Chapter 2
Multi-User Communication and Interference Cancellation

As the first effort to cope with interference, we try to minimize its effect in physical layer. One of the most common cases where interference will be crucial is when the base station is receiving data from several users at the same time. In this chapter we will propose a method to cancel out interference in this case using antennas at the base station.

1 The Channel Model

We model a multi-user wireless communication system where the receiving unit is equipped with $M = J + r - 1$ receive antennas, where $r \geq 1$ is the receive antenna redundancy. There are J transmitter units and each unit j, $j = 1, 2, \ldots, J$ is equipped with N antennas.

Let $c_{t,n}(j)$ denote the transmitted symbol from the n-th antenna of user j at transmission interval t and $r_{t,m}$ be the received word at the receive antenna m at the base unit. Then, for the received signal we will have

$$r_{t,m} = \sum_{j=1}^{J} \sum_{n=1}^{N} \alpha_{n,m}(j) c_{t,n}(j) + \eta_{t,m} \tag{2.1}$$

2 Interference Cancellation Using Space-Time Block Coding

It is well-known that one can separate signals sent from J different users each equipped with N transmit antennas, with $(J - 1)N + 1$ receive antennas [1]. We can simply form a decoding matrix that is orthogonal to the space spanned by channel coefficients of the users to be eliminated:

$$\mathbf{R}_t = \mathbf{C}_t \mathbf{H} + \mathcal{N}_t \tag{2.2}$$

S.J. Kazemitabar, *Coping with Interference in Wireless Networks*,
Signals and Communication Technology,
DOI 10.1007/978-90-481-9990-7_2, © Springer Science+Business Media B.V. 2011

where

$$\begin{aligned} \mathbf{C}_t &= (\mathbf{C}_t(1), \mathbf{C}_t(2), \ldots, \mathbf{C}_t(J)) \\ \mathbf{R}_t &= (r_{t,1}, r_{t,2}, \ldots, r_{t,M}) \\ \mathcal{N}_t &= (\eta_{t,1}, \eta_{t,2}, \ldots, \eta_{t,M}) \\ \mathbf{H} &= \left(\mathbf{H}(1)^T | \mathbf{H}(2)^T | \cdots | \mathbf{H}(J)^T\right) \end{aligned} \tag{2.3}$$

with

$$\mathbf{C}_t(j) = (c_{t,1}(j), c_{t,2}(j), \ldots, c_{t,N}(j))$$

$$\mathbf{H}(j) = \begin{pmatrix} \alpha_{1,1}(j) & \alpha_{1,2}(j) & \cdots & \alpha_{1,M}(j) \\ \alpha_{2,1}(j) & \alpha_{2,2}(j) & \cdots & \alpha_{2,M}(j) \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N,1}(j) & \alpha_{N,2}(j) & \cdots & \alpha_{N,M}(j) \end{pmatrix} \tag{2.4}$$

Therefore, one can rewrite (3.4) as follows:

$$\mathbf{R}_t = \sum_{j=1}^{J} \mathbf{C}_t(j)\mathbf{H}(j) + \mathcal{N}_t \tag{2.5}$$

To decode user 1, one can simply find a zero-forcing (ZF) matrix $\mathbf{Z}$ such that

$$\begin{aligned} &\mathbf{H}(1)\mathbf{Z} \neq \mathbf{0} \\ &\mathbf{H}(j)\mathbf{Z} = \mathbf{0} \quad \text{for } j \neq 1 \end{aligned} \tag{2.6}$$

In other words, $\mathbf{Z}$ should null the space spanned by the row vectors of all $\mathbf{H}(j)$s, for $j = 2, 3, \ldots, J$. Also, it should not null at least one row vector of $\mathbf{H}(1)$. Since all the rows of $\mathbf{H}(j)$s might be linearly independent, the dimension of $\mathbf{Z}$, i.e. M, must be at least equal to the number of these rows, or $(J-1)N + 1$. Each antenna group (user) can employ a modulation scheme to benefit transmit diversity; as if it is the only group that is sending data.

One might naturally think that using some smart coding in space and/or time, may reduce the number of required receive antennas. In fact that is true; in [15] it is shown that when $N = 2$ and users are equipped with Alamouti code, i.e. Orthogonal Space-Time Block Code (OSTBC) for $N = 2$, the number of required receive antennas is reduced to J. Also, in [16], same task is accomplished for a larger group of OSTBCs when only $J = 2$ users exist. To the best of our knowledge, for users with more than 2 transmit antennas and for general number of users, such an example does not exist in the literature. In what follows we describe the algorithm in [15] except that we use ZF instead of minimum mean square error (MMSE) that requires matrix inversion. We then show why it is not possible to repeat what Naguib et al. did for higher order OSTBCs for an arbitrary number of users[1] and then offer an alternative.

Consider 2 users each transmitting an Alamouti block to a receiver unit equipped with at least 2 receive antennas. The received signal can be written in the following format:

$$\begin{pmatrix} r_{1,m} \\ r_{2,m} \end{pmatrix} = \begin{pmatrix} s_1(1) & s_2(1) \\ -s_2^*(1) & s_1^*(1) \end{pmatrix} \cdot \begin{pmatrix} \alpha_{1,m}(1) \\ \alpha_{2,m}(1) \end{pmatrix} + \begin{pmatrix} s_1(2) & s_2(2) \\ -s_2^*(2) & s_1^*(2) \end{pmatrix} \cdot \begin{pmatrix} \alpha_{1,m}(2) \\ \alpha_{2,m}(2) \end{pmatrix} + \begin{pmatrix} \eta_{1,m} \\ \eta_{2,m} \end{pmatrix} \tag{2.7}$$

The idea behind interference cancellation arises from separate decodability of each symbol; at each receive antenna we perform the decoding algorithm as if there is only one user. This user will be the one the effect of whom we want to cancel out. Then, we simply subtract the soft-decoded value of each symbol in one of the receive antennas from the rest and as a result remove the effect of that user. This procedure is presented in the following lines:

$$\begin{pmatrix} r_{1,m} \\ r_{2,m}^* \end{pmatrix} = \begin{pmatrix} \alpha_{1,m}(1) & \alpha_{2,m}(1) \\ \alpha_{2,m}^*(1) & -\alpha_{1,m}^*(1) \end{pmatrix} \cdot \begin{pmatrix} s_1(1) \\ s_2(1) \end{pmatrix} + \begin{pmatrix} \alpha_{1,m}(2) & \alpha_{2,m}(2) \\ \alpha_{2,m}^*(2) & -\alpha_{1,m}^*(2) \end{pmatrix} \cdot \begin{pmatrix} s_1(2) \\ s_2(2) \end{pmatrix} + \begin{pmatrix} \eta_{1,m} \\ \eta_{2,m}^* \end{pmatrix} \tag{2.8}$$

As one can easily check, the matrix multiplied to $(s_1(1), s_2(1))^T$ is multiple of a unitary matrix. Therefore, we can simply write

$$\begin{pmatrix} \alpha_{1,m}^*(1) & \alpha_{2,m}(1) \\ \alpha_{2,m}^*(1) & -\alpha_{1,m}(1) \end{pmatrix} \begin{pmatrix} r_{1,m} \\ r_{2,m}^* \end{pmatrix} = (|\alpha_{1,m}(1)|^2 + |\alpha_{2,m}(1)|^2) \begin{pmatrix} s_1(1) \\ s_2(1) \end{pmatrix} + \begin{pmatrix} \alpha_{1,m}^*(1) & \alpha_{2,m}(1) \\ \alpha_{2,m}^*(1) & -\alpha_{1,m}(1) \end{pmatrix} \begin{pmatrix} \alpha_{1,m}(2) & \alpha_{2,m}(2) \\ \alpha_{2,m}^*(2) & -\alpha_{1,m}^*(2) \end{pmatrix} \begin{pmatrix} s_1(2) \\ s_2(2) \end{pmatrix} + \begin{pmatrix} \eta'_{1,m} \\ \eta'_{2,m} \end{pmatrix} \tag{2.9}$$

for $m = 1, 2, \ldots, M$. $\eta'_{i,m}$ is a zero mean Gaussian random variable, with variance $\frac{2(|\alpha_{1,m}(1)|^2+|\alpha_{2,m}(1)|^2)}{SNR}$. The above analysis is a part of decoding for any orthogonal space-time block code [25, Chap. 4]. To completely eliminate the effect of user No. 1, it remains to divide (9) by $(|\alpha_{1,m}(1)|^2 + |\alpha_{2,m}(1)|^2)$ and subtract the terms

[1]When there are only 2 users, it is possible to do interference cancellation with a class of OST-BCs [16].

