Contents

1 Background to Research .......................................................... 1
   1.1 Introduction ........................................................................ 1
   1.2 The DFT and Its Efficient Computation ............................... 2
   1.3 Twentieth Century Developments of the FFT ......................... 4
   1.4 The DHT and Its Relation to the DFT ................................. 6
   1.5 Attractions of Computing the Real-Data DFT via the FHT ......... 7
   1.6 Modern Hardware-Based Parallel Computing Technologies .... 8
   1.7 Hardware-Based Arithmetic Units .................................... 9
   1.8 Performance Metrics ................................................... 10
   1.9 Basic Definitions ....................................................... 11
   1.10 Organization of the Monograph ....................................... 12
   References ...................................................................... 13

2 Fast Solutions to Real-Data Discrete Fourier Transform .............. 15
   2.1 Introduction ........................................................................ 15
   2.2 Real-Data FFT Algorithms ............................................ 16
       2.2.1 The Bergland Algorithm ...................................... 16
       2.2.2 The Brunn Algorithm ......................................... 18
   2.3 Real-From-Complex Strategies ........................................ 19
       2.3.1 Computing One Real-Data DFT via One
            Full-Length Complex-Data FFT .............................. 20
       2.3.2 Computing Two Real-Data DFTs via One
            Full-Length Complex-Data FFT .............................. 20
       2.3.3 Computing One Real-Data DFT via One
            Half-Length Complex-Data FFT ............................. 22
   2.4 Data Re-ordering ....................................................... 23
   2.5 Discussion .............................................................. 24
   References ...................................................................... 25

3 The Discrete Hartley Transform ............................................ 27
   3.1 Introduction ........................................................................ 27
   3.2 Normalization of DHT Outputs ........................................ 28
   3.3 Decomposition into Even and Odd Components .................... 29
3.4 Connecting Relations Between DFT and DHT .................. 29
    3.4.1 Real-Data DFT ............................................. 30
    3.4.2 Complex-Data DFT ........................................ 30
3.5 Fundamental Theorems for DFT and DHT ....................... 31
    3.5.1 Reversal Theorem ........................................ 32
    3.5.2 Addition Theorem ........................................ 33
    3.5.3 Shift Theorem ........................................... 34
    3.5.4 Convolution Theorem .................................... 34
    3.5.5 Product Theorem ........................................ 35
    3.5.6 Autocorrelation Theorem ................................ 35
    3.5.7 First Derivative Theorem ................................ 35
    3.5.8 Second Derivative Theorem .............................. 36
    3.5.9 Summary of Theorems .................................... 36
3.6 Fast Solutions to DHT ......................................... 37
3.7 Accuracy Considerations ...................................... 39
3.8 Discussion ..................................................... 39
References ............................................................ 40

4 Derivation of the Regularized Fast Hartley Transform ........ 41
    4.1 Introduction .................................................. 41
    4.2 Derivation of the Conventional Radix-4 Butterfly Equations 42
    4.3 Single-to-Double Conversion of the Radix-4 Butterfly Equations 45
    4.4 Radix-4 Factorization of the FHT ............................ 46
    4.5 Closed-Form Expression for Generic Radix-4 Double Butterfly .... 48
    4.5.1 Twelve-Multiplier Version of Generic Double Butterfly .... 54
    4.5.2 Nine-Multiplier Version of Generic Double Butterfly ....... 54
    4.6 Trigonometric Coefficient Storage, Accession and Generation .... 56
        4.6.1 Minimum-Arithmetic Addressing Scheme ............... 57
        4.6.2 Minimum-Memory Addressing Scheme .................. 57
        4.6.3 Trigonometric Coefficient Generation via Trigonometric Identities 58
    4.7 Comparative Complexity Analysis with Existing FFT Designs .... 59
    4.8 Scaling Considerations for Fixed-Point Implementation ......... 61
    4.9 Discussion .................................................. 62
References ............................................................ 63

