</td>
<td>Dual Tableau for Region Connection Calculus</td>
<td>348</td>
</tr>
<tr>
<td>18.5</td>
<td>Dual Tableaux for Spatial Theories of Proximity Relation</td>
<td>354</td>
</tr>
<tr>
<td>19</td>
<td>Dual Tableaux for Logics of Programs</td>
<td>359-376</td>
</tr>
<tr>
<td>19.1</td>
<td>Introduction</td>
<td>359</td>
</tr>
<tr>
<td>19.2</td>
<td>Relational Formalization of Propositional Dynamic Logic</td>
<td>360</td>
</tr>
<tr>
<td>19.3</td>
<td>Relational Formalization of Dynamic Logic with Program Specifications</td>
<td>366</td>
</tr>
<tr>
<td>19.4</td>
<td>Relational Formalization of Logics of Demonic Nondeterministic Programs</td>
<td>371</td>
</tr>
<tr>
<td>19.5</td>
<td>Relational Formalization of Event Structure Logics</td>
<td>376</td>
</tr>
</tbody>
</table>
Part VI  Beyond Relational Theories

20 Dual Tableaux for Threshold Logics ........................................385
   20.1 Introduction ...........................................................385
   20.2 Threshold Logics .....................................................385
   20.3 Dual Tableaux for Threshold Logics .............................388
   20.4 Mutual Interpretability of a Threshold Logic
      and Classical First-Order Logic .....................................393

21 Signed Dual Tableau for Gödel–Dummett Logic ......................397
   21.1 Introduction ...........................................................397
   21.2 Gödel–Dummett Logic ...............................................398
   21.3 Signed Dual Tableau Decision Procedure
      for Gödel–Dummett Logic ...........................................398

22 Dual Tableaux for First-Order Post Logics ..........................407
   22.1 Introduction ...........................................................407
   22.2 Post Algebras of Order $n$ ........................................407
   22.3 First-Order $n$-Valued Post Logic ...............................408
   22.4 Dual Tableaux for Post Logics ....................................410

23 Dual Tableau for Propositional Logic with Identity ................417
   23.1 Introduction ...........................................................417
   23.2 A Propositional Logic with Identity ..............................418
   23.3 Axiomatic Extensions of the Propositional Logic with Identity ...420
   23.4 Dual Tableau for the Propositional Logic with Identity ......424
   23.5 Dual Tableaux for Axiomatic Extensions
      of the Propositional Logic with Identity ..........................428

24 Dual Tableaux for Logics of Conditional Decisions .................433
   24.1 Introduction ...........................................................433
   24.2 Logic of Conditional Decisions and Its Dual Tableau
      Decision Procedure ....................................................434
   24.3 Algebras of Conditional Decisions .................................438
   24.4 Relational Interpretation of the Logic of Conditional Decisions ..441
   24.5 Logics of Conditional Decisions of Order $n$ and Their
      Dual Tableau Decision Procedures .................................444

Part VII  Conclusion

25 Methodological Principles of Dual Tableaux ..........................455
   25.1 Introduction ...........................................................455
   25.2 Theories Interpreted Relationally ................................456
   25.3 Relational Logics .....................................................458
   25.4 Relational Languages Versus First-Order Languages ..........460
25.5 Dual Tableaux ..........................................................461
25.6 Constraint–Rule Correspondence .................................463
25.7 Definition–Rule Correspondence ..............................468
25.8 Branch Model and Completeness Proof ......................473
25.9 Alternative Forms of Rules ........................................476
25.10 Implementations ..........................................................487
25.11 Towards Decision Procedures ................................492
25.12 Conclusion .................................................................492

References ...........................................................................495

Index ...................................................................................519
Dual Tableaux: Foundations, Methodology, Case Studies
Orłowska, E.; Golińska Pilarek, J.
2011, XVI, 523 p., Hardcover
ISBN: 978-94-007-0004-8