Preface

“In other words, the invention of a mechanism will be to the scientific kinematist a synthetic problem, - which he can solve by the use of systematic, if also difficult, methods.”

Reuleaux, F., Theoretische Kinematik, Braunschweig: Vieweg, 1875

This book represents the third part of a larger work dedicated to the structural synthesis of parallel robots. Part 1 (Gogu 2008a) presented the methodology of structural synthesis and the systematisation of structural solutions of simple and complex limbs with two to six degrees of connectivity systematically generated by the structural synthesis approach. Part 2 (Gogu 2009a) presented structural solutions of translational parallel robotic manipulators with two and three degrees of mobility. This book focuses on various topologies of parallel robotic manipulators with planar motion of the moving platform systematically generated by using the structural synthesis approach proposed in Part 1.

The originality of this work resides in the fact that it combines the new formulae for mobility connectivity, redundancy and overconstraints, and the evolutionary morphology in a unified approach of structural synthesis giving interesting innovative solutions for parallel mechanisms.

Parallel robotic manipulators can be considered a well-established option for many different applications of manipulation, machining, guiding, testing, control, tracking, haptic force feedback, etc. A typical parallel robotic manipulator consists of a mobile platform connected to the base (fixed platform) by at least two kinematic chains called limbs. The mobile platform can achieve between one and three independent translations (T) and one to three independent rotations (R).

Parallel manipulators have been the subject of study of much robotic research during the last two decades. Early research on parallel manipulators has concentrated primarily on six degrees of freedom (DoFs) Gough-
Stewart-type PMs introduced by Gough for a tire-testing device, and by Stewart for flight simulators. In the last decade, PMs with fewer than 6-DoFs attracted researchers’ attention. Lower mobility PMs are suitable for many tasks requiring less than six DoFs.

The motion freedoms of the end-effector are usually coupled together due to the multi-loop kinematic structure of the parallel manipulator. Hence, motion planning and control of the end-effector for PMs usually become very complicated. With respect to serial manipulators, such mechanisms can offer advantages in terms of stiffness, accuracy, load-to-weight ratio, dynamic performances. Their disadvantages include a smaller workspace, complex command and lower dexterity due to a high motion coupling, and multiplicity of singularities inside their workspace. Uncoupled, fully-isotropic and maximally regular PMs can overcome these disadvantages.

Isotropy of a robotic manipulator is related to the condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is equal to one. We know that the Jacobian matrix of a robotic manipulator is the matrix mapping (i) the actuated joint velocity space on the end-effector velocity space, and (ii) the static load on the end-effector and the actuated joint forces or torques. The isotropic design aims at ideal kinematic and dynamic performance of the manipulator.

We distinguish five types of PMs (i) maximally regular PMs, if the Jacobian \( J \) is an identity matrix throughout the entire workspace, (ii) fully-isotropic PMs, if the Jacobian \( J \) is a diagonal matrix with identical diagonal elements throughout the entire workspace, (iii) PMs with uncoupled motions if \( J \) is a diagonal matrix with different diagonal elements, (iv) PMs with decoupled motions, if \( J \) is a triangular matrix and (v) PMs with coupled motions if \( J \) is neither a triangular nor a diagonal matrix. Maximally regular and fully-isotropic PMs give a one-to-one mapping between the actuated joint velocity space and the external velocity space.

The first solution for a fully-isotropic \( T3 \)-type translational parallel robot was developed at the same time and independently by Carricato and Parenti-Castelli at University of Genoa, Kim and Tsai at University of California, Kong and Gosselin at University of Laval, and the author of this work at the French Institute of Advanced Mechanics. In 2002, the four groups published the first results of their works.

The general methods used for structural synthesis of parallel mechanisms can be divided into three approaches: the method based on displacement group theory, the methods based on screw algebra, and the
method based on the theory of linear transformations. The method proposed in this work is based on the theory of linear transformations and the evolutionary morphology and allows us to obtain the structural solutions of decoupled, uncoupled, fully-isotropic and maximally regular PMs with two to six DoFs in a systematic way. The new formulae for mobility, connectivity (spatiality), redundancy and overconstraint of PMs proposed recently by the author are integrated into the synthesis approach developed in this work.

