Preface

It is not very common to embark upon the assiduous project of writing a book on a research topic within 5 years of its inception. One usually needs somewhat longer time to let the dust settle, allowing a clear picture of the relevant mathematical structures and the simplest forms and elegant proofs of the theorems to emerge. In fact, when we began our ambitious programme of defining an analogue of the group of isometries in the framework of noncommutative geometry and quantum groups around 2008–2009, we did not even dream of a book in near future. However, the enthusiasm and encouragement with which the new-born theory of quantum isometry groups was received among different quarters of researchers from around the globe and the flurry of papers on different aspects of it and several Ph.D. theses being written up within 3–4 years were really overwhelming. It makes us feel that it is perhaps the right time to collect the main results and present them in the form of a book to make it convenient for those who are working in this area. There are two important criteria for deciding the timing of a research monograph. On the one hand, the topic should still be "hot" enough in terms of number of active researchers working on it and recognition of importance of the theme among them. On the other hand, there should be enough developments of the topic, such as some major breakthroughs and some deep and difficult theorems which can pave directions of future research. While there can be no doubt that the theme of the book fulfills the first condition, we opine that the recent proof of the conjecture about nonexistence of genuine isometric (compact) quantum group coaction on classