Chapter 2
Characteristics of Hydrologic Systems

Abstract The dynamics of hydrologic systems are governed by the interactions between climate inputs and the landscape. Due to the spatial and temporal variability in climate inputs and the heterogeneity in the landscape, hydrologic systems exhibit a wide range of characteristics. While some characteristics may be specific to certain systems and situations, most hydrologic systems often exhibit a combination of these characteristics. This chapter discusses many of the salient characteristics of hydrologic systems, including complexity, correlation, trend, periodicity, cyclicity, seasonality, intermittency, stationarity, nonstationarity, linearity, nonlinearity, determinism, randomness, scale and scale-invariance, self-organization and self-organized criticality, threshold, emergence, feedback, and sensitivity to initial conditions. The presentation focuses on the occurrence, form, and role of each of these characteristics in hydrologic system dynamics and the methods for their identification. At the end, a particularly interesting property of hydrologic systems, wherein simple nonlinear deterministic systems with sensitive dependence on initial conditions can give rise to complex and ‘random-looking’ dynamic behavior, and popularly known as ‘chaos,’ is also highlighted.

2.1 Introduction

Hydrologic systems are often complex heterogeneous systems and function in synchronization with other Earth systems. Hydrologic phenomena arise as a result of interactions between climate inputs and landscape characteristics that occur over a wide range of space and time scales. Considering the spatial scale, our interest in hydrologic studies may be the entire hydrologic cycle or a continental-scale river basin or a medium-size catchment or a small creek in a forest or a 10 m × 10 m plot on a farm or some area even finer. Similarly, considering the temporal scale, we may be interested in studying processes at decadal or annual or monthly or daily or hourly or even finer intervals. The appropriate spatial and temporal scales for hydrologic studies are often dictated by the purpose at hand. For example, for medium- to long-term water resources planning and management for a region,
monthly or annual or decadal scale in time and river-basin scale in space are normally more appropriate than other scales; on the other hand, for design of drainage structures in a city, hourly or even finer-resolution temporal scale and a spatial scale in the order of a kilometer or even less are generally more appropriate than others.

Models that can adequately mimic real hydrologic systems are vital for reliable assessment of the overall landscape changes, understanding of the specific processes, and forecasting of the future events. However, development of such models crucially depends on our ability to properly identify the level of complexity of hydrologic systems and understand the associated processes in the first place. This has always been an extremely difficult task, and will probably become even more challenging, especially with the continuing explosion in human population and changes to our landscapes and rivers, not to mention the influence of various external factors, including those related to climate and other Earth systems with which hydrologic systems interact.

This chapter presents information about some of the inherent and salient characteristics of hydrologic systems, which could offer important clues as to the development of appropriate models. The term ‘system’ herein is defined as a combination of hydrologic process, scale, and purpose, as appropriate, in addition to ‘catchment’ in its general sense. These features may include, among others: complexity, correlation, trend, seasonality, cyclicity, stationarity, nonstationarity, linearity, nonlinearity, periodicity, quasi-periodicity, non-periodicity, intermittency, determinism, randomness, scaling or fractals, self-organized criticality, thresholds, emergence, and sensitivity to initial conditions and chaos. Depending upon the system under consideration, any or all of these characteristics may come into the picture. The discussion herein on these characteristics is with particular emphasis on hydrologic time series, consistent with the focus of this book. Extensive details on these characteristics with particular reference to hydrologic systems can be found in many books, including those by Yevjevich (1972), Chow et al. (1988), Isaaks and Srivastava (1989), Maidment (1993), Haan (1994), Salas et al. (1995), Rodriguez-Iturbe and Rinaldo (1997), and McCuen (2003).

### 2.2 Complexity

Although the words ‘complex’ and ‘complexity’ are widely used both in scientific theory and in common practice, there is no general consensus on the definition. The difficulty in arriving at a consensus definition comes from the fact that it is oftentimes subjective; what is ‘complex’ for one person may not be complex at all for another person, or even when viewed by the same person from a different perspective or at a different time. Nevertheless, one workable definition may be: “something (or some situation) with inter-connected or inter-woven parts.” Such a definition is often tied to the concept of a ‘system’ (see Chap. 1, Sect. 1.4). For physical and dynamic systems, such as the ones encountered in hydrology, the term
`complexity’ often refers to the ‘degree’ to which the components engage in organized structured interactions. With this definition, however, it is also important to clarify why the nature of a complex system is inherently related to its parts, since simple systems are also formed out of parts. Therefore, to explain the difference between ‘simple’ and ‘complex’ systems, the terms ‘interconnected’ or ‘interwoven’ are essential.

Qualitatively, to understand the behavior of a complex system, we must understand not only the behavior of the parts but also how they act together to form the behavior of the whole. This is because: (1) we cannot describe the whole without describing each part; and (2) each part must be described also in relation to other parts. As a result, complex systems are often difficult to understand. This is relevant to another definition of ‘complex’: ‘not easy to understand or analyze.’ These qualitative ideas about what a complex system is can be made more quantitative. Articulating them in a clear way is both essential and fruitful in pointing out the way toward progress in understanding the universal properties of these systems.

For a quantitative description, the central issue again is defining quantitatively what ‘complexity’ means. In the context of systems, it may perhaps be useful to ask: (1) What do we mean when we say that a system is complex? (2) What do we mean when we say that one system is more complex than another? and (3) Is there a way to identify the complexity of one system and to compare it with the complexity of another system? To develop a quantitative understanding of complexity, a variety of tools can be used. These may include: statistical (e.g. coefficient of variation), nonlinear dynamic (e.g. dimension), information theoretic (e.g. entropy) or some other measure.

In this book, the complexity of a system is essentially taken to be a quantitative measure of the variability of time series under consideration. Further, in the specific context of nonlinear dynamic methods (see Part B) and their applications in hydrology (see Part C), the variability is generally represented by reconstruction of a time series and determination of the ‘dimensionality’ (or related measure) and, thus, the complexity is related to the number of variables dominantly governing the system that produced the time series; in other words, the amount of information necessary to describe the system.

During the past few decades, numerous attempts have been made to define, qualify, and quantify ‘complexity’ and also to apply complexity-based theories for studying natural and physical systems. Extensive details on these can be found in Ferdinand (1974), Cornacchio (1977), Nicolis and Prigogine (1989), Waldrop (1992), Cilliers (1998), Buchanan (2000), Barabási (2002), McMillan (2004), Johnson (2007), and Érdi (2008), among others.

Due to the tremendous variabilities and heterogeneities in climatic inputs and landscape properties, hydrologic systems are often highly variable and complex at all scales (although simplicity is also possible). Consequently, they are not fully understood; indeed, it is even hard to arrive at an acceptable definition of a ‘hydrologic system.’ Sivakumar (2008a) suggests that hydrologic systems may be viewed from three different, but related, angles: process, scale, and purpose of interest. Examples of hydrologic processes are rainfall, streamflow, groundwater
flow, and evaporation. Scale is generally considered in terms of space (e.g. plot scale, landscape scale, river basin scale) and time (e.g. daily, monthly, annual). Examples of purposes in hydrology are characterization, prediction, and aggregation/disaggregation or upscaling/downscaling.