Various solutions of $TaRb$-type PMs are known today. In this notation, $a=1,2,3$ indicates the number of independent translations and $b=1,2,3$ the number of independent rotations of the moving platform. The parallel robots actually proposed by the robot industry have coupled and decoupled motions and just some isotropic positions in their workspace. As far as we are aware, this is the first work on robotics presenting solutions of uncoupled, fully-isotropic and maximally regular PMs along with coupled solutions obtained by a systematic approach of structural synthesis.

Non-redundant/redundant, overconstrained/isostatic solutions of uncoupled and fully-isotropic/maximally regular PMs with elementary/complex limbs actuated by linear/rotary actuators with/without idle mobilities and two to six DoFs are present in a systematic approach of structural synthesis. A serial kinematic chain is associated with each elementary limb and at least one closed loop is integrated in each complex limb.

The synthesis methodology and the solutions of PMs presented in this work represent the outcome of some recent research developed by the author in the last years in the framework of the projects ROBEA-MAX and ROBEA-MP2 supported by the National Center for Scientific Research (CNRS). These results have been partially published by the author in the last years. In these works the author has proposed the following for the first time in the literature:

a) new formulae for calculating the degree of mobility, the degree of connectivity (spatiality), the degree of redundancy and the number of overconstraints of parallel robotic manipulators that overcome the drawbacks of the classical Chebychev-Grübler-Kutzbach formulae,

b) a new approach to systematic innovation in engineering design called evolutionary morphology,

c) solutions of $TaRb$-type fully-isotropic and maximally regular PMs for any combination of $a$ independent translations and $b$ independent rotations of the moving platform.

The various solutions of maximally regular PMs proposed by the author belong to a modular family called Isoglide-$TaRb$ with $a+b=n$ with $2 \leq n \leq 6$, $a=1,2,3$ and $b=1,2,3$. The mobile platform of these robots can have any combination of $n$ independent translations ($T$) and rotations ($R$).
The Isoglide-TaRb modular family was developed by the author and his research team of the Mechanical Engineering Research Group (LaMI), Blaise Pascal University and French Institute of Advanced Mechanics (IFMA) in Clermont-Ferrand.

Part 1 of this work (Gogu, 2008a) was organized in ten chapters. The first chapter introduced the main concepts, definitions and components of the mechanical robotic system. Chapter 2 reviewed the contributions in mobility calculation systematized in the so called Chebychev-Grübler-Kutzbach mobility formulae. The drawbacks and the limitations of these formulae are discussed, and the new formulae for mobility, connectivity, redundancy and overconstraint are demonstrated via an original approach based on the theory of linear transformations. These formulae are applied in chapter 3 for the structural analysis of parallel robots with simple and complex limbs. The new formulae are also applied to calculate the mobility and other structural parameters of single and multi-loop mechanisms that do not obey the classical Chebychev-Grübler-Kutzbach formulae, such as the mechanisms proposed by De Roberval, Sarrus, Bennett, Bricard and other so called “paradoxical mechanisms”. We have shown that these mechanisms completely obey the definitions, the theorems and the formulae proposed in the previous chapter. There is no reason to continue to consider them as “paradoxical”. Chapter 4 presented the main models and performance indices used in parallel robots. We put particular emphasis on the Jacobian matrix, which is the main issue in defining robot kinematics, singularities and performance indices. New kinetostatic performance indices are introduced in this section to define the motion decoupling and input-output propensity in parallel robots. Structural parameters introduced in the second chapter are integrated in the structural synthesis approach founded on the evolutionary morphology (EM) presented in chapter 5. The main paradigms of EM are presented in a closed relation with the biological background of morphological approaches and the synthetic theory of evolution. The main difference between the evolutionary algorithms and the EM are also discussed. The evolutionary algorithms are methods for solving optimization-oriented problems, and are not suited to solving conceptual design-oriented problems. They always start from a given initial population of solutions and do not solve the problem of creating these solutions.