for $m = 1$ from that of $m = 2, \ldots, M$. The resulting terms will be

$$\left\{ \frac{1}{|\alpha_{1,m}(1)|^2 + |\alpha_{2,m}(1)|^2} \begin{pmatrix} \alpha^*_{1,m}(1) & \alpha_{2,m}(1) \\ \alpha^*_{2,m}(1) & -\alpha_{1,m}(1) \end{pmatrix} \begin{pmatrix} \alpha_{1,m}(2) & \alpha_{2,m}(2) \\ \alpha_{2,m}(2)^* & -\alpha_{1,m}(2)^* \end{pmatrix} \right.$$
$$\left. - \frac{1}{|\alpha_{1,1}(1)|^2 + |\alpha_{2,1}(1)|^2} \begin{pmatrix} \alpha^*_{1,1}(1) & \alpha_{2,1}(1) \\ \alpha^*_{2,1}(1) & -\alpha_{1,1}(1) \end{pmatrix} \begin{pmatrix} \alpha_{1,1}(2) & \alpha_{2,1}(2) \\ \alpha^*_{2,1}(2) & -\alpha^*_{1,1}(2) \end{pmatrix} \right\}$$
$$\times \begin{pmatrix} s_1(2) \\ s_2(2) \end{pmatrix} + \begin{pmatrix} \eta''_{1,m} \\ \eta''_{2,m} \end{pmatrix} \tag{2.10}$$

for $m = 2, \ldots, M$. The distribution of $\eta''_{i,m}$ is Gaussian, and its variance is equal to $\frac{2}{SNR(|\alpha_{1,m}(1)|^2+|\alpha_{2,m}(1)|^2)} + \frac{2}{SNR(|\alpha_{1,1}(1)|^2+|\alpha_{2,1}(1)|^2)}$. It can be easily shown that the overall matrix multiplied by $(s_1(2), s_2(2))^T$ above is also a multiple of a unitary matrix. Therefore, the system in (2.10) is equivalent to one that transmits an orthogonal design. We can simply apply Maximum-Likelihood (ML) decoding, which is separate for each of the symbols due to having a unitary channel matrix. Note that only $M = J = 2$ receive antennas are needed for gaining full transmit diversity, $N = 2$; using more receive antennas will result in a multiplicative receive diversity, such that the overall diversity will be $2r$, with $r = M - J + 1$.

Lemma 1 *The interference cancellation technique performed on Alamouti structure cannot be extended to higher order OSTBCs when having more than* 2 *users.*

Proof After eliminating the first user there will be K equations left each produced by eliminating one of the symbols $(s_1(1), s_2(1), \ldots, s_K(1))^T$, where K is the number of symbols in the block code. The algorithm described above requires that after elimination of the first user, the resulting equations be in the following format

$$\sum_{i=2}^{J} \mathbf{U_i} \cdot \underline{\mathbf{s_i}} \tag{2.11}$$

where $\underline{\mathbf{s_i}} = (s_1(i), s_2(i), \ldots, s_K(i))^T$ and $\mathbf{U_i}$ is a multiple of a unitary matrix. A set of equations with the form of $\mathbf{U} \cdot \underline{\mathbf{s}}$ is equivalent to usage of a $K \times N$ orthogonal block code in the transmitter. A block code sending K symbols in K time slots is rate one. However, we know that rate one complex orthogonal design is impossible for more than 2 transmit antennas [11, 24]. Therefore, after eliminating the first user, the resulting system cannot enjoy simple decoding as it did for the case of 2 transmit antennas in [15]. Therefore, users No. 2 and above cannot be cancelled out. This means that the algorithm used in [15] cannot be extended to more than 2 transmit antennas for an arbitrary number of users. □

Note that all this discussion happens when the first user is cancelled. Therefore, we might be able to perform interference cancellationinterference cancellation using

OSTBCs for the case of $J = 2$ as [16] suggests. In addition, since there exist rate one *real* orthogonal designs, we can apply the same algorithm to them as we show in the following lemmas.

Definition A generalized real orthogonal design is a $T \times N$ matrix $\mathbf{C}$ with real entries $s_1, -s_1, s_2, -s_2, \ldots, s_K, -s_K$ such that

$$\mathbf{C}^T\mathbf{C} = \kappa\left(s_1^2 + s_2^2 + \cdots + s_K^2\right)I_N \tag{2.12}$$

Lemma 2 *A generalized real orthogonal design can be written as*

$$\mathbf{C} = \sum_{k=1}^{K} s_k \mathbf{E}_k \tag{2.13}$$

where matrices $\mathbf{E}_k$ *are* $T \times N$ *real matrices and satisfy*

$$\begin{cases} \mathbf{E}_k^T\mathbf{E}_{k'} + \mathbf{E}_{k'}^T\mathbf{E}_k = 0_N & k \neq k' \\ \mathbf{E}_k^T\mathbf{E}_k = I_N & k = 1, 2, \ldots, K \end{cases} \tag{2.14}$$

Proof Refer to [25]. □

Lemma 3 *For any number of transmit antennas, with usage of real orthogonal space-time block codes, and using a real constellation, we can simply decode a group of J users given the same number of receive antennas.*

Proof Consider a real channel with channel matrix $\mathbf{H}$ and noise vector $\mathcal{N}$. Assume we have J users each equipped with N antennas and transmitting a $T \times N$ real orthogonal code $\mathbf{C}$. At each receive antenna m, for $m = 1, 2, \ldots, M$ we have

$$\begin{aligned} \mathbf{R}_m^T &= \sum_{j=1}^{J} \mathbf{H}_m^T(j) \cdot \mathbf{C}^T(j) + \mathcal{N}_m^T = \sum_{j=1}^{J} \mathbf{H}_m^T(j) \sum_{k=1}^{K} s_k(j)\mathbf{E}_k^T + \mathcal{N}_m^T \\ &= \sum_{j=1}^{J}\sum_{k=1}^{K} s_k(j)\mathbf{\Omega}_k^m(j) + \mathcal{N}_m^T \\ &= \sum_{j=1}^{J} (s_1(j), s_2(j), \ldots, s_K(j)) \cdot \mathbf{\Omega}_m(j) + \mathcal{N}_m^T \end{aligned} \tag{2.15}$$

where $\mathbf{\Omega}_k^m(j) = \mathbf{H}_m^T(j)\mathbf{E}_k^T$ is the kth row of a $K \times T$ matrix $\mathbf{\Omega}_m(j)$ and contains T elements. These elements are a function of N path gains $\alpha_{1m}(j), \alpha_{2m}(j), \ldots,$ $\alpha_{Nm}(j)$. It can be easily shown that [25]

$$\mathbf{\Omega}_m(j) \cdot \mathbf{\Omega}_m^T(j) = \left(\sum_{n=1}^{N} \alpha_{nm}^2(j)\right) I_K \tag{2.16}$$

For example, let us assume we want to eliminate the effect of user No. 1. We simply multiply each $\mathbf{R}_m$ by $\mathbf{\Omega}_m^T(1)$, divide it by $\sum_{n=1}^N \alpha_{nm}^2(1)$ and subtract the term for $m = 1$ from that of $m = 2, \ldots, M$:

$$\frac{\mathbf{R}_m^T \cdot \mathbf{\Omega}_m^T(1)}{\sum_{n=1}^N \alpha_{nm}^2(1)} - \frac{\mathbf{R}_1^T \cdot \mathbf{\Omega}_1^T(1)}{\sum_{n=1}^N \alpha_{n1}^2(1)} = \sum_{j=2}^{J} (s_1(j), s_2(j), \ldots, s_K(j)) \cdot \left\{ \frac{\mathbf{\Omega}_m(j) \cdot \mathbf{\Omega}_m^T(1)}{\sum_{n=1}^N \alpha_{nm}^2(1)} - \frac{\mathbf{\Omega}_1(j) \cdot \mathbf{\Omega}_1^T(1)}{\sum_{n=1}^N \alpha_{n1}^2(1)} \right\} + \mathcal{N}' \tag{2.17}$$

To prove that we can continue eliminating other users, we need to show that the following matrix is also multiple of a unitary matrix:

$$\frac{\mathbf{\Omega}_m(j) \cdot \mathbf{\Omega}_m^T(1)}{\sum_{n=1}^N \alpha_{nm}^2(1)} - \frac{\mathbf{\Omega}_1(j) \cdot \mathbf{\Omega}_1^T(1)}{\sum_{n=1}^N \alpha_{n1}^2(1)} \tag{2.18}$$

For that, it would be enough to show $\mathbf{\Omega}_{m_1}(j_1) \cdot \mathbf{\Omega}_{m_2}^T(j_2)$ is a multiple of a unitary matrix.[2] The latter is true due to the following:

$$\left[\mathbf{\Omega}_{m_1}(j_1) \cdot \mathbf{\Omega}_{m_2}^T(j_2)\right]^T \left[\mathbf{\Omega}_{m_1}(j_1) \cdot \mathbf{\Omega}_{m_2}^T(j_2)\right] = \mathbf{\Omega}_{m_2}(j_2) \cdot \mathbf{\Omega}_{m_1}^T(j_1) \cdot \mathbf{\Omega}_{m_1}(j_1) \cdot \mathbf{\Omega}_{m_2}^T(j_2) \tag{2.19}$$