Depending upon the angle at which they are viewed, hydrologic systems may be either simple or complex; for example, the rainfall occurrence in a desert (even at different spatial and temporal scales) may be treated as an extremely simple system since there may be no rainfall at all, while the runoff system in a large river basin may be highly complex due to the basin complexities and heterogeneities, in addition to rainfall variability. The complexity of the hydrologic systems has important implications for hydrologic modeling, since it is the property that essentially dictates the complexity of the model to be developed (and type and amount of data to be collected and computational power required) to obtain reliable results. Consequently, hydrologic modeling must also be viewed from the above three angles; in other words, the appropriate model to represent a given hydrologic system may also be either simple or complex. The obvious question, however, is: how simple or how complex the models should be? There is a plethora of literature on the question of complexity of hydrologic models; see, for example, Jakeman and Hornberger (1993), Young et al. (1996), Grayson and Blöschl (2000), Perrin et al. (2001), Beven (2002), Young and Parkinson (2002), Sivakumar (2004b, 2008a, b), Wainwright and Mulligan (2004), Sivakumar et al. (2007), Sivakumar and Singh (2012), and Jenerette et al. (2012), among others. This issue of complexity in hydrologic systems is extensively addressed in this book in the discussion of nonlinear dynamic and chaos methods and their applications in hydrology, especially through estimation of variability of time series ('dimensionality') and, thus, determination of the number of variables dominantly governing the underlying system dynamics.

2.3 Correlation and Connection

Generally speaking, correlation refers to relation between two or more things, say variables. However, measuring correlation between totally unrelated variables (at least for well-known situations) has no practical relevance. Therefore, correlation must be essentially viewed in the context of dependence or connection between variables; this dependence may be in only one direction (i.e. a variable either influences another or is influenced by another) or in both directions (i.e. a variable influences another and is also influenced by another). In other words, correlation must be associated with causation, at least in one direction (e.g. streamflow is correlated to rainfall). Therefore, in scientific and engineering studies, the purpose of correlation analysis is to measure how strongly pairs of variables (or entities, more broadly) are related or connected, if at all.

Hydrologic systems are complex systems governed by a large number of influencing variables that are also often interdependent. The nature and extent of
interdependencies among hydrologic variables are often different for different systems. Indeed, such interdependencies may even be different for the same system under different conditions or when different components are considered. The hydrologic cycle is a perfect example: each and every component is connected to (i.e. influences and is influenced by) every other component, but the way the components are connected among themselves (e.g. direct or indirect) and the strength of such connections (e.g. strong or weak) vary greatly. For instance, rainfall and streamflow, being components of the hydrologic cycle, are connected, influencing and being influenced by each other. However, the influence of rainfall on streamflow is direct, far more pronounced, and often immediate (notwithstanding evaporation and infiltration), whereas the influence of streamflow on rainfall is not easily noticeable and also very slow (often having to pass through many steps in the functioning of the hydrologic cycle). Unraveling the nature and extent of connections in hydrologic systems, as well as their interactions with other systems, has always been a fundamental challenge in hydrology.

Correlation analysis plays a basic and vital role in the study of hydrologic systems, in almost every context imaginable. For instance, correlation analysis between data collected for a single hydrologic variable at successive times (i.e. temporal correlation) is useful for prediction. Correlation analysis between data collected at different locations for the same hydrologic variable (i.e. spatial correlation) is useful for interpolation and extrapolation. Similarly, correlation analysis between data collected for different hydrologic variables at the same location or at different locations is useful for very many purposes.

Correlation analysis may be performed in different ways and using different methods, depending upon the variable, data, and task at hand. Correlation analysis may involve a single variable, two variables, or multiple variables, and similarly a single location, two locations, or multiple locations. The existing methods for correlation analysis may broadly be classified into linear and nonlinear methods, and include autocorrelation function (Yule 1896), Pearson product moment correlation (Pearson 1895), Spearman’s rank correlation (Spearman 1904), Kendall tau rank correlation (Kendall 1938), regression (e.g. Legendre 1805; Galton 1894), mutual information function (Shannon 1948), and distance correlation (Székely et al. 2007), among others. Extensive accounts of these methods are already available in the literature and, therefore, details are not reported herein. However, the autocorrelation function and mutual information function methods will be briefly discussed in Chap. 7, in the specific context of delay time selection for data reconstruction for chaos analysis.

Due to the significance of correlation in establishing possible connections, correlation analysis has been a key component of research on hydrologic systems. Numerous studies have applied a wide range of correlation and regression techniques to explore hydrologic connections in space, time, space-time, and between/among variables and to use such information for various applications; see, for example, Douglas et al. (2000), McMahon et al. (2007), Skøien and Blöschl (2007), and Archfield and Vogel (2010), among others.
2.4 Trend

A trend is a slow, gradual change in a system over a period of observation, and can be upward, downward or other. A trend thus generally represents a deterministic component and is a useful property to identify system changes and to make future predictions. At the same time, a trend does not repeat, or at least does not repeat within the time range of our system observations (which is normally not that long). A trend may be in a linear form or in a nonlinear form or even a combination of both. In hydrologic systems (and in many other real systems), trend is most often in a nonlinear form.

There are no proven ‘automatic’ methods to identify the trend component in a system. In fact, the presence of a trend cannot be readily identified, since trends and minor system fluctuations are oftentimes indistinguishable. Many methods to identify trends (in time series) have been developed, including the Mann-Kendall test, the linear and nonlinear least-squares regression methods, the moving average method, the exponential smoothing, and the spectral method. Extensive details of these are already available in the literature (e.g. Mann 1945; Box and Jenkins 1970; Kendall 1975; Haan 1994; Chatfield 1996).

In hydrologic systems, trends may be observed in various ways, and a few example situations are as follows. Natural climatic changes may result in gradual changes to the hydrologic environment over a very long period of time. Gradual changes to the landscape over a long or medium timescale, whether natural or man-made, may cause gradual changes to the hydrologic processes. Changes in basin conditions over a period of several years can result in corresponding changes in streamflow characteristics, mainly because of the basin’s response to rainfall. Urbanization and deforestation on a large scale may result in gradual changes in precipitation amounts over time.

Since the presence of trend helps in identifying system evolution and prediction, examination of trend forms one of the most basic analyses in hydrology. Numerous studies have applied the above different trend analysis methods (and their variants) to identify trends in hydrologic systems, in many different contexts and for many different purposes. Some studies have also proposed modifications to the standard trend analysis methods, to be appropriate for hydrologic systems. Extensive details of these studies can be found in, for example, Hirsch et al. (1982), Lettenmaier et al. (1994), Lins and Slack (1999), Douglas et al. (2000), Burn and Hag Elnur (2002), Hamed (2008), and Şen (2014).

2.5 Periodicity, Cyclicity, and Seasonality

In addition to correlation and trend, a system may exhibit a host of properties that are generally deterministic in nature and, thus, facilitate prediction of its evolution. The exact nature and extent of determinism associated with each of such properties
may vary, especially depending upon the scale of consideration. However, a
commonality among almost all of these properties is the presence of some kind of
‘repetition’ or ‘cycle.’ These properties include periodicity, cyclicity, and seasonality. The absence of such properties are then called aperiodicity, acyclicity, and non-seasonality. These are defined below.