The first stage in structural synthesis of parallel robots is the generation of the kinematic chains called limbs used to give some constrained or unconstrained motion to the mobile platform. The constrained motion of the mobile platform is obtained by using limbs with less than six degrees of connectivity. The various solutions of simple and complex limbs with two to six degrees of connectivity are systematically generated by the structural
synthesis approach and presented in chapters 6-10. We focused on the solutions with a unique basis of the operational velocity space that are useful for generating various topologies of decoupled, uncoupled, fully-isotropic and maximally regular parallel robots presented in Parts 2 and 3. Limbs with multiple bases of the operational velocity space and redundant limbs are also presented in these chapters. These limb solutions are systematized with respect to various combinations of independent motions of the distal link. They are defined by symbolic notations and illustrated in about 250 figures containing more than 1500 structural diagrams. The kinematic chains presented in chapters 6-10 are useful as innovative solutions of limbs in parallel, serial and hybrid robots. In fact, serial and hybrid robots may be considered as a particular case of parallel robots with only one limb which can be a simple, complex or hybrid kinematic chain. Many serial robots actually combine closed loops in their kinematic structure.

The various types of kinematic chains generated in chapters 6-10 of Part 1 are combined in Parts 2 and 3 and the following parts to set up innovative solutions of parallel robots with two to six degrees of mobility and various sets of independent motions of the moving platform.

Part 2 of this work (Gogu, 2009a) was organised in 7 chapters. The first chapter recalled the main concepts, the new formulae used to calculate the main structural parameters of PMs, and the original approach of structural synthesis. Chapter 2 focused on the structural synthesis of $T_2$-type translational parallel manipulators (TPMs) with two degrees of freedom used in pick-and-place operations. Overconstrained/isostatic solutions of coupled, decoupled, uncoupled and fully-isotropic/maximally regular PMs with elementary/complex limbs actuated by linear/rotary actuators with/without idle mobilities are presented. Chapter 3 presented the structural synthesis of overconstrained $T_3$-type translational parallel manipulators with three degrees of freedom and coupled motions. Basic and derived solutions with linear or rotating actuators are presented. The basic solutions do not combine idle mobilities. Idle mobilities are used to reduce the degree of overconstraint in the derived solutions. The structural synthesis of non-overconstrained $T_3$-type TPMs with decoupled motions is presented in chapter 4. Basic and derived solutions with linear or rotating actuators are on hand. Chapters 5 and 6 presented the structural synthesis of overconstrained and non-overconstrained $T_3$-type TPMs with uncoupled motions. Basic and derived solutions with rotating actuators and identical limbs are presented. Chapter 7 focused on the structural synthesis of overconstrained and non-overconstrained maximally regular $T_3$-type TPMs. Basic and derived solutions with linear actuators and identical limbs are on hand. About 1000 solutions of TPMs are illustrated in 550 figures. The structural parameters of these solutions are systematized in 134 tables.
This book representing Part 3 is organised in 8 chapters. The first chapter recalls, the main concepts, the new formulae used to calculate the main structural parameters of PMs, and the original approach of structural synthesis applied to parallel robots with planar motion of the moving platform. In such a robot, the moving platform can undergo two independent translational motions $T_2$ and one rotational motion $R_1$ around an axis perpendicular to the plane of translations. This motion can be obtained by using planar or spatial parallel mechanisms. Chapters 2 and 3 present the structural synthesis of overconstrained and non-overconstrained planar parallel robots with coupled motions. Basic and derived fully-parallel and non fully-parallel solutions are on hand. The structural synthesis of overconstrained and non-overconstrained planar parallel robots with uncoupled motions is presented in Chapter 4. Chapter 5 focuses on the structural synthesis of overconstrained and non-overconstrained maximally regular planar parallel robots. Chapters 6 and 7 present the structural synthesis of basic and derived solutions of overconstrained and non-overconstrained spatial parallel robots with coupled and uncoupled planar motions of the moving platform. Chapter 8 focuses on the structural synthesis of overconstrained and non-overconstrained maximally regular spatial parallel robots with planar motion of the moving platform. About 750 solutions are illustrated in 400 figures. The structural parameters of these solutions are systematized in 150 tables.