Since $K = T$ [25], $\mathbf{\Omega}_m$s are $K \times K$ square matrices and (2.16) implies

$$\mathbf{\Omega}_m^T(j) \cdot \mathbf{\Omega}_m(j) = \left(\sum_{n=1}^N \alpha_{nm}^2(j) \right) I_K \tag{2.20}$$

Therefore, (2.19) can be simplified as following

$$\mathbf{\Omega}_{m_2}(j_2) \cdot \left(\sum_{n=1}^N \alpha_{nm_1}^2(j_1) \right) I_K \cdot \mathbf{\Omega}_{m_2}^T(j_2) = \left(\sum_{n=1}^N \alpha_{nm_1}^2(j_1) \right) \left(\sum_{n=1}^N \alpha_{nm_2}^2(j_2) \right) I_K \tag{2.21}$$

This concludes the proof that the matrix in (2.18) is a multiple of a unitary matrix. Therefore, once we eliminate the first user we can apply the same method on the remaining $M - 1$ signals and eliminate the second user. The reason is because the corresponding Ω matrix at each stage of elimination is square and multiple of a unitary matrix and the above proof guarantees the same property for the matrix of the next stage. □

---

[2] Since we know that the two subtracted terms will have the same format and will keep it after subtraction.

However, as discussed earlier, *complex* OSTBCs, except for $N = 2$, are not good candidates for interference cancellation when the goal is reducing the number of receive antennas. In next section, we offer another modulation scheme that fulfills the task for higher number of transmit antennas.

3 Interference Cancellation Using Quasi-Orthogonal Space-Time Block Coding

We use *Quasi-Orthogonal Space-Time Block Codes (QOSTBCs)* as follows [17]:

$$\mathbf{C} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{pmatrix} \tag{2.22}$$

In this design each column of the generator matrix is orthogonal to all other columns except one. As a result a pairwise decoding of the symbols is possible. Using rotation for some of the symbols, full diversity QOSTBCs are realizable [18, 23].

We describe the decoding when there is one user. The multi-user case is next to be studied. Assuming perfect channel state information is available, the receiver computes the decision metric

$$\sum_{m=1}^{M} \sum_{t=1}^{4} \left| r_{t,m} - \sum_{n=1}^{4} \alpha_{n,m}(1) c_{t,n}(1) \right|^2 \tag{2.23}$$

over all possible symbols to replace $s_1, \ldots, s_4$ in $\mathbf{C}$ and decides in favor of constellation symbols that minimize this sum. Since we have only one user and for simplicity specify one receive antenna, we do not mention indexing of group or receive antenna in the rest of this section. Simple algebraic manipulation shows that ML decoding for the code in (2.22) is equivalent to minimizing the following sum [17]:

$$f_{13}(s_1, s_3) + f_{24}(s_2, s_4) \tag{2.24}$$

where $f_{13}(s_1, s_3)$ is independent of (s_2, s_4) and $f_{24}(s_2, s_4)$ is independent of (s_1, s_3). Therefore, the pairs (s_1, s_3) and (s_2, s_4) can be decoded separately and the scheme is pairwise decodable.

In [27] a new decoding method was introduced by which differential decoding of a QOSTBC for non-coherent systems became possible. Let us review this decoding method. For $M = 1$ receive antenna, let us define the received signals at four time slots by r_1, r_2, r_3, r_4. Then, the set of input-output equations is equivalent to [27]:

$$\mathbf{R}_1 = \mathbf{S}_1 \mathbf{H}_1 + \mathcal{N}_1 \tag{2.25}$$

where

$$\mathbf{R}_1 = (r_{13,1}, r_{24,1})^T = (r_1 + r_3, r_2 + r_4)^T$$
$$\mathbf{H}_1 = (\alpha_{13,1}, \alpha_{24,1})^T = (\alpha_1 + \alpha_3, \alpha_2 + \alpha_4)^T$$
$$\mathcal{N}_1 = (\eta_1 + \eta_3, \eta_2 + \eta_4)^T$$
$$\mathbf{S}_1 = \begin{pmatrix} s_{13,1} & s_{24,1} \\ -s_{24,1}^* & s_{13,1}^* \end{pmatrix} = \begin{pmatrix} s_1 + s_3 & s_2 + s_4 \\ -(s_2^* + s_4^*) & s_1^* + s_3^* \end{pmatrix}$$

and

$$\mathbf{R}_2 = \mathbf{S}_2\mathbf{H}_2 + \mathcal{N}_2 \tag{2.26}$$

where

$$\mathbf{R}_2 = (r_{13,2}, r_{24,2})^T = (r_1 - r_3, r_2 - r_4)^T$$
$$\mathbf{H}_2 = (\alpha_{13,2}, \alpha_{24,2})^T = (\alpha_1 - \alpha_3, \alpha_2 - \alpha_4)^T$$
$$\mathcal{N}_2 = (\eta_1 - \eta_3, \eta_2 - \eta_4)^T$$
$$\mathbf{S}_2 = \begin{pmatrix} s_{13,2} & s_{24,2} \\ -s_{24,2}^* & s_{13,2}^* \end{pmatrix} = \begin{pmatrix} s_1 - s_3 & s_2 - s_4 \\ -(s_2^* - s_4^*) & s_1^* - s_3^* \end{pmatrix}$$

We can consider (2.25) and (2.26) as two equivalent subsystems, each of which has two transmit antennas. We can see that $\mathbf{S}_1$ and $\mathbf{S}_2$ have the structure of Alamouti code [10]. This is the key property in multi-user decoding as we discuss in the next section. The ML decoding metric for this system is[3]

$$\begin{aligned} X = &|r_{13,1} - \alpha_{13,1}s_{13,1} - \alpha_{24,1}s_{24,1}|^2 + |r_{24,1} + \alpha_{13,1}s_{24,1}^* + \alpha_{24,1}s_{13,1}|^2 \\ &+ |r_{13,2} - \alpha_{13,2}s_{13,2} - \alpha_{24,2}s_{24,2}|^2 \\ &+ |r_{24,2} + \alpha_{13,2}s_{24,2}^* - \alpha_{24,2}s_{13,2}^*|^2 \end{aligned} \tag{2.27}$$

We need to find symbols s_1, s_2, s_3, s_4 that minimize X. Expanding the expression for X we get

$$X = |r_1|^2 + |r_2|^2 + |r_3|^2 + |r_4|^2 + 2f_{13}(s_1, s_3) + 2f_{24}(s_2, s_4) \tag{2.28}$$

Therefore, the choice of $\{s_1, s_2, s_3, s_4\}$ that minimize X, will minimize the ML metric of the original system as well. In other words, our transformation is lossless, and the error performance is the same as the optimal decoding of the system in (2.22). Then, using the method in Sect. III for multi-user detection of Alamouti equipped transmitters, we decode the whole group.

[3]The reason we can write ML metric like this is because the new noise terms are still independent and Gaussian.

Recalling the model in Sect. II and having in mind that users are sending QOST-BCs, at each receive antenna m we receive the following signals in the four time slots

$$\begin{aligned}
r_{1,m} &= \sum_{j=1}^{J} \alpha_{1,m}(j)s_1(j) + \alpha_{2,m}(j)s_2(j) + \alpha_{3,m}(j)s_3(j) + \alpha_{4,m}(j)s_4(j) + \eta_{1,m} \\
r_{2,m} &= \sum_{j=1}^{J} -\alpha_{1,m}(j)s_2^*(j) + \alpha_{2,m}(j)s_1^*(j) - \alpha_{3,m}(j)s_4^*(j) + \alpha_{4,m}(j)s_3^*(j) + \eta_{2,m} \\
r_{3,m} &= \sum_{j=1}^{J} \alpha_{1,m}(j)s_3(j) + \alpha_{2,m}(j)s_4(j) + \alpha_{3,m}(j)s_1(j) + \alpha_{4,m}(j)s_2(j) + \eta_{3,m} \\
r_{4,m} &= \sum_{j=1}^{J} -\alpha_{1,m}(j)s_4^*(j) + \alpha_{2,m}(j)s_3^*(j) - \alpha_{3,m}(j)s_2^*(j) + \alpha_{4,m}(j)s_1^*(j) + \eta_{4,m}
\end{aligned} \tag{2.29}$$

where $\eta_{i,m}$s are i.i.d. zero mean Gaussian random variables with variance $\frac{4}{SNR}$. Similarly, for the signals at each receive antenna we form two equivalent 2-antenna systems as follows