Periodicity refers to the property of being repeated at certain regular and periodic intervals, but without exact repetition. Aperiodicity refers to the property of lacking any kind of periodicity. Quasi-periodicity is a property that displays irregular periodicity. Cyclicity refers to the property of being repeated in a cyclic manner. When cyclicity is absent, the property is called acyclic. Seasonality refers to the property of being repeated according to seasons. When seasonality is absent, then the property is called non-seasonality.

Almost all natural, physical, and social systems exhibit, in one way or another, periodic, cyclic, or seasonal properties, depending upon the temporal and spatial scales. The hydroclimate system is a good example for this, including its somewhat repetitive nature at annual, seasonal, and daily scales that allow modeling and prediction of its evolution. Indeed, these properties are intrinsic to the functioning of complex systems. Due to the often nonlinear interactions among the various system components, these properties drive and are driven by several other key properties of complex systems, including self-organization, threshold, emergence, and feedback.

There exist numerous methods and models for identifying periodic, cyclic, and seasonal properties of dynamic systems. Such methods range from very simple ones to highly sophisticated ones, including those based on autocorrelation function, power spectrum, maximum likelihood, runs test, information decomposition, Rayleigh analysis, wavelets, singular value decomposition, and empirical mode decomposition, as well as their variants and hybridizations. These methods have been extensively used to study systems in many different fields.

The properties of periodicity, cyclicity, and seasonality are commonplace in hydrologic systems at many different scales, due to the influence of the climate system that drives hydrologic processes and the nature of the hydrologic cycle in itself. For instance, rainy and flow seasons indicate one form of periodicity, as they occur and span over a certain time every year. Periodicity is reflected in the occurrence of high precipitation and high runoff during some months (e.g. summer) and low precipitation and low runoff during some other months (e.g. winter). Evaporation is another form of periodicity, as it is predominantly a day-time process. Similarly, cyclicity in hydrologic processes also exists within a give year. On the other hand, monthly streamflow series are marked by the presence of seasonality according to the geophysical year.

As these properties play crucial roles in the evolution of a system, their identification is often a fundamental step in the analysis of hydrologic systems. To this end, numerous studies have employed various methods to identify these properties in hydrologic systems. Such an identification is also particularly prominent in the application of stochastic time series methods. There exists a plethora of literature on this, and the interested reader is referred to Yevjevich (1972), Rao and Jeong (1992), Maidment (1993), and Salas et al. (1995), among others.
2.6 Intermittency

Intermittency is the irregular alternation of phases of behavior, i.e. occurrence (say, non-zero) and non-occurrence (say, zero) at irregular intervals. Intermittency is observed in numerous natural, physical, and socio-economic systems.

Due to its basic nature of irregular changes in behavior, intermittency is an extremely challenging property to understand, model, and predict. Conventional time series approaches generally adopt separation of a system (time series) into structural components (trend, periodicity, cyclicity, seasonality) and error (noise) that facilitates the use of standard modeling techniques; see, for example, Box and Jenkins (1970). While such approaches generally work well for non-intermittent systems, they are not suitable for intermittent systems. The fact that there might also be different degrees of intermittency brings additional challenges to modeling intermittent systems. Research over the past few decades have resulted in many different approaches and numerous models to study intermittency. Notable among the models are the point process model, cluster process model, Cox process model, renewal process model, random multiplicative cascades, resampling, normal quantile transform, and many others.

Intermittency is very common in hydrologic systems. For instance, rainfall that is observed in a recording raingage is an intermittent time series. Hourly, daily, and weekly rainfall in many parts of the world are typically intermittent time series. In semi-arid and arid regions, even monthly and annual rainfall and monthly and annual runoff are also often intermittent. In view of this, numerous studies have addressed the issue of intermittency in hydrologic systems. Many studies have also proposed different methods for modeling intermittent time series. Extensive details of such studies are available in Todorovic and Yevjevich (1969), Richardson (1977), Kavvas and Delleur (1981), Waymire and Gupta (1981), Smith and Carr (1983, 1985), Yevjevich (1984), Rodriguez-Iturbe et al. (1987), Delleur et al. (1989), Isham et al. (1990), Copertwait (1991, 1994), Gupta and Waymire (1993), Salas et al. (1995), Gyasi-Agyei and Willgoose (1997), Verhoest et al. (1997), Montanari (2005), Burton et al. (2010), Pui et al. (2012), and Paschalis et al. (2013), among others.

2.7 Stationarity and Nonstationarity

Stationarity is defined as a property of a system where the statistical properties of the system (e.g. mean, variance, autocorrelation) do not change with time. This means that there are no trends. The most important property of a stationary system is that the autocorrelation function depends on lag alone and does not change with the time at which the function is calculated. Weak stationarity refers to a constant mean and variance. True stationarity or strong stationarity means that all higher-order moments (including variance and mean) are constant.
Stationarity is a very common assumption in stochastic methods; see, for example, Cramer (1940), Yevjevich (1972), and Box and Jenkins (1970) for some early details. Such an assumption, however, may not be appropriate for all systems under all conditions, as the statistical properties of most systems, especially complex systems, change over time. In view of this, there has been an increasing attention, in recent years, to address the nonstationarity conditions in stochastic methods. In particular, attempts have been made to extend the classical stochastic approaches to accommodate certain types of nonstationarity. On the other hand, if the type of nonstationarity can be identified and modeled, one can remove such to arrive at a stationary time series to suit the stationarity assumption. However, since nonstationarity can take various forms, such a separation may also be tremendously challenging.

There are two broad approaches for testing stationarity/nonstationarity in a system: parametric and nonparametric. Parametric approaches are based on certain prior assumptions about the nature of the data and are usually used when working in the time domain. Nonparametric approaches do not make any prior assumptions about the nature of the data and are usually used when working in the frequency domain. In recent years, there have also been advances in the time-frequency domain analysis.

There exist many different ways and methods for testing stationarity/nonstationarity in a time series. Among the commonly used methods are the augmented Dickey-Fuller (ADF) unit root test (Dickey and Fuller 1979; Said and Dickey 1984), multi-taper method (Thomson 1982), maximum entropy method (Childers 1978), evolutionary spectral analysis (Priestley 1965), wavelet analysis (Daubechies 1992), and the KPSS test (Kwiatkowski et al. 1992; Shin and Schmidt 1992).