Special attention was paid to graphic quality of structural diagrams to ensure a clear correspondence between the symbolic and graphic notation of joints and the relative position of their axes. The graphic illustration of the various solutions is associated with the author’s conviction that a good structural diagram really “is worth a thousand words”, especially when you are trying to disseminate the result of the structural synthesis of kinematic chains.

The following parts of this work will present the structural synthesis of other PMs with two and three degrees of freedom (Part 4) and PMs with four, five and six degrees of freedom (Part 5). The writing of Parts 4 and 5 is still in progress and will soon be finalized.

Many solutions for parallel robots obtained through this systematic approach of structural synthesis are presented, in this work, for the first time in the literature. The author had to make a difficult and challenging choice between protecting these solutions through patents, and releasing them directly into the public domain. The second option was adopted by publishing them in various recent scientific publications and mainly in this work. In this way, the author hopes to contribute to a rapid and widespread implementation of these solutions in future industrial products.
Acknowledgements

The scientific environment of the projects ROBEA-MAX and ROBEA-MP2 supported by the CNRS was the main source of encouragement and motivation to pursue the research on the structural synthesis of parallel robots and to finalize this work. Deep gratitude is expressed here to Dr. François Pierrot, Deputy Director of LIRMM and coordinator of both ROBEA projects, and also to all colleagues involved in these projects from the research laboratories LIRMM, INRIA, IRCCyN LASMEA and LaMI for the valuable scientific exchanges during the joint work on these projects.

Moreover, financial support from the CNRS, FR TIMS and IFMA for developing the innovative Isoglide-family of parallel robots is duly acknowledged.

Furthermore, Prof. Graham M.L. Gladwell, the series editor of Solids Mechanics and Its Applications, and Mrs. Nathalie Jacobs, Springer Dordrecht Engineering Editor are gratefully acknowledged for their availability and encouragement in pursuing this publishing project.

Ms. Sarah Davey is also gratefully acknowledged for the linguistic reviewing of this manuscript.

May I also acknowledge the excellent facilities and research environment provided by LaMI and IFMA which contributed actively to the completion of this project.

To conclude, I cannot forget my wife Iléana and my son Christian for their love, affection and encouragement, providing the fertile ambience for this sustained work very often prolonged late into the evening and mostly during week-ends and holidays.
**List of abbreviations and notations**

*C* - cylindrical joint
*C* - cylindrical joint with one or two idle mobilities
CNRS - Centre National de la Recherche Scientifique (National Center for Scientific Research)
DoF - degree-of-freedom
\(e_A\) and \(e_{G1}\) - link of \(G_1\)-limb \((e=1,2,3,...,n)\)
\(e_B\) and \(e_{G2}\) - link of \(G_2\)-limb \((e=1,2,3,...,n)\)
\(e_C\) and \(e_{G3}\) - link of \(G_3\)-limb \((e=1,2,3,...,n)\)
\(e_D\) and \(e_{G4}\) - link of \(G_4\)-limb \((e=1,2,3,...,n)\)
EM - evolutionary morphology
\(f_i\) - degree of mobility of the \(i^{th}\) joint
\(F \leftarrow G_{1}\rightarrow G_{2}\rightarrow ... \rightarrow G_{k}\) general notation for the kinematic chain associated to a parallel mechanism with \(k\) simple and/or complex limbs \(G_i\) \((i=1,2,...,k)\)
FR TIMS - Fédération de Recherche Technologies de l’Information, de la Mobilité et de la Sûreté
\(G_i\) \((G_{1i}\rightarrow G_{2i}\rightarrow ... \rightarrow G_{ni})\) - kinematic chain associated to the \(i^{th}\) limb
\(H\) - characteristic point of the distal link/end-effector
IFMA - Institut Français de Mécanique Avancée (French Institute of Advanced Mechanics)
IFToMM - International Federation for the Promotion of Mechanism and Machine Science
INRIA - Institut National de Recherche en Informatique et en Automatique (The French National Institute for Research in Computer Science and Control)
IRCCyN - Institut de Recherche en Communications et Cybernétique de Nantes
\(I_{n\times n}\) - \(n \times n\) identity matrix
\(J\) - Jacobian matrix
\(k\) - total number of limbs in the parallel manipulator
\(k_1\) - number of simple limbs in the parallel manipulator
\(k_2\) - number of complex limbs in the parallel manipulator
LaMI - Laboratoire de Mécanique et Ingénieries (Mechanical Engineering Research Group)
LASMEA - Laboratoire des Sciences et Matériaux pour l’Electronique, et d’Automatique (Laboratory of Sciences and Materials for Electronic, and of Automatic)
LIRMM - Laboratoire d’Informatique, de Robotique et de Micro-électronique de Montpellier (Montpellier Laboratory of Computer Science, Robotics, and Microelectronics)