$$\begin{aligned}
\mathbf{R}_{1,m} = \begin{pmatrix} r_{1,m} + r_{3,m} \\ r_{2,m} + r_{4,m} \end{pmatrix} &= \sum_{j=1}^{J} \begin{pmatrix} s_1(j) + s_3(j) & s_2(j) + s_4(j) \\ -(s_2(j) + s_4(j))^* & (s_1(j) + s_3(j))^* \end{pmatrix} \\
&\times \begin{pmatrix} \alpha_{1,m}(1) + \alpha_{3,m}(1) \\ \alpha_{2,m}(1) + \alpha_{4,m}(1) \end{pmatrix} + \begin{pmatrix} \eta_{1,m} + \eta_{3,m} \\ \eta_{2,m} + \eta_{4,m} \end{pmatrix} \\
\mathbf{R}_{2,m} = \begin{pmatrix} r_{1,m} - r_{3,m} \\ r_{2,m} - r_{4,m} \end{pmatrix} &= \sum_{j=1}^{J} \begin{pmatrix} s_1(j) - s_3(j) & s_2(j) - s_4(j) \\ -(s_2(j) - s_4(j))^* & (s_1(j) - s_3(j))^* \end{pmatrix} \\
&\times \begin{pmatrix} \alpha_{1,m}(1) - \alpha_{3,m}(1) \\ \alpha_{2,m}(1) - \alpha_{4,m}(1) \end{pmatrix} + \begin{pmatrix} \eta_{1,m} - \eta_{3,m} \\ \eta_{2,m} - \eta_{4,m} \end{pmatrix}
\end{aligned} \tag{2.30}$$

Without loss of generality we assume that we eliminate the effect of user No. 1 first. By applying complex conjugation on the second row of both systems we get

$$\mathbf{R}'^{T}_{1,m} = \begin{pmatrix} r_{1,m} + r_{3,m} \\ r_{2,m}^* + r_{4,m}^* \end{pmatrix}^T = \sum_{j=1}^{J} (s_1(j) + s_3(j), s_2(j) + s_4(j))$$

$$\cdot \begin{pmatrix} \alpha_{1,m}(j) + \alpha_{3,m}(j) & (\alpha_{2,m}(j) + \alpha_{4,m}(j))^* \\ \alpha_{2,m}(j) + \alpha_{4,m}(j) & -(\alpha_{1,m}(j) + \alpha_{3,m}(j))^* \end{pmatrix} + \begin{pmatrix} \eta_{1,m} + \eta_{3,m} \\ \eta_{2,m}^* + \eta_{4,m}^* \end{pmatrix}^T$$

$$= \sum_{j=1}^{J} (s_1(j) + s_3(j), s_2(j) + s_4(j)) \cdot \mathcal{G}(\alpha_{1,m}(j) + \alpha_{3,m}(j), \alpha_{2,m}(j) + \alpha_{4,m}(j)) + \mathcal{N}_{1,m} \tag{2.31}$$

and

$$\mathbf{R}'^{T}_{2,m} = \begin{pmatrix} r_{1,m} - r_{3,m} \\ r^*_{2,m} - r^*_{4,m} \end{pmatrix}^T = \sum_{j=1}^{J} (s_1(j) - s_3(j), s_2(j) - s_4(j)) \cdot \begin{pmatrix} \alpha_{1,m}(j) - \alpha_{3,m}(j) & (\alpha_{2,m}(j) - \alpha_{4,m}(j))^* \\ \alpha_{2,m}(j) - \alpha_{4,m}(j) & -(\alpha_{1,m}(j) - \alpha_{3,m}(j))^* \end{pmatrix} + \begin{pmatrix} \eta_{1,m} - \eta_{3,m} \\ \eta^*_{2,m} - \eta^*_{4,m} \end{pmatrix}^T$$
$$= \sum_{j=1}^{J} (s_1(j) - s_3(j), s_2(j) - s_4(j)) \cdot \mathcal{G}(\alpha_{1,m}(j) - \alpha_{3,m}(j), \alpha_{2,m}(j) - \alpha_{4,m}(j)) + \mathcal{N}_{2,m} \tag{2.32}$$

where

$$\mathcal{G}(x, y) = \begin{pmatrix} x & y^* \\ y & -x^* \end{pmatrix} \tag{2.33}$$

The advantage of representing the received signals in the above format is hidden in the structure of the equivalent channel matrices:

$$\mathbf{\Omega}_{1,m}(j) = \begin{pmatrix} \alpha_{1,m}(j) + \alpha_{3,m}(j) & (\alpha_{2,m}(j) + \alpha_{4,m}(j))^* \\ \alpha_{1,m}(j) + \alpha_{3,m}(j) & -(\alpha_{1,m}(j) + \alpha_{3,m}(j))^* \end{pmatrix} = \mathcal{G}(\alpha_{1,m}(j) + \alpha_{3,m}(j), \alpha_{2,m}(j) + \alpha_{4,m}(j))$$

$$\mathbf{\Omega}_{2,m}(j) = \begin{pmatrix} \alpha_{1,m}(j) - \alpha_{3,m}(j) & (\alpha_{2,m}(j) - \alpha_{4,m}(j))^* \\ \alpha_{1,m}(j) - \alpha_{3,m}(j) & -(\alpha_{1,m}(j) - \alpha_{3,m}(j))^* \end{pmatrix} = \mathcal{G}(\alpha_{1,m}(j) - \alpha_{3,m}(j), \alpha_{2,m}(j) - \alpha_{4,m}(j))$$

Since both $\mathbf{\Omega}_{1,m}(j)$ and $\mathbf{\Omega}_{2,m}(j)$ are multiples of a unitary matrix, we can simply separate and therefore eliminate the effect of each group from the received signal.

$$\mathbf{R}'^{T}_{1,m} \cdot \mathbf{\Omega}^{\dagger}_{1,m}(1)$$
$$= (|\alpha_{1,m}(1) + \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) + \alpha_{4,m}(1)|^2)(s_1(j) + s_3(1), s_2(1) + s_4(1))$$
$$+ \sum_{j=2}^{J} (s_1(j) + s_3(j), s_2(j) + s_4(j)) \cdot \mathcal{G}'((\alpha_{1,m}(j) + \alpha_{3,m}(j))$$

$$\times (\alpha_{1,m}(j) + \alpha_{3,m}(j))^* + (\alpha_{2,m}(j) + \alpha_{4,m}(j))^*(\alpha_{2,m}(1) + \alpha_{4,m}(1))$$
$$\times (\alpha_{1,m}(j) + \alpha_{3,m}(j))(\alpha_{2,m}(j) + \alpha_{4,m}(j))^*$$
$$- (\alpha_{2,m}(j) + \alpha_{4,m}(j))^*(\alpha_{1,m}(1) + \alpha_{3,m}(1))) + \mathcal{N}'_{1,m} \quad (2.34)$$

$$\mathbf{R}'^T_{2,m} \cdot \mathbf{\Omega}^\dagger_{2,m}(1)$$
$$= (|\alpha_{1,m}(1) - \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) - \alpha_{4,m}(1)|^2)(s_1(j) - s_3(1), s_2(1) - s_4(1))$$
$$+ \sum_{j=2}^{J} (s_1(j) - s_3(j), s_2(j) - s_4(j)) \cdot \mathcal{G}'((\alpha_{1,m}(j) - \alpha_{3,m}(j))$$
$$\times (\alpha_{1,m}(j) - \alpha_{3,m}(j))^* + (\alpha_{2,m}(j) - \alpha_{4,m}(j))^*(\alpha_{2,m}(1) - \alpha_{4,m}(1))$$
$$\times (\alpha_{1,m}(j) - \alpha_{3,m}(j))(\alpha_{2,m}(j) - \alpha_{4,m}(j))^*$$
$$- (\alpha_{2,m}(j) - \alpha_{4,m}(j))^*(\alpha_{1,m}(1) - \alpha_{3,m}(1))) + \mathcal{N}'_{2,m} \quad (2.35)$$

where

$$\mathcal{G}'(x, y) = \begin{pmatrix} x & y \\ -y^* & x^* \end{pmatrix} \quad (2.36)$$

$\mathcal{N}'_{1,m}$ and $\mathcal{N}'_{2,m}$ are the new noise vectors corresponding to the mth receive antenna. Noting that $\mathbf{\Omega}_{i,m}$ matrices are multiples of unitary, the distribution of each element of $\mathcal{N}'_{1,m}$ will be Gaussian with variance $\frac{8(|\alpha_{1,m}(1)+\alpha_{3,m}(1)|^2+|\alpha_{2,m}(1)+\alpha_{4,m}(1)|^2)}{SNR}$. The two elements will still be i.i.d. Same thing is true for $\mathcal{N}'_{2,m}$ with a change of sign.