The assumption of stationarity is very common in hydrology, especially with the applications of stochastic methods that have been dominant over several decades now; see, for instance, Thomas and Fiering (1962), Chow (1964), Dawdy and Matalas (1964), Yevjevich (1972), Kottegoda (1980), Hipel and McLeod (1994), Salas et al. (1995), Hubert (2000), Chen and Rao (2002) for details. However, with the recognition of nonstationarity in real time series, studies on nonstationarity in hydrology have also been growing in recent decades (e.g. Potter 1976; Kottegada 1985; Rao and Hu 1986; Hamed and Rao 1998; Young 1999; Cohn and Lins 2005; Coulbaly and Baldwin 2005; Koutsoyiannis 2006; Clarke 2007; Kwon et al. 2007). With recognition of the significant changes in global climate and the anticipated impacts on hydrology and water resources, especially in the form of extreme hydroclimatic events, the need to move away from the traditional stationarity-based approaches and to develop nonstationarity-based approaches for hydrologic modeling, prediction, and design is increasingly realized at the current time (e.g. Milly et al. 2008).
2.8 Linearity and Nonlinearity

In simple terms, linearity represents a situation where changes in inputs will result in proportional changes in outputs. For instance, linearity is a situation in which if a change in any variable at some initial state produces a change in the same or some other variable at some later time, then twice as large a change at the same initial time will produce twice as large a change at the same later time. It follows that if the later values of any variable are plotted against the associated initial values of any variable on graph paper, the points will be on a straight line—hence the name. Nonlinearity, on the other hand, represents a situation where changes in inputs would not produce proportional changes in outputs.

Although the above definitions of linearity and nonlinearity are theoretically accurate, they cannot, and are not, strictly adopted in practice. This is because, according to these definitions, true linearity may exist only in (simple) artificially-created systems; it does not exist in natural systems at all. For instance, any change in the quantity of food purchased will result in a proportional change in the cost (provided that there is no discount for purchasing large quantities!). However, any change in rainfall amount would not result in a proportional change in streamflow (even in catchments that are not complex), since many other factors also influence the conversion of rainfall (input) to streamflow (output).

The fact that a strict definition of linearity does not apply to natural systems does not mean that the assumption of linearity and development of linear models are not relevant and useful for such systems. The importance of the assumption of linearity lies in a combination of two circumstances. First, many tangible phenomena behave approximately linearly over restricted periods of time or restricted areas of space or restricted ranges of variables, so that useful linear mathematical models can simulate their behavior. Second, linear equations can be handled by a wide variety of techniques that do not work with nonlinear equations. These aspects, not to mention the constraints in computational power and measurement technology, mostly led, until at least the mid-twentieth century, to the development and application of linear approaches to natural systems. In recent decades, however, nonlinear approaches have been gaining consideration attention.

Depending on the specific definition of a system (in terms of process, scale, and purpose; see Sect. 2.2), a hydrologic system may be treated as either linear or nonlinear; for example, overland flow in a desert over a few hours of time may be treated as a purely linear system (since a small change in rainfall may not result in any overland flow), while overland flow in a partially-developed or fully-developed catchment over the same period of time is almost always nonlinear (due to the influence of both rainfall and land use properties). In a holistic perspective of “inter-connected or inter-woven parts,” however, all hydrologic systems are inherently nonlinear. The nonlinear behavior of hydrologic systems is evident in various ways and at almost all spatial and temporal scales. For instance, the hydrologic cycle itself is an example of a system exhibiting nonlinear behavior, with almost all of the individual components themselves exhibiting nonlinear
behavior at different temporal and spatial scales. Nevertheless, linearity is also present over certain parts, depending upon the variable and time period, among others.

For the reasons mentioned above, much of the past research in hydrologic systems, including those developing and applying time series methods, had essentially resorted to linear approaches (e.g. Thomas and Fiering 1962; Harms and Campbell 1967; Yevjevich 1972; Valencia and Schaake 1973; Klemes 1978; Beaumont 1979; Kavvas and Delleur 1981; Salas and Smith 1981; Srikanthan and McMahon 1983; Bras and Rodriguez-Iturbe 1985; Salas et al. 1995), although the nonlinear nature of hydrologic systems had already been known for some time (e.g. Minshall 1960; Jacoby 1966; Amorocho 1967, 1973; Dooge 1967; Amorocho and Brandstetter 1971; Bidwell 1971; Singh 1979). In the last few decades, however, a number of nonlinear approaches have been developed and applied for hydrologic systems; see Young and Beven (1994), Kumar and Foufoula-Georgiou (1997), Singh (1997, 1998, 2013), ASCE Task Committee (2000a, b), Govindaraju and Rao (2000), Dibike et al. (2001), Kavvas (2003), Sivakumar (2000, 2004a, 2009), Gupta et al. (2007), Young and Ratto (2009), Şen (2009), Abrahart et al. (2010), and Sivakumar and Berndtsson (2010), among others. More details about the linear approaches and nonlinear approaches, especially in the context of time series methods, are presented in Chaps. 3 and 4, respectively. Among the nonlinear approaches, a particular class is that of nonlinear determinism and chaos, i.e. systems with sensitivity to initial conditions (e.g. Lorenz 1963; Gleick 1987). Section 2.17 presents a brief account of this class, which is also the main focus of this book, as can be seen from the extensive details presented in Part B, Part C, and Part D.

It is appropriate to mention, at this point, that there is still some confusion on the definition of ‘nonlinearity’ in hydrology, and perhaps in many other fields as well. This is highlighted, for example, by Sivapalan et al. (2002), who discuss two definitions of nonlinearity that appear in the hydrologic literature, especially with respect to catchment response. One is with respect to the dynamic property, such as the rainfall-runoff response of a catchment, where nonlinearity refers to a nonlinear dependence of the storm response on the magnitude of the rainfall inputs (e.g. Minshall 1960; Wang et al. 1981), which is also generally the basis in time series methods. The other definition is with respect to the dependence of a catchment statistical property, such as the mean annual flood, on the area of the catchment (e.g. Goodrich et al. 1997). The ideas presented in this book are mainly concerned with the dynamic property of hydrologic processes.

2.9 Determinism and Randomness

Determinism represents a situation where the evolution from an earlier state to a later state(s) occurs according to a fixed law. Randomness (or stochasticity), on the other hand, represents a situation where the evolution from one state to another is
not according to any fixed law but is independent. In natural systems, it is almost impossible to find a completely deterministic or a completely random situation, especially when considered over a long period of time. In such systems, determinism and randomness often co-exist, even if over different times.

In light of determinism and randomness in nature, there have been two corresponding dominant approaches in modeling. In the deterministic modeling approach, deterministic mathematical equations based on well-known scientific laws are used to describe system evolution. In the stochastic approach, probability distributions based on probability concepts are used to assure that certain properties of the system are reproduced. Either approach has its own merits and limitations when applied to natural systems. For instance, the deterministic approach is particularly useful if the purpose is to make accurate prediction of the system evolution, but it also requires accurate knowledge of the governing equations and system details. The stochastic approach, on the other hand, is particularly useful when one is interested in generating possible future scenarios of system properties, but it cannot reproduce important physical processes.