$m$ - total number of links including the fixed base

$M_F$ - mobility of parallel mechanism $F$

$M_{Gi}$ - mobility of the kinematic chain associated with limb $G_i$

$N_F$ - number of overconstraints in the parallel mechanism $F$

$n=\sum_{i} n_{Gi}$ - moving platform in the parallel mechanism $F: G_1-G_2-\ldots-G_k$

$O_{0x0y0z0}$ - reference frame

$p$ - total number of joints in the parallel mechanism

$p_{Gi}$ - number of joints in $G_i$-limb

$P$ - prismatic joint

$P_a$ - actuated prismatic joint

$P^*$ - prismatic joint with idle mobility

$Pa - R||R||R||R$-type planar parallelogram loop

$Pa - R||R||R||R$-type parallelogram loop with an actuated revolute joint

$Pa^*$ or $Pa^{cs}$ - $R||R||C$-type parallelogram loop with three idle mobilities combined in a cylindrical and a spherical joint

$Pa^c$ - $R||R||R||C$-type parallelogram loop with one idle mobility combined in a cylindrical joint

$Pa^{su}$ - parallelogram loop with three idle mobilities combined in a spherical and a revolute joint

$Pa^s$ - $R||R||R$-type parallelogram loop with two idle mobilities combined in a spherical joint

$Pa^{ss}$ - $R||R$-type parallelogram loop with idle mobilities combined in two spherical joints adjacent to the same link

$Pa^t$ - $R\perp R\perp R$-type telescopic planar parallelogram loop

$Pa^{tcs}$ - telescopic parallelogram loop with three idle mobilities combined in a cylindrical and a spherical pair

$Pa^u$ - parallelogram loop with one idle mobility combined in a universal joint

$Pa^{uu}$ - parallelogram loop with two idle mobilities combined in two universal joints

$PM$ - parallel manipulator

$Pn2$ - planar close loop with two degrees of mobility

$Pn2^*$ or $Pn2^{cs}$ - close loop with two degrees of mobility and three idle mobilities combined in a cylindrical and a spherical pair

$Pn3$ - planar close loop with three degrees of mobility

$Pn3^*$ or $Pn3^{cs}$ - close loop with three degrees of mobility and three idle mobilities combined in a cylindrical and a spherical pair

$PPM$ - planar parallel manipulator

$q$ - number of independent closed loops in the parallel mechanism
\( \dot{q} \) - joint velocity vector

\( q_i \) - finite displacement in the \( i \)th actuated joint

\( r_F \) - total number of joint parameters that lose their independence in the closed loops combined in parallel mechanism \( F \)

\( r_l \) - total number of joint parameters that lose their independence in the closed loops combined in the \( k \) limbs

\( r_{Gi} \) - number of joint parameters that lost their independence in the closed loops combined in \( G_i \)-limb,

\( R \) - revolute joint

\( R^{\text{actuated}} \) - actuated revolute joint

\( R^* \) - revolute joint with idle mobility

\( R_b \) - rhombus loop

\( R_b^* \) or \( R_b^{cs} \) - planar rhombus loop with three idle mobilities combined in a cylindrical and a spherical joint