Let us reconsider (2.34) and (2.35) for all receive antennas $m = 1, 2, \ldots, J + r - 1$. If we subtract the expression for the first receive antenna, $m = 1$, from that of the other antennas we will have $J + r - 2$ equations as follows

$$\frac{\mathbf{R}'^T_{1,m} \cdot \mathbf{\Omega}^\dagger_{1,m}(1)}{|\alpha_{1,m}(1) + \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) + \alpha_{4,m}(1)|^2}$$
$$- \frac{\mathbf{R}'^T_{1,1} \cdot \mathbf{\Omega}^\dagger_{1,1}(1)}{|\alpha_{1,1}(1) + \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) + \alpha_{4,1}(1)|^2}$$
$$= \sum_{j=2}^{J} (s_1(j) + s_3(j), s_2(j) + s_4(j)) \cdot \mathcal{G}'(A_{1m}(j), B_{1m}(j))$$
$$+ \frac{\mathcal{N}'_{1,m}}{|\alpha_{1,m}(1) + \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) + \alpha_{4,m}(1)|^2}$$
$$- \frac{\mathcal{N}'_{1,1}}{|\alpha_{1,1}(1) + \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) + \alpha_{4,1}(1)|^2} \quad (2.37)$$

and

$$\frac{\mathbf{R}'^{T}_{2,m} \cdot \mathbf{\Omega}^{\dagger}_{2,m}(1)}{|\alpha_{1,m}(1) - \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) - \alpha_{4,m}(1)|^2} - \frac{\mathbf{R}'^{T}_{2,1} \cdot \mathbf{\Omega}^{\dagger}_{2,1}(1)}{|\alpha_{1,1}(1) - \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) - \alpha_{4,1}(1)|^2}$$
$$= \sum_{j=2}^{J} (s_1(j) - s_3(j), s_2(j) - s_4(j)) \cdot \mathcal{G}'(A_{2m}(j), B_{2m}(j))$$
$$+ \frac{\mathcal{N}'_{2,m}}{|\alpha_{1,m}(1) - \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) - \alpha_{4,m}(1)|^2} - \frac{\mathcal{N}'_{2,1}}{|\alpha_{1,1}(1) - \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) - \alpha_{4,1}(1)|^2} \tag{2.38}$$

where

$$A_{1m}(j) = \frac{(\alpha_{1,m}(j) + \alpha_{3,m}(j))(\alpha_{1,m}(1) + \alpha_{3,m}(1))^* + (\alpha_{2,m}(j) + \alpha_{4,m}(j))^*(\alpha_{2,m}(j) + \alpha_{4,m}(j))}{|\alpha_{1,m}(1) + \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) + \alpha_{4,m}(1)|^2}$$
$$- \frac{(\alpha_{1,1}(j) + \alpha_{3,1}(j))(\alpha_{1,1}(1) + \alpha_{3,1}(1))^* + (\alpha_{2,1}(j) + \alpha_{4,1}(j))^*(\alpha_{2,1}(j) + \alpha_{4,1}(j))}{|\alpha_{1,1}(1) + \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) + \alpha_{4,1}(1)|^2}$$
$$B_{1m}(j) = \frac{(\alpha_{1,m}(j) + \alpha_{3,m}(j))(\alpha_{2,m}(1) + \alpha_{4,m}(1))^* - (\alpha_{2,m}(j) + \alpha_{4,m}(j))^*(\alpha_{1,m}(1) + \alpha_{3,m}(1))}{|\alpha_{1,m}(1) + \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) + \alpha_{4,m}(1)|^2}$$
$$- \frac{(\alpha_{1,1}(j) + \alpha_{3,1}(j))(\alpha_{2,1}(1) + \alpha_{4,1}(1))^* - (\alpha_{2,1}(j) + \alpha_{4,1}(j))^*(\alpha_{1,1}(1) + \alpha_{3,1}(1))}{|\alpha_{1,1}(1) + \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) + \alpha_{4,1}(1)|^2} \tag{2.39}$$

and

$$A_{2m}(j) = \frac{(\alpha_{1,m}(j) - \alpha_{3,m}(j))(\alpha_{1,m}(1) - \alpha_{3,m}(1))^* + (\alpha_{2,m}(j) - \alpha_{4,m}(j))^*(\alpha_{2,m}(j) - \alpha_{4,m}(j))}{|\alpha_{1,m}(1) - \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) - \alpha_{4,m}(1)|^2}$$
$$- \frac{(\alpha_{1,1}(j) - \alpha_{3,1}(j))(\alpha_{1,1}(1) - \alpha_{3,1}(1))^* + (\alpha_{2,1}(j) - \alpha_{4,1}(j))^*(\alpha_{2,1}(j) - \alpha_{4,1}(j))}{|\alpha_{1,1}(1) - \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) - \alpha_{4,1}(1)|^2}$$
$$B_{2m}(j) = \frac{(\alpha_{1,m}(j) - \alpha_{3,m}(j))(\alpha_{2,m}(1) - \alpha_{4,m}(1))^* - (\alpha_{2,m}(j) - \alpha_{4,m}(j))^*(\alpha_{1,m}(1) - \alpha_{3,m}(1))}{|\alpha_{1,m}(1) - \alpha_{3,m}(1)|^2 + |\alpha_{2,m}(1) - \alpha_{4,m}(1)|^2}$$
$$- \frac{(\alpha_{1,1}(j) - \alpha_{3,1}(j))(\alpha_{2,1}(1) - \alpha_{4,1}(1))^* - (\alpha_{2,1}(j) - \alpha_{4,1}(j))^*(\alpha_{1,1}(1) - \alpha_{3,1}(1))}{|\alpha_{1,1}(1) - \alpha_{3,1}(1)|^2 + |\alpha_{2,1}(1) - \alpha_{4,1}(1)|^2} \tag{2.40}$$

where m can be $2,\ldots,J+r-1$.[4] The variance will be equal to $\frac{8}{SNR(|\alpha_{1,m}(1)\pm\alpha_{3,m}(1)|^2+|\alpha_{2,m}(1)\pm\alpha_{4,m}(1)|^2)} + \frac{8}{SNR(|\alpha_{1,1}(1)\pm\alpha_{3,1}(1)|^2+|\alpha_{2,1}(1)\pm\alpha_{4,1}(1)|^2)}$ respectively. Equations (2.37) and (2.38) have the same structure as (2.34) and (2.35). We note that $\mathcal{G}'$ is also multiple of a unitary matrix as[5] $\mathcal{G}$. It is as if, in (2.34) and (2.35) we have $J+r-2$ receive antennas and $J-1$ user groups. Therefore, we can separate and eliminate user No. 2 similar to user No. 1; except that the number of equivalent receive antennas will be one less, i.e. $(J+r-2)$. Eliminating user No. 2 gives us $J+r-3$ equations with $J-2$ users. If we continue this procedure, we will have r equations for user No. J with no interference from other users. We can simply decode this user. Since we have r equations and our metric is ML, diversity gain will be equal to $4\times r$. One should note that better performance is possible, if we use more complex decoding methods. For example, since we break down our code to two Alamouti structures, the MMSE-ZF method used in [15] for 2 transmit antenna systems can be applied here. This method that involves a matrix inversion, will provide a better error rate, compared with our simple ZF method. Also, one can decode users on the order of the strength of their respective channel and improve the performance. Our proposed algorithm can be combined with different decoding methods. The goal in here is to show it is possible to do multi-user detection for any number of transmit antennas,[6] using very few number of receive antennas. It is not meant to find the best performance, and there is no claim of optimality.

The flow of decoding is summarized below. In what follows we assume we want to decode user No. J; we can then simply change the ordering of users in case we are interested in decoding another user.

Algorithm

(1) Initialization. Let $i=1$. Also, for all values of $j=1,2,\ldots,J$ and $m=1,2,\ldots,M$ let:

$$\begin{aligned}A_{1m}(j)&=\alpha_{1,m}(j)+\alpha_{3,m}(j)\\ B_{1m}(j)&=\alpha_{2,m}(j)+\alpha_{4,m}(j)\\ A_{2m}(j)&=\alpha_{1,m}(j)-\alpha_{3,m}(j)\quad\text{and}\\ B_{2m}(j)&=\alpha_{2,m}(j)-\alpha_{4,m}(j)\end{aligned}$$

Construct $\mathbf{R}'_{1,1},\mathbf{R}'_{1,2},\ldots,\mathbf{R}'_{1,M}$ and $\mathbf{R}'_{2,1},\mathbf{R}'_{2,2},\ldots,\mathbf{R}'_{2,M}$ from the M received signal vectors $\mathbf{R}_1,\mathbf{R}_2,\ldots,\mathbf{R}_M$.

(2) For all values of $j=i,i+1,\ldots,J$ and $m=i,i+1,\ldots,M$: if $i=1$ let $\Omega_{1,m}(j)=\mathcal{G}(A_{1m}(j),B_{1m}(j))$ and $\Omega_{2,m}(j)=\mathcal{G}(A_{2m}(j),B_{2m}(j))$ else let $\Omega_{1,m}(j)=\mathcal{G}'(A_{1m}(j),B_{1m}(j))$ and $\Omega_{2,m}(j)=\mathcal{G}'(A_{2m}(j),B_{2m}(j))$.