Both the deterministic approach and the stochastic approach have clear merits for studying hydrologic systems. For instance, the deterministic approach has merits considering the ‘permanent’ nature of the Earth and the ‘cyclical’ nature of the associated processes. Similarly, the stochastic approach has merits considering that hydrologic systems are often governed by complex interactions among various components in varying degrees and that we have only ‘limited ability to observe’ the detailed variations. In view of their relevance and usefulness, both these approaches have been extensively employed to study hydrologic systems, but almost always independently (e.g. Darcy 1856; Richards 1931; Sherman 1932; Horton 1933, 1945; Nash 1957; Thomas and Fiering 1962; Yevjevich 1963, 1972; Fiering 1967; Mandelbrot and Wallis 1969; Woolhiser 1971; Srikanthan and McMahon 1983; Bras and Rodriguez-Iturbe 1985; Dooge 1986; Gelhar 1993; Salas and Smith 1981; Salas et al. 1995; Govindaraju 2002).

In spite of, and indeed because of, their differences, the deterministic approach and the stochastic approach can actually be complementary to each other to study hydrologic systems. For instance, in the context of river flow, the deterministic approach is useful to represent the significant deterministic nature present in the form of seasonality and annual cycle, while the stochastic approach is useful to represent the randomness brought by the varying degrees of nonlinear interactions among the various components involved. Therefore, the question of whether the deterministic or the stochastic approach is better for hydrologic systems is often meaningless, and is really a philosophical one. What is more meaningful is to ask whether the two approaches can be coupled to increase their advantages and limit their limitations, for practical applications to specific situations of interest (e.g. Yevjevich 1974; Sivakumar 2004a). This is where ideas of nonlinear deterministic dynamic and chaos theories can be particularly useful to bridge the gap, as they encompass nonlinear interdependence, hidden determinism and order, and sensitivity to initial conditions (e.g. Sivakumar 2004a). Additional details on this are provided in Sect. 2.17 and in Part B.
2.10 Scale, Scaling, and Scale-invariance

The term ‘scale’ may be defined as a characteristic dimension (or size) in either space or time or both. For instance, $1 \text{ km} \times 1 \text{ km}$ ($1 \text{ km}^2$) area is a scale in space, a day is a scale in time, and their combination is a scale in space-time. The term ‘scaling’ is used to represent the link (and transformation) of things between different scales. For instance, the link in a process between $1 \text{ km} \times 1 \text{ km}$ ($1 \text{ km}^2$) and $10 \text{ km} \times 10 \text{ km}$ ($100 \text{ km}^2$) is scaling in space, between daily and monthly scales is scaling in time, and their combination is scaling in space-time. The term ‘scale-invariance’ is used to represent a situation where such links do not change across different scales.

Scale, scaling, and scale-invariance are key concepts and properties in studying natural systems. Their relevance and significance can be explained as follows. Natural phenomena occur at a wide range of spatial and temporal scales. They are generally governed by a large number of components (e.g. variables) that often interact in complex ways. Each of these components and the interactions among themselves may or may not change across different spatial and temporal scales. Therefore, for an adequate understanding of such systems, observations at many different temporal and spatial scales are necessary. Although significant progress has been made in measurement technology and data collection, it is almost impossible to make observations at all the relevant scales, due to technological, financial, and many other constraints.

An enormous amount of effort has been made to study scale-related issues in natural systems, especially in the development and application of methods for downscaling (transferring information from a given scale to a smaller scale) and upscaling (transferring information from a given scale to a larger scale). In this regard, ideas gained from the modern concept of ‘fractal’ or ‘self-similarity’ (e.g. Mandelbrot 1977, 1983) combined with the earlier ideas from the concept of ‘topology’ (e.g. Cantor 1874; Poincaré 1895; Hausdorff 1919) have been extensively used. There exist several methods for identifying scale-invariant behavior and for transformation of data from one scale to another. These methods may largely be grouped under mono-fractal and multi-fractal methods, and include box counting method, power spectrum method, variogram method, empirical probability distribution function method, statistical moment scaling method, and probability distribution multiple scaling method, among others.

The scale-related concepts and issues are highly relevant for hydrologic systems, since hydrologic phenomena arise as a result of interactions between climate inputs and landscape characteristics that occur over a wide range of space and time scales. For instance, unsaturated flows occur in a 1 m soil profile, while floods in major river systems occur over millions of kilometers; similarly, flash floods occur over several minutes only, while flows in aquifers occur over hundreds of years. Hydrologic processes span about eight orders of magnitude in space and time (Klemeš 1983). At least six causes of scale problems with regard to hydrologic responses have been identified (Bugmann 1997; Harvey 1997): (1) spatial
heterogeneity in surface processes; (2) nonlinearity in response; (3) processes require threshold scales to occur; (4) dominant processes change with scale; (5) evolution of properties; and (6) disturbance regimes.

In the study of hydrologic systems, three dominant types of scales are relevant: (1) Process scales—Process scales are defined as the scales at which hydrologic processes occur. These scales are not fixed, but vary with process; (2) Observational scales—Observational scales are the scales at which we choose to collect samples of observations and to study the phenomenon concerned. They are determined by logistics (e.g. access to places of observation), technology (e.g. cost of state-of-the-art instrumentation), and individuals’ perception (i.e. what is perceived to be important for a study at a given point in time); and (3) Operational scales—Operational scales are the working scales at which management actions and operations focus. These are the scales at which information is available. These three scales seldom coincide with each other: we are not able to make observations at the scales hydrologic processes actually occur, and the operational scales are determined by administrative rather than by purely scientific considerations.

Yet another type is the ‘modeling scales,’ which are also ‘working scales.’ They are generally agreed upon within the scientific community and are partly related to processes and partly to the applications of hydrologic models. Typical modeling scales in space are: the local scale—1 m; the hillslope (reach) scale—100 m; the catchment scale—10 km; and the regional scale—1000 km. Typical modeling scales in time are: the event scale—1 day; the seasonal scale—1 year; and the long-term scale—100 years; see, for example, Dooge (1982, 1986). With increasing need to understand hydrologic processes at very large and very small scales, and with the availability of observations at these scales, there are also changes to our modeling scales. However, oftentimes, the modeling scale is much larger or much smaller than the observation scale. Therefore, ‘scaling’ is needed to bridge this gap.

During the past few decades, an extensive amount of research has been devoted for studying the scale issues in hydrologic systems and for transferring hydrologic information from one scale to another (e.g. Mandelbrot and Wallis 1968, 1969; Klemeš 1983; Gupta and Waymire 1983, 1990; Gupta et al. 1986; Stedinger and Vogel 1984; Salas et al. 1995; Kalma and Sivapalan 1996; Puente and Obregon 1996; Tessier et al. 1996; Dooge and Bruen 1997; Rodriguez-Iturbe and Rinaldo 1997; Tarboton et al. 1998; Sivakumar et al. 2001; Sposito 2008). In addition to downscaling and upscaling for transfer of hydrologic information from one scale to another, regionalization is also used to transfer information from one catchment (location) to another, including in the context of ungaged basins (e.g. Merz and Blöschl 2004; Oudin et al. 2008; He et al. 2011); see also Razavi and Coulibaly (2013) for a review. Regionalization may be satisfactory if the catchments are similar (in some sense), but error-prone if they are not (Pilgrim 1983). To this end, there has also been great interest in recent years to identify similar catchments, within the specific context of catchment classification (e.g. Olden and Poff 2003; Snelder et al. 2005; Isik and Singh 2008; Moliere et al. 2009; Kennard et al. 2010; Ali et al. 2012; Sivakumar and Singh 2012), although catchment classification had been attempted in many past studies (e.g. Budyko 1974; Gottschalk et al. 1979;
Haines et al. 1988; Nathan and McMahon 1990); see also Sivakumar et al. (2015) for a review. With increasing interest in studying the impacts of global climate change on water resources, downscaling of coarse-scale global climate model outputs to fine-scale hydrologic variables has also been gaining considerable attention; see Wilby and Wigley (1997), Prudhomme et al. (2002), Wood et al. (2004), and Fowler et al. (2007) for details. Despite our progress in addressing the scale issues in hydrologic systems, many challenges still remain. One of the factors that make scaling so difficult is the heterogeneity of catchments and the variability of hydrologic processes, not to mention the uncertainties in climate inputs and hydrologic data measurements.