\( R_F \) - the vector space of relative velocities between the mobile and the reference platforms in the parallel mechanism \( F \leftarrow G_1-G_2-...-G_k \),

\( (R_F) \) - the basis of vector space \( R_F \)

\( R_{Gi} \) - the vector space of relative velocities between the mobile and the reference platforms in the kinematic chain \( G_i \) disconnected from the parallel mechanism \( F \leftarrow G_1-G_2-...-G_k \),

\( (R_{Gi}) \) - the basis of vector space \( R_{Gi} \)

\( S \) - spherical joint

\( S^* \) - spherical joint with idle mobilities

\( S_F \) - the connectivity between the mobile and the reference platforms in the parallel mechanism \( F \leftarrow G_1-G_2-...-G_k \),

\( S_{Gi} \) - the connectivity between the mobile and the reference platforms in the kinematic chain \( G_i \) disconnected from the parallel mechanism \( F \leftarrow G_1-G_2-...-G_k \).

SPM - spatial parallel manipulator

\( T_F \) - degree of structural redundancy of parallel mechanism \( F \)

TPM - translational parallel manipulator

\( U \) - universal joint

\( U^* \) - universal joint with an idle mobility

\( v, v_1, v_2, v_3 \) - translational velocity vectors

\( x, y, z \) - coordinates of characteristic point \( H \)

\( \dot{x}, \dot{y}, \dot{z} \) - time derivatives of coordinates

\( \alpha, \beta, \delta \) - rotation angles

\( \dot{\alpha}, \dot{\beta}, \dot{\delta} \) - time derivatives of the rotation angles

\( \omega, \omega_\alpha, \omega_\beta, \omega_\delta \) - angular velocity vectors

\( \theta \) - fixed base of a kinematic chain/mechanism
1\equiv l_{G_i} - fixed platform in the parallel mechanism \( F \leftarrow G_1 - G_2 - \ldots - G_k \),
\( l_{G_i} - 2_{G_i} - \ldots - n_{G_i} \) - links of limb \( G_i \),
\( l_A - 2_A - \ldots - n_A \) - links of limb \( G_1 \),
\( l_{B_i} - 2_{B_i} - \ldots - n_{B_i} \) - links of limb \( G_2 \),
\( l_C - 2_C - \ldots - n_C \) - links of limb \( G_3 \),
\( l_D - 2_D - \ldots - n_D \) - links of limb \( G_4 \).

1 and 2 in the notation 2PRR-1RPaPa - the parallel mechanism has two limb of type PRR and one limb of type RPaPa.

\( || \) - parallel position of joint axes/directions; for example the notation \( Pa||Pa^{ss} \) indicates the fact that the axes of the revolute joints of the parallelogram loops \( Pa \) and \( Pa^{ss} \) are parallel.

\( \perp \) - perpendicular position of joint axes/directions; for example the notation \( P \perp Pa \) indicates the fact that the axes of revolute joints in the parallelogram loop are perpendicular to the direction of the prismatic joint.

\( \perp || \) in the notation \( R \perp Pa \perp ||C \) - the axis of the cylindrical joint is perpendicular to the direction of the actuated prismatic joint and parallel to the direction of the revolute joint.

\( \perp || \) in the notation \( R \perp Pa \perp Pa \) - the revolute axes of the second parallelogram loop are perpendicular to the revolute axes of the first parallelogram loop and parallel to the axis of the actuated revolute joint.

\( \perp \perp \) in the notation \( R \perp Pa \perp Pa \) - the revolute axes of the second parallelogram loop are perpendicular to the revolute axes of the first parallelogram loop and also to the axis of the actuated prismatic joint.

\( \perp \perp \) in the notation \( Pa^{ss} \perp R||R \perp Pa \) - the revolute axes of parallelogram loop \( Pa \) are perpendicular to the axes of the parallel revolute joints \( R || R \) and also to the axes of the revolute joints of parallelogram loop \( Pa^{ss} \).
Structural Synthesis of Parallel Robots
Part 3: Topologies with Planar Motion of the Moving Platform
Gogu, G.
2010, XVIII, 686 p., Hardcover