[4]The distribution of noise will not be i.i.d. for $M>2$. We describe this in the next chapter along with he optimal decoding method for it.

[5]One can easily show that had the channel matrix been $\mathcal{G}'$ instead of $\mathcal{G}$, we would have ended with $\mathcal{G}'$ as our equivalent channel, after suppressing the interference of the first user. Therefore, we will face $\mathcal{G}'$ at cancellation of all users except the first.

[6]As will be seen in the end of this section.

(3) For all values of $j = i+1, i+2, \ldots, J$ and $m = i+1, i+2, \ldots, M$:

$$\mathbf{R}'_{1,m} = \frac{\Omega^*_{1,m}(i)}{\frac{1}{2}\|\Omega_{1,m}(i)\|_F^2} \cdot \mathbf{R}'_{1,m} - \frac{\Omega^*_{1,i}(i)}{\frac{1}{2}\|\Omega_{1,i}(i)\|_F^2} \cdot \mathbf{R}'_{1,i}$$

$$\mathbf{R}'_{2,m} = \frac{\Omega^*_{2,m}(i)}{\frac{1}{2}\|\Omega_{2,m}(i)\|_F^2} \cdot \mathbf{R}'_{2,m} - \frac{\Omega^*_{2,i}(i)}{\frac{1}{2}\|\Omega_{2,i}(i)\|_F^2} \cdot \mathbf{R}'_{2,i}$$

if $i = 1$

$$A_{1m}(j) = \frac{A_{1m}(j)A^*_{1m}(i) + B^*_{1m}(j)B_{1m}(i)}{|A_{1m}(i)|^2 + |B_{1m}(i)|^2} - \frac{A_{1i}(j)A^*_{1i}(i) + B^*_{1i}(j)B_{1i}(i)}{|A_{1i}(i)|^2 + |B_{1i}(i)|^2}$$

$$B_{1m}(j) = \frac{A_{1m}(j)B^*_{1m}(i) - B^*_{1m}(j)A_{1m}(i)}{|A_{1m}(i)|^2 + |B_{1m}(i)|^2} - \frac{A_{1i}(j)B^*_{1i}(i) - B^*_{1i}(j)A_{1i}(i)}{|A_{1i}(i)|^2 + |B_{1i}(i)|^2}$$

$$A_{2m}(j) = \frac{A_{2m}(j)A^*_{2m}(i) + B^*_{2m}(j)B_{2m}(i)}{|A_{2m}(i)|^2 + |B_{2m}(i)|^2} - \frac{A_{2i}(j)A^*_{2i}(i) + B^*_{2i}(j)B_{2i}(i)}{|A_{2i}(i)|^2 + |B_{2i}(i)|^2}$$

$$B_{2m}(j) = \frac{A_{2m}(j)B^*_{2m}(i) - B^*_{2m}(j)A_{2m}(i)}{|A_{2m}(i)|^2 + |B_{2m}(i)|^2} - \frac{A_{2i}(j)B^*_{2i}(i) - B^*_{2i}(j)A_{2i}(i)}{|A_{2i}(i)|^2 + |B_{2i}(i)|^2}$$

else

$$A_{1m}(j) = \frac{A_{1m}(j)A^*_{1m}(i) + B_{1m}(j)B^*_{1m}(i)}{|A_{1m}(i)|^2 + |B_{1m}(i)|^2} - \frac{A_{1i}(j)A^*_{1i}(i) + B_{1i}(j)B^*_{1i}(i)}{|A_{1i}(i)|^2 + |B_{1i}(i)|^2}$$

$$B_{1m}(j) = \frac{-A_{1m}(j)B_{1m}(i) + B_{1m}(j)A_{1m}(i)}{|A_{1m}(i)|^2 + |B_{1m}(i)|^2} - \frac{-A_{1i}(j)B_{1i}(i) + B_{1i}(j)A_{1i}(i)}{|A_{1i}(i)|^2 + |B_{1i}(i)|^2}$$

$$A_{2m}(j) = \frac{A_{2m}(j)A^*_{2m}(i) + B_{2m}(j)B^*_{2m}(i)}{|A_{2m}(i)|^2 + |B_{2m}(i)|^2} - \frac{A_{2i}(j)A^*_{2i}(i) + B_{2i}(j)B^*_{2i}(i)}{|A_{2i}(i)|^2 + |B_{2i}(i)|^2}$$

$$B_{2m}(j) = \frac{-A_{2m}(j)B_{2m}(i) + B_{2m}(j)A_{2m}(i)}{|A_{2m}(i)|^2 + |B_{2m}(i)|^2} - \frac{-A_{2i}(j)B_{2i}(i) + B_{2i}(j)A_{2i}(i)}{|A_{2i}(i)|^2 + |B_{2i}(i)|^2}$$

(4) Let $i = i + 1$. If $i < J$ go to Step 2, otherwise stop the algorithm and decode the following system

$$\mathbf{R}'^{T}_{1,m} = (s_1(J) + s_3(J), s_2(J) + s_4(J)) \cdot \mathcal{G}'(A_{1m}(J), B_{1m}(J)) + \text{noise}$$

$$\mathbf{R}'^{T}_{2,m} = (s_1(J) - s_3(J), s_2(J) - s_4(J)) \cdot \mathcal{G}'(A_{2m}(J), B_{2m}(J)) + \text{noise}$$

for all $m = J, J+1, \ldots, M$. □

As can be noticed the complexity of this algorithm is linear in terms of the number of users and receive antennas. Also, the diversity provided in the last equation is $4 \times (M - J + 1)$ as discussed earlier.[7]

The relationship between the power of noise, at the beginning of each state and that of the end of the stage can be derived by comparing the ones in the first stage. Let us assume we are at Step i of detection and the noise power at the beginning of the step is σ^2. Based on what we found for the noise distribution in Step 2 and the recursive algorithm above, the noise power at the end of the step i at receive antenna m will be

$$\frac{\sigma^2}{|A_{1m}(i)|^2 + |B_{1m}(i)|^2} + \frac{\sigma^2}{|A_{1i}(i)|^2 + |B_{1i}(i)|^2} \tag{2.41}$$

One can use a similar approach and perform the task of interference cancellation for users equipped with 3 transmit antennas. We should simply remove one of the columns of the code matrix, e.g. fourth column, in encoding. In decoding, i.e. cancellation, we can use the above algorithm except that we put zero for every $\alpha_{4,m}(j)$. Note that the resulting code from column removal may not be optimal in terms of delay.

Theorem *Using modulation schemes in the form of* $\begin{pmatrix} A & B \\ B & A \end{pmatrix}$, *one can extend the above interference cancellation technique to users with any number of transmit antennas in the form of power of* 2.

Proof We use induction. Due to the structure of the code, we can convert the original 2^n transmit antenna system to two 2^{n-1} transmit antenna systems. Then, we use interference cancellation for each of those systems using $M = J$ receive antennas, and finally decode the two systems together. □

Corollary *We can extend this algorithm to users with any number of transmit antennas, by finding the first power of* 2 *not smaller than* N, *and the modulation scheme designed for that many transmit antennas using the method in the above theorem. Then, we can use the column removal method mentioned earlier to get the desired number of transmit antennas.*

4 Interference Cancellation Using Minimum Decoding Complexity Quasi-Orthogonal Space-Time Block Codes (MDC-QOSTBC)

In [28], a new Quasi-Orthogonal design has been introduced that trades a small amount of performance loss with simpler decoding. These codes, provide separate

[7]The diversity claims will be proved in the next chapter rigorously.