2.11 Self-organization and Self-organized Criticality

Self-organization has various and often conflicting definitions. In its most general sense, however, self-organization refers to the formation of patterns arising out of the internal dynamics of a system (i.e. local interactions between the system components), independently of external controls or inputs. Because such an organizing process may offset or intensify the effects of external forcings and boundary conditions, self-organization is often a source of nonlinearity in a system.

The original principle of self-organization was formulated by Ashby (1947), which states that any deterministic dynamic system will automatically evolve towards a state of equilibrium that can be described in terms of an attractor in a basin of surrounding states; see also Ashby (1962). According to von Foerster (1960), self-organization is facilitated by random perturbations (i.e. noise) that let the system explore a variety of states in its state space, which, in turn, increases the chance that the system would arrive into the basin of a ‘strong’ and ‘deep’ attractor, from which it would then quickly enter the attractor itself. This, in other words, is ‘order from noise.’ A similar principle was formulated by Nicolis and Prigogine (1977) and Prigogine and Stenders (1984). The concept of self-organization was further advanced by Bak et al. (1987, 1988), through the introduction of ‘self-organized criticality’ (SOC) to explain the behavior of a cellular automaton (CA) model. Self-organized criticality is a property of dynamic systems that have a critical, or emergent, point as an attractor, through natural processes. Self-organized criticality is linked to fractal structure, $1/f$ noise, power law, and other signatures of dynamic systems; see also Bak (1996), Tang and Bak (1988a, b), and Vespignani and Zapperi (1998) for some additional details.

Hydrologic systems generally exhibit complex nonlinear behaviors and are also often fractal. These properties combine to make hydrologic systems organize themselves in many different ways. For instance, drainage patterns in a landscape and its properties, such as slope, topography, channel properties, soil texture, and vegetation, are the result of very long-term and nonlinear interactions between geology, soils, climate, and the biosphere and, therefore, often organize themselves. The hydroclimatic system, governed by the numerous individual components of
land, ocean, atmosphere and their complex nonlinear interactions, organizes itself as well. Similar observations can be made regarding the ecologic-hydrologic interactions. Therefore, self-organization and self-organized criticality are highly relevant for hydrologic systems and their interactions with other Earth systems.

In light of these, numerous studies have investigated the existence of self-organization and self-organized critical behavior in hydrologic systems. While a significant majority of these studies have been on river networks, many other systems have also been studied, including land-atmosphere interactions, soil moisture, and rainfall; see Rinaldo et al. (1993), Rigon et al. (1994), Rodriguez-Iturbe et al. (1994), Stolum (1996), Rodriguez-Iturbe and Rinaldo (1997), Andrade et al. (1998), Rodriguez-Iturbe et al. (1998, 2006), Phillips (1999), Sapozhnikov and Foufoula-Georgiou (1996, 1997, 1999), Talling (2000), Baas (2002), Garcia-Marin et al. (2008), Caylor et al. (2009), Jenerette et al. (2012), and Bras (2015), among others.

Another concept that is also highly relevant in the context of self-organization and SOC in hydrologic systems, especially in the context of river networks, is ‘optimal channel networks’ (OCNs) (Rodriguez-Iturbe et al. 1992a). Drainage basins organize themselves to convey water and sediment from upstream to downstream in the most efficient way possible. Optimum channel networks are based on three principles: (1) minimum energy expenditure in any link of the network; (2) equal energy expenditure per unit area of channel anywhere in the network; and (3) minimum energy expenditure in the network as a whole. Since the study by Rodriguez-Iturbe et al. (1992a), there has been significant interest in the study of optimal channel networks in hydrology (e.g. Rinaldo et al. 1992, 2014; Rodriguez-Iturbe et al. 1992b; Rigon et al. 1993, 1998; Maritan et al. 1996; Colaiori et al. 1997; Molnar and Ramirez 1998; Banavar et al. 2001; Briggs and Krishnamoorthy 2013).

2.12 Threshold

In simple terms, a threshold is the point at which a system’s behavior changes. More accurately, however, it refers to the point where the system abruptly changes its behavior from one state to another even when the influencing factors change only progressively. Thresholds in a system may be either extrinsic or intrinsic. Extrinsic thresholds are associated with, and responses to, an external influence, i.e. a progressive change in an external factor triggers abrupt changes or failure within the system. In this case, the threshold exists within the system, but it will not be crossed and change will not occur without the influence of an external factor. Intrinsic thresholds, on the other hand, are associated with the inherent structure or dynamics of the system, without any external influences whatsoever.

Threshold behavior in a system can be deemed as an extreme form of nonlinear dynamics, such as, for example, when the system dynamics are highly intermittent. Consequently, threshold behavior drastically reduces our ability to make predictions at different levels, including at the level of: (1) an individual process; (2) the
response of larger units that involve interactions of many processes; and (3) the long-term functioning of the whole systems. The fact that threshold behavior can often be different at different levels of a system, its identification at each and every level of the system of interest is often a tremendously challenging problem.

There are many different ways and methods to identify thresholds. These include: (1) histogram-based methods (e.g. convex hull, peak-and-valley, shape-modeling); (2) clustering-based methods (iterative, clustering, minimum error, fuzzy clustering); (3) entropy-based methods (entropic, cross-entropic, fuzzy entropic); (4) attribute similarity-based methods (e.g. moment preserving, edge field matching, fuzzy similarity, maximum information); (5) spatial methods (co-occurrence, higher-order entropy, 2-D fuzzy partitioning); and (6) locally adaptive methods (e.g. local variance, local contrast, kriging), among others. However, one of the most effective ways to study threshold behavior is through catastrophe theory (Thom 1972; Zeeman 1976). Catastrophe theory describes the discontinuities (sudden changes) in dependent variables of a dynamic system as a function of continuous changes (progressive changes) in independent variables.

Hydrologic systems exhibit abrupt changes in behavior in many ways, even when the influencing factors change only progressively. Indeed, threshold behavior in the form of intermittency is one of the most important characteristics of hydrologic systems, especially at finer scales. Therefore, the concept of thresholds is highly relevant for hydrologic systems. Thresholds in hydrologic systems may be either intrinsic or extrinsic. They are observed in many different ways and at many different levels. For instance, surface and subsurface runoff generation processes at the local, hillslope, and catchment scales exhibit threshold behavior. Similarly, particle detachment and soil erosion (influenced by rainfall intensity, shear stress due to overland flow, and soil stability) is a threshold process. Threshold behavior is also discussed in the context of the long-term development of soil structures and landforms, fluvial morphology, rill and gully erosion, and formation and growth of channel networks. Soil moisture and land-atmosphere interaction processes also exhibit threshold behavior.