5 Algorithm Design for Hardware-Based Computing Technologies ... 65
    5.1 Introduction .................................................. 65
    5.2 The Fundamental Properties of FPGA and ASIC Devices ......... 66
    5.3 Low-Power Design Techniques ................................ 67
        5.3.1 Clock Frequency ........................................ 68
        5.3.2 Silicon Area ............................................ 68
        5.3.3 Switching Frequency ................................... 70
    5.4 Proposed Hardware Design Strategy .......................... 70
        5.4.1 Scalability of Design .................................. 71
6 Derivation of Area-Efficient and Scalable Parallel Architecture .... 77
6.1 Introduction .................................................................. 77
6.2 Single-PE Versus Multi-PE Architectures ...................... 78
6.3 Conflict-Free Parallel Memory Addressing Schemes .......... 80
6.3.1 Data Storage and Accession .................................. 80
6.3.2 Trigonometric Coefficient Storage, Accession and Generation ................................................. 84
6.4 Design of Pipelined PE for Single-PE Architecture ........... 89
6.4.1 Internal Pipelining of Generic Double Butterfly .......... 90
6.4.2 Space Complexity Considerations ............................ 91
6.4.3 Time Complexity Considerations ............................. 92
6.5 Performance and Requirements Analysis of FPGA Implementation ................................................................. 93
6.6 Constraining Latency Versus Minimizing Update-Time ....... 95
6.7 Discussion .................................................................. 97
References ...................................................................... 98

7 Design of Arithmetic Unit for Resource-Constrained Solution ....101
7.1 Introduction .................................................................. 101
7.2 Accuracy Considerations .............................................. 102
7.3 Fast Multiplier Approach ........................................... 103
7.4 CORDIC Approach .................................................... 104
7.4.1 CORDIC Formulation of Complex Multiplier ............... 104
7.4.2 Parallel Formulation of CORDIC-Based PE ................. 105
7.4.3 Discussion of CORDIC-Based Solution ..................... 106
7.4.4 Logic Requirement of CORDIC-Based PE .................. 109
7.5 Comparative Analysis of PE Designs ................................. 110
7.6 Discussion .................................................................. 112
References ...................................................................... 115

8 Computation of $2^n$-Point Real-Data Discrete Fourier Transform ... 117
8.1 Introduction .................................................................. 117
8.2 Computing One DFT via Two Half-Length Regularized FHTs .......... 118
8.2.1 Derivation of $2^n$-Point Real-Data FFT Algorithm .......... 119
8.2.2 Implementational Considerations ............................. 122
8.3 Computing One DFT via One Double-Length Regularized FHT ....129
8.3.1 Derivation of $2^n$-Point Real-Data FFT Algorithm .......... 129
8.3.2 Implementational Considerations ............................. 130
9 Applications of Regularized Fast Hartley Transform

9.1 Introduction ............................................................. 135
9.2 Fast Transform-Space Convolution and Correlation .............. 136
9.3 Up-Sampling and Differentiation of Real-Valued Signal .......... 137
  9.3.1 Up-Sampling via Hartley Space ................................ 138
  9.3.2 Differentiation via Hartley Space............................... 139
  9.3.3 Combined Up-Sampling and Differentiation ................. 139
9.4 Correlation of Two Arbitrary Signals ............................... 140
  9.4.1 Computation of Complex-Data Correlation via Real-Data Correlation ........................................ 141
  9.4.2 Cross-Correlation of Two Finite-Length Data Sets .......... 142
  9.4.3 Auto-Correlation: Finite-Length Against Infinite-Length Data Sets ............................................. 143
  9.4.4 Cross-Correlation: Infinite-Length Against Infinite-Length Data Sets ............................................. 145
  9.4.5 Combining Functions in Hartley Space ....................... 147
9.5 Channelization of Real-Valued Signal ................................ 149
  9.5.1 Single Channel: Fast Hartley-Space Convolution .......... 149
  9.5.2 Multiple Channels: Conventional Polyphase DFT Filter Bank ............................................. 151
9.6 Discussion .............................................................. 155
References ...................................................................... 156
Appendix B  Source Code Listings for Regularized Fast Hartley Transform ................................................173
  B.1 Listings for Main Program and Signal Generation Routine .......... 173
  B.2 Listings for Pre-processing Functions ....................................185
  B.3 Listings for Processing Functions .........................................189

Glossary .........................................................................................221

Index .............................................................................................223
The Regularized Fast Hartley Transform
Optimal Formulation of Real-Data Fast Fourier
Transform for Silicon-Based Implementation in
Resource-Constrained Environments
Jones, K.
2010, XVII, 200 p. With online files/update., Hardcover