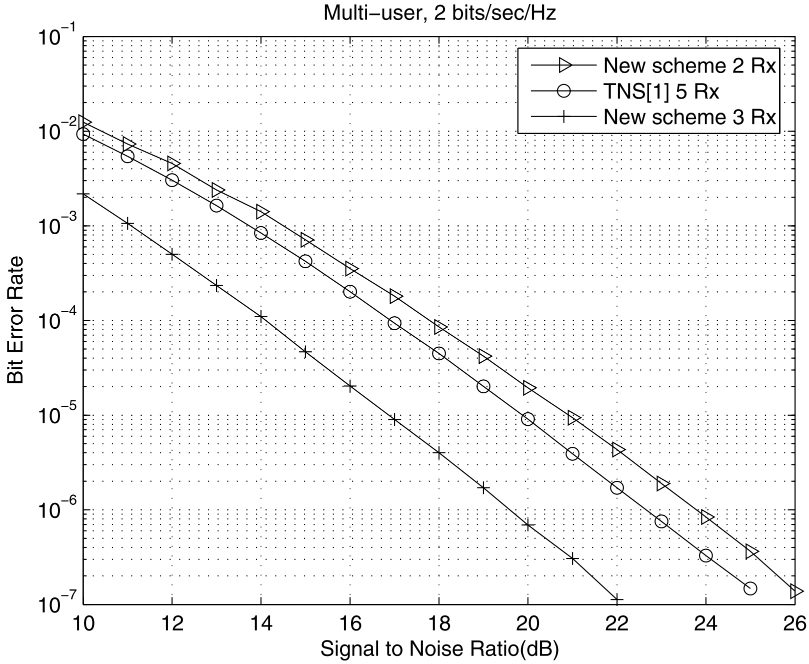

Fig. 2.1 Simulation results after interference cancellation when there are 2 users each transmitting QOSTBC with QPSK modulation

decoding complexity rather than pairwise, while maintaining full diversity and unit rate. In this section we briefly show that our introduced algorithm will work for this family of codes as well. The main idea is that, these codes can be presented as a mapping of the original QOSTBCs. We consider the code presented in [28] as an example

$$\mathbf{C}'(x_1,x_2,x_3,x_4)=\begin{pmatrix} x_1^R+jx_3^R & x_2^R+jx_4^R & -x_1^I+jx_3^I & -x_2^I+jx_4^I \\ -x_2^R+jx_4^R & x_1^R-jx_3^R & x_2^I+jx_4^I & -x_1^I-jx_3^I \\ -x_1^I+jx_3^I & -x_2^I+jx_4^I & x_1^R+jx_3^R & x_2^R+jx_4^r \\ x_2^I+jx_4^I & -x_1^I-jx_3^I & -x_2^R+jx_4^R & x_1^R-jx_3^R \end{pmatrix} \tag{2.42}$$

where $x_i^R=\Re\{x_i\}$ and $x_i^I=\Im\{x_i\}$. Recalling the definition of $\mathbf{C}$ used in the algorithm from (22), we realize that

$$\begin{aligned} &\mathbf{C}'(x_1,x_2,x_3,x_4)=\mathbf{C}(s_1,s_2,s_3,s_4)\\ &s_1=x_1^R+jx_3^R\\ &s_2=x_2^R+jx_4^R \end{aligned} \tag{2.43}$$

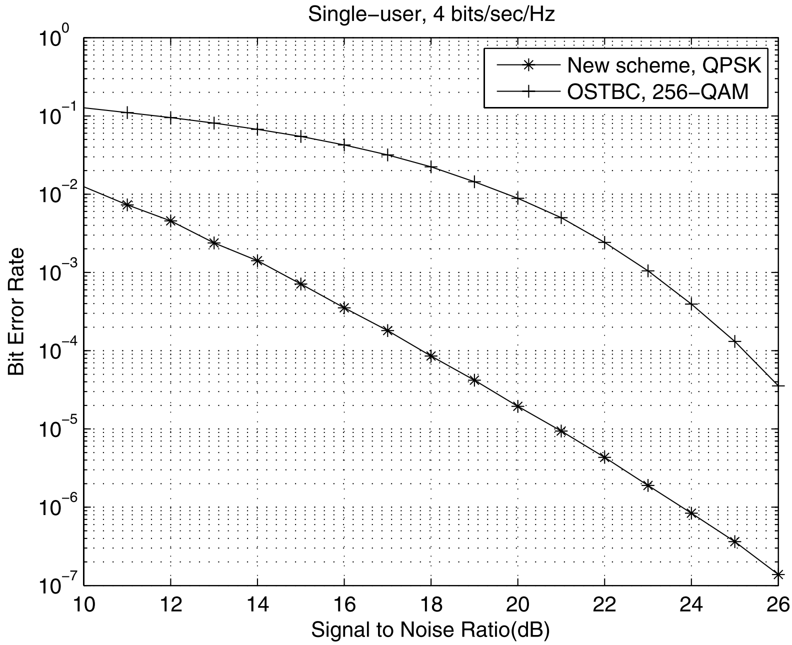

Fig. 2.2 Bit error probability vs. SNR for the new array processing scheme, and OSTBC at 2 bits/s/Hz; 8 transmit and 2 receive antennas

$$s_3 = -x_1^I + jx_3^I$$

$$s_4 = -x_2^I + jx_4^I$$

Therefore, in order to cancel out the interference of users equipped with MDC-QOSTBC, we will only need to map it to the corresponding QOSTBC. From there, we can apply the algorithm of previous section and detect the desired user. Then, we do the reverse mapping on the detected symbols and get the original ones.

5 Application of the New Interference Cancellation Scheme in Array Processing

Assume we have an interference cancellation scheme. The scheme does not put any assumption on the location of the users. Therefore, even if the users are all together, unified as part of a bigger user, the proposed decoding works and provides transmit diversity. If we unify J of such users, the spatial multiplexing of the overall system will be J times that of the individual users alone. In other words, if we have a system with JN transmit antennas, we can use a combination of J space-time codes with interference cancellation decoding, that sends J times more symbols per time slot

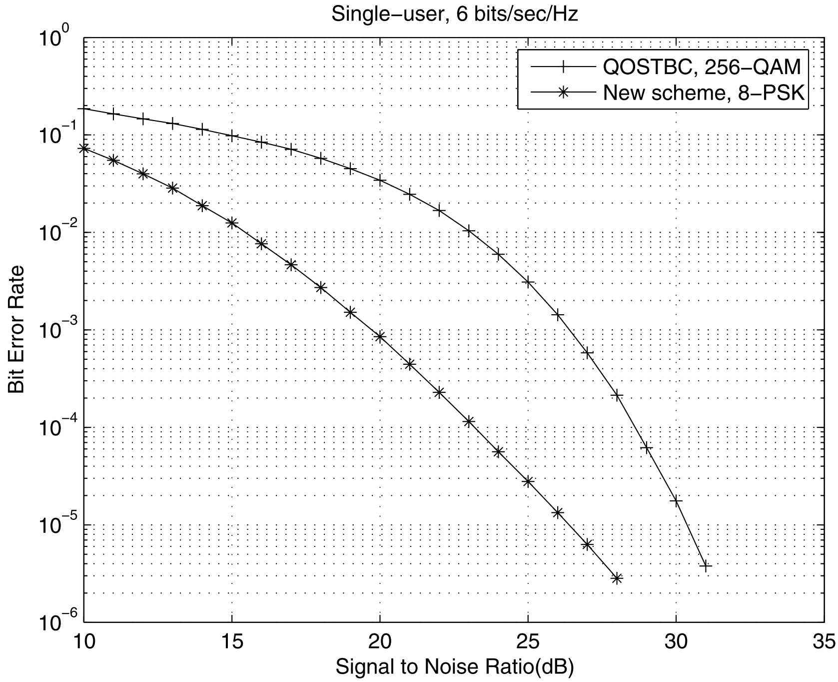

Fig. 2.3 Bit error probability vs. SNR for the new array processing scheme, and QOSTBC at 6 bits/s/Hz; 8 transmit and 2 receive antennas

than each of the individual space-time codes. This gives us an idea to apply the newly introduced scheme for increasing the rate in multiple antenna systems.

For example, let us assume we have 8 transmit antennas. We can always use a space-time code designed for 8 transmit antennas with full diversity, $8M$, and send at most one symbol per time slot [9]. Instead, we can exchange diversity for rate and use 2 separate QOSTBCs as mentioned in the previous section. This way we can send twice as many symbols and enjoy an acceptable diversity gain equal to $4(M-1)$.

As it is seen in the example above, our scheme provides a trade-off between rate and diversity. Since this scheme requires relatively less number of receive antennas, one may think of comparing its performance with popular multiple antenna schemes like OSTBC, QOSTBC, or BLAST. This task is performed in the next section.