The significance of threshold behavior in hydrologic systems has led to a large number of studies on its identification, nature, causes, and effects in such systems and other systems with which they interact, in the specific context of thresholds (e.g. Dunne et al. 1991; Grayson et al. 1997; Hicks et al. 2000; Toms and Lesperance 2003; Blöschl and Zehe 2005; Sivakumar 2005; Phillips 2006, 2014; Pitman and Stouffer 2006; Tromp-Van-Meerveld and McDonnell 2006a, b; Emanuel et al. 2007; Lehmann et al. 2007; McGrath et al. 2007; O’Kane and Flynn 2007; Zehe et al. 2007; Andersen et al. 2009; Zehe and Sivapalan 2009), while numerous other studies have addressed the role of thresholds in many other contexts as well. Towards the identification of thresholds, the use of the ‘range of variability approach’ (RVA), which considers the ecologic flow regime characteristics (i.e. magnitude, frequency, duration, timing, and rate of change of flow) (Richter et al. 1996, 1997, 1998), has been gaining considerable attention in recent years (e.g. Shiau and Wu 2008; Kim et al. 2011; Yin et al. 2011; Yang et al. 2014). The usefulness of catastrophe theory to study hydrologic systems has also been investigated (Ghorbani et al. 2010).
2.13 Emergence

In simple terms, emergence is a property of a system in which larger entities, patterns, and regularities occur through interactions among smaller ones that themselves do not exhibit such properties. Emergence is a key property of complex systems. It is impossible to understand complex systems without recognizing that simple and separate entities (e.g. atoms) in large numbers give rise to complex collective behaviors that have patterns and regularity, i.e. emergence. How and when this occurs is the simplest and yet the most profound problem in studying complex systems. There are two types of emergence: (1) local emergence, where collective behavior appears in a small part of the system; and (2) global emergence, where collective behavior pertains to the system as a whole. The significance of the concept of emergence in studying natural, physical, and social systems can be clearly recognized when considered against our traditional view of reductionism, where the part defines the whole.

Although the concept of emergence has a long history, recent developments in complex systems science, especially in the areas of nonlinear dynamics, chaos, and complex adaptive systems, have provided a renewed impetus to its studies; see, for example, Waldrop (1992), Crutchfield (1994), O’Connor (1994), Holland (1998), Kim (1999), and Goldstein (2002) for some details. Such developments have led to a number of approaches for studying emergence, especially in the context of agent-based modeling, including genetic algorithms and artificial life simulations; see Holland (1975), Goldberg (1989), and Fogel (1995), among others. Dimensionality-reduction methods, such as self-organizing maps, local linear embedding and its variants, and isomap, are among the useful tools for an initial, exploratory investigation of the dynamics, or in the subsequent visual representation and description of the dynamics.

Emergent properties are inherent in hydrologic systems, as such systems are governed by various Earth-system components and their interactions in nonlinear ways at different spatial and temporal scales. For instance, vertical vadose zone processes or macropore influences are dominant at small plot scales, whereas topography begins to dominate runoff processes at the hillslope scale, and the stream network may begin to dominate catchment organization, spatial soil moisture variations, and patterns of runoff generation at the catchment scale (e.g. Blöschl and Sivapalan 1995). A similar situation occurs through timescale changes; for instance, from the diurnal scale to the event scale to the annual scale to the decadal scale. Consequently, many studies have attempted to address the emergent properties in hydrologic systems and associated ones in different ways (e.g. Levin 1992; Lansing and Kremer 1993; Young 1998, 2003; Eder et al. 2003; Lehmann et al. 2007; Rodriguez-Iturbe et al. 2009; Phillips 2011, 2014; Yeakel et al. 2014; Moore et al. 2015).

Emergent properties are generally associated with many other properties of complex systems, such as scale, nonlinear interactions, self-organization, threshold, and feedback. As all these properties play vital roles in hydrologic system dynamics, study of emergent properties is key to advance our understanding, modeling, and prediction of such systems.
2.14 Feedback

Feedback is a mechanism by which a change in something (e.g. a variable) results in either an amplification or a dampening of that change. When the change results in an amplification, it is called ‘positive feedback;’ when the change results in dampening, it is called ‘negative feedback.’ Complex systems are influenced by countless interacting processes at many scales and levels of system organization. These interactions mean that changes rarely occur in linear and incremental ways but happen in a nonlinear way, often driven by feedbacks. Due to their nature, and especially in amplification, feedbacks are also described as a threshold concept for understanding complex systems.

A positive feedback, due to amplification of changes, generally leads to destabilization of the system and moves it into another state, such as a regime shift. For example, when the atmospheric temperature rises, evaporation increases. This causes an increase in atmospheric water vapor concentration, resulting in an additional rise in atmospheric temperature through the greenhouse effect, which causes more evaporation, and the process continues. If there were no other factors contributing to atmospheric temperature, then this rise in temperature would spiral out of control. Therefore, a positive feedback is generally not good for a system.

A negative feedback, due to suppression of changes, generally leads to a stabilizing effect on a system. For example, if increased water in the atmosphere leads to greater cloud cover, there will be an increase in the percentage of sunlight reflected away from the Earth (albedo). This leads to a fall in the atmospheric temperature and a decrease in the rate of evaporation (Schmidt et al. 2010). A negative feedback is, therefore, generally good for a system.

The significance of feedback mechanisms in hydrologic systems has been known for a long time (e.g. Dooge 1968, 1973). Indeed, the hydrologic cycle itself serves as a perfect example of the feedback mechanisms, since every component in this cycle is connected to every other component, which leads to feedback processes, both positive and negative, at different times and at different scales. With the increasing recognition of anthropogenic influences on hydrologic systems and our increasing interest in understanding the interactions between hydrologic systems and the associated Earth and socio-economic systems, which bring their own and additional feedback mechanisms, the significance of feedbacks in hydrology has been increasingly realized in recent times. Consequently, there have been numerous attempts to study the causes, nature, and impacts of feedbacks in hydrologic systems and in their interactions with others (e.g. Dooge 1986; Brubaker and Entekhabi 1996; Hu and Islam 1997; Dooge et al. 1999; Yang et al. 2001; Hall 2004; Steffen et al. 2004; Dirmeyer 2006; Maxwell and Kollet 2008; Francis et al. 2009; Kastens et al. 2009; Roe 2009; Ferguson and Maxwell 2010, 2011; Brimelow et al. 2011; Runyan et al. 2012; Van Walsum and Supit 2012; D’Odorico et al. 2013; Butts et al. 2014; Blair and Buytaert 2015; Di Baldassarre et al. 2015).
Although the term ‘feedback’ has not been specifically used in a large number of these studies, the study of feedback mechanisms and the reported outcomes clearly indicate the advances made in understanding feedbacks in hydrology.

### 2.15 Sensitivity to Initial Conditions

The concept of sensitive dependence on initial conditions has been popularized in the so-called “butterfly effect;” i.e. a butterfly flapping its wings in one location (say, New York) could change the weather in a far off location (say, Tokyo). The underlying message in this is that even as inconsequent as the simple flap of a butterfly’s wings could be enough to change the initial conditions of the Earth’s atmosphere and, consequently, could have profound effects on global weather patterns. Edward Lorenz discovered this effect when he observed that runs of his weather model with initial condition data that was rounded in a seemingly inconsequential manner would fail to reproduce the results of runs with the unrounded initial condition data (Lorenz 1963); see also Gleick (1987) for additional details. The main reason for this effect is the presence of a strong level of interdependence among the components of the underlying (climate) system and deterministic non-linearity in each and every component and in their interactions, as well as the possibility for signal amplification via feedback. Sensitive dependence on initial conditions of a system may place serious limits on the predictability of its dynamic evolution.

The property of sensitive dependence on initial conditions is highly relevant for complex systems, since such systems often exhibit a strong level of interdependence, nonlinearity, and feedback mechanisms among the components. The existence of such a property has, consequently, far reaching implications for the modeling, understanding, prediction, and control of complex systems. This led to the investigation of this property in complex dynamic systems, especially in fluid turbulence (Ruelle 1978; Farmer 1985; Lai et al. 1994; Faisst and Eckhardt 2004). There exist many ways to quantify the sensitive dependence of initial conditions. One of the most popular methods is the Lyapunov exponent method (e.g. Wolf et al. 1985; Eckhardt and Yao 1993). Lyapunov exponents are the average exponential rates of divergence (expansion) or convergence (contraction) of nearby orbits in the phase space; see Chap. 6 for details.

Since hydrologic systems are made up of highly interconnected components that also exhibit nonlinearity, the property of sensitive dependence on initial conditions are certainly relevant to such systems and their modeling and predictions. Such a property may be observed in different ways in different hydrologic systems at different scales. For instance, overland flow is highly sensitive not only to small changes in rainfall but also small changes in catchment properties. Similarly, contaminant transport phenomena in surface and sub-surface waters largely depend upon the time (e.g. rainy or dry season) at which the contaminants were released at the source. In light of the signifinance of the property of sensitive dependence, a
number of studies have investigated such a property in hydrologic systems and associated ones (e.g. Stephenson and Freeze 1974; Rabier et al. 1996; Zehe and Blöschl 2004; Zehe et al. 2007; DeChant and Moradkhani 2011; Fundel and Zappa 2011). Some studies have also addressed this property in the specific context of nonlinear dynamic and chaotic properties, especially using the Lyapunov exponent method (e.g. Rodriguez-Iturbe et al. 1989; Jayawardena and Lai 1994; Puente and Obregon 1996; Shang et al. 2009; Dhanya and Nagesh Kumar 2011).

2.16 The Class of Nonlinear Determinism and Chaos

Although nonlinearity represents a situation where changes in inputs would not produce proportional changes in outputs (see Sect. 2.8), it does not necessarily mean that there is complete absence of determinism/predictability. Indeed, nonlinearity may contain inherent determinism on one hand, but may also be sensitively dependent on initial conditions on the other. While the former allows accurate predictions in the short term, the latter eliminates the possibility of accurate predictions in the long term. This class of nonlinearity is popularly termed as ‘deterministic chaos’ or simply ‘chaos’ (e.g. Lorenz 1963). This class of nonlinearity is also particularly interesting because it, despite the inherent determinism, is essentially ‘random-looking.’ For instance, time series generated from such nonlinear deterministic systems are visually indistinguishable from those generated from purely random systems, and some basic (linear) tools that are widely used for identification of system behavior (e.g. autocorrelation function, power spectrum) often cannot distinguish the two time series either; see, for example, Lorenz (1963), Henon (1976), May (1976), Rössler (1976), Tsonis (1992), and Kantz and Schreiber (2004) for some details.

The intriguing nature of chaos and its possible existence in various natural, physical, and socio-economic systems led to the development of many different methods for its identification, since the 1980s. These include correlation dimension method (e.g. Grassberger and Procaccia 1983a), Kolmogorov entropy method (Grassberger and Procaccia 1983b), Lyapunov exponent method (Wolf et al. 1985), nonlinear prediction method (e.g. Farmer and Sidorowich 1987; Casdagli 1989), false nearest neighbor method (e.g. Kennel et al. 1992), and close returns plot (e.g. Gilmore 1993), among others. These methods have been extensively applied in numerous fields; see, for example, Tsonis (1992), Strogatz (1994), Kaplan and Glass (1995), Kiel and Elliott (1996), and Kantz and Schreiber (2004), among others.

The fundamental properties inherent in the definition of ‘chaos:’ (1) nonlinear interdependence; (2) hidden determinism and order; and (3) sensitivity to initial conditions, are highly relevant for hydrologic systems, as is also clear from the observations made in the preceding sections. For example: (1) components and mechanisms involved in the hydrologic cycle act in a nonlinear manner and are also interdependent; (2) daily cycle in temperature and annual cycle in river flow possess
determinism and order; and (3) contaminant transport phenomena in surface and sub-surface waters largely depend upon the time (i.e. rainy or dry season) at which the contaminants were released at the source, which themselves may not be known (Sivakumar 2004a). The first property represents the ‘general’ nature of hydrologic systems, whereas the second and third represent their ‘deterministic’ and ‘stochastic’ natures, respectively. Further, despite their complexity and random-looking behavior, hydrologic systems may also be governed by a very few degrees of freedom (e.g. runoff in a well-developed urban catchment depends essentially on rainfall), another fundamental idea of chaos theory.

In view of these, numerous studies have applied the concepts and methods of chaos theory in hydrology. Such studies have analyzed various hydrologic time series (e.g. rainfall, streamflow, lake volume, sediment), addressed different hydrologic problems (e.g. system identification, prediction, scaling and disaggregation, catchment classification), and examined a host of data-related issues in chaos studies in hydrology (e.g. data size, data noise, presence of zeros); see Sivakumar (2000, 2004a, 2009) for comprehensive reviews. Chaos theory and its applications in hydrology are the focus of this book.

### 2.17 Summary

Hydrologic phenomena arise as a result of interactions between climate inputs and the landscape. The significant spatial and temporal variability in climate inputs and the complex heterogeneous nature of the landscape often give rise to a wide range of characteristics in the resulting phenomena. This chapter has discussed many of these characteristics. Some of these characteristics are easy to identify, but some others are far more difficult. Over the past century, numerous methods have been developed to identify, model, and predict these characteristics, especially based on data (i.e. time series). Early methods were mostly based on the assumption of linearity. In recent decades, however, advances in computational power and data measurements have allowed development of a number of nonlinear methods. Both types of methods have been found to be very useful and are now extensively applied in hydrology. The next two chapters discuss many of these methods, with Chap. 3 focusing on the linear methods (and also methods that make no prior assumption regarding linearity/nonlinearity) and Chap. 4 presenting the nonlinear methods.

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