6 Simulation Results

In this section we provide simulation results that confirm our analysis explained in the previous sections. The performance of our multi-user detection scheme is shown in Fig. 2.1. We consider 2 users each equipped with 4 antennas and transmitting

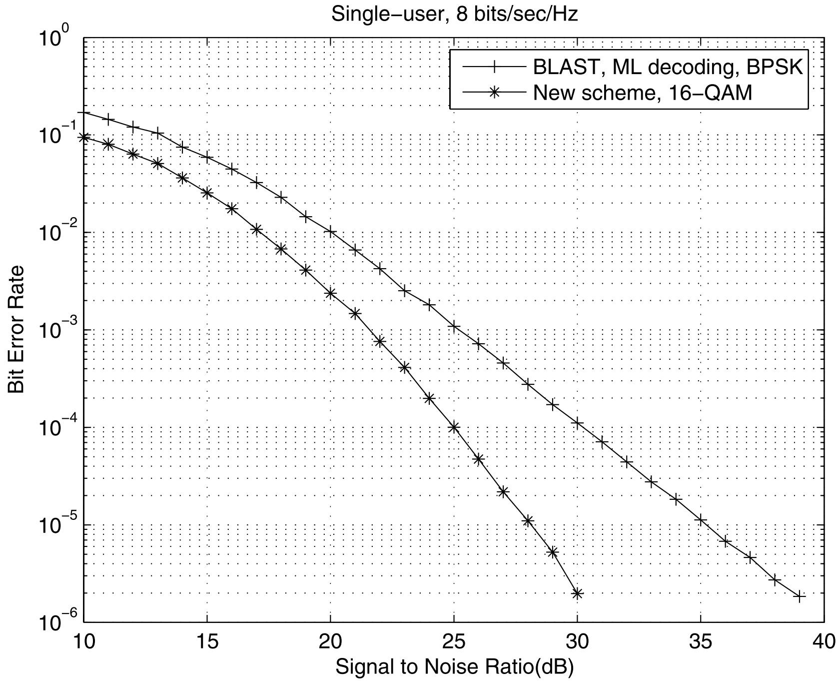

Fig. 2.4 Bit error probability vs. SNR for the new array processing scheme, and BLAST-ML at 8 bits/s/Hz; 8 transmit and 2 receive antennas

QOSTBCs. We compare this result with the one offered by [1] when it uses the same code, i.e. QOSTBC.[8] The constellation used is QPSK which provides a rate equal to 2 bits per channel use. The method offered in [1] requires at least 8 receive antennas for cancelling the interference whereas ours requires 2. It can be seen that our algorithm with usage of 3 receive antennas outperforms the old method when it uses 8 receive antennas.

Figures 2.2, 2.3, and 2.4 represent the comparison of the array processing scheme discussed in Sect. 5 with orthogonal space-time block codes, quasi-orthogonal space-time block codes, and BLAST respectively. We use ML for the BLAST code in Fig. 2.4. In Figs. 2.5 and 2.6, however, we are comparing the performance of our array processing scheme with ZF-BLAST with 8 and 12 antennas respectively. The array processing system in Figs. 2.1–2.5 consists of two 4-transmit antenna QOSTBC transmitters unified in one unit. In Fig. 2.2, we compare this system with the 16×8 orthogonal design in [11] and [25] using 256-QAM modulation. We use QPSK for the array processing scheme so that both systems provide a rate equal to 4 bits per channel use. In Fig. 2.3 we have used the 8-transmit antenna quasi-orthogonal design mentioned in [17] and [18], using rotation to provide full

[8]When [1] was published, QOSTBCs were not known; however, the method in that work allows usage of any modulation scheme including QOSTBCs.

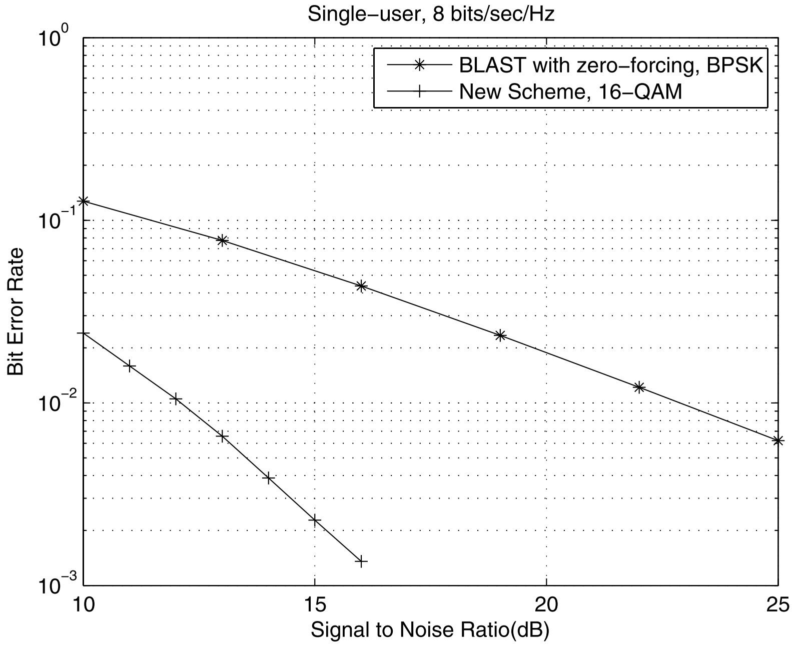

Fig. 2.5 Bit error probability vs. SNR for the new array processing scheme, and BLAST-ZF at 8 bits/s/Hz; 8 transmit and 8 receive antennas

diversity. For this experiment we have used 256-QAM for the 8-transmit antenna QOSTBC and 8PSK for the array processing system to provide the desired rate of 6 bits per channel use. In the case of BLAST in Fig. 2.4 we have used the regular 8-transmit antenna V-BLAST encoding with ML decoding. Note that the performance of the BLAST using the usual nulling and cancellation methods is much worse than that of BLAST using ML decoding. The constellation used for BLAST is BPSK and the one used for array processing system is 16-QAM; both systems transmit at a rate equal to 8 bits per channel use. In Fig. 2.5 both systems have 8 transmit and 8 receive antennas both using ZF as decoding method. The array processing system is using 16-QAM, and the BLAST code uses BPSK. Figure 2.6 shows comparison of a BLAST code with 12 transmit and receive antennas and the array processing system with same number of antennas. The array processing system consists of 3 unified 4 antenna users. Decoding method and constellation size is chosen the same as those of Fig. 2.5. For the experiments shown in Figs. 2.2, 2.3, and 2.4 we have considered 2 receive antennas at the receiver. In Figs. 2.5 and 2.6, however, we used the same number of receive antennas as that of the transmit antennas. The reason for this is that, many codes in the literature need at least as many receive antennas as the number of transmit antennas. This simulation gives a chance to compare the new scheme with those codes.

Note that both OSTBC and QOSTBC schemes have higher diversity order than our proposed scheme, and will take over at high enough Signal-to-Noise-Ratio.

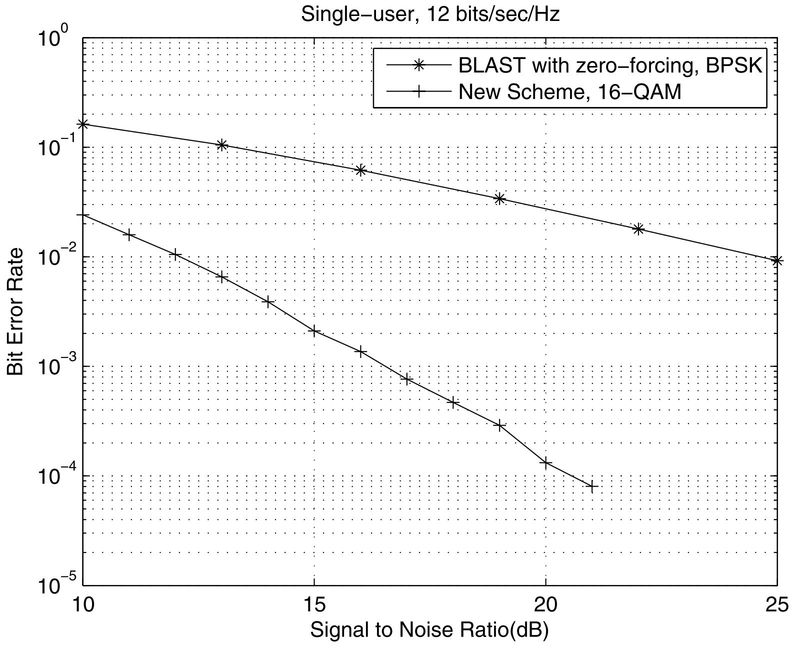

Fig. 2.6 Bit error probability vs. SNR for the new array processing scheme, and BLAST-ZF at 12 bits/s/Hz; 12 transmit and 12 receive antennas

However, our scheme performs better in a practical range of SNR and error rate. In the following, we summarize the main results of this chapter.

1. When there is $J > 2$ users each with $N > 2$ transmit antennas using a complex constellation, and orthogonal design, we show that one cannot perform interference cancellation with $M = J$ or less receive antennas.
2. When using a *real* constellation, we offer an interference cancellation technique that works for any number of users, J, having any number of transmit antennas; our algorithm only requires $M = J$ receive antennas, using real orthogonal designs.
3. For $N > 2$ transmit antenna equipped users using a complex constellation, we offer an interference cancellation technique that works for any number of users; again, our technique only requires receive antennas as many as the number of users.
4. We offer a joint array processing and space-time coding scheme for point to point communication (single user). The resulting code outperforms a number of popular modulation schemes, e.g. BLAST, OSBTC, and QOSTBC.

http://www.springer.com/978-90-481-9989-1

Coping with Interference in Wireless Networks
Kazemitabar, S.J.
2011, XV, 102 p., Hardcover
ISBN: 978-90-481-9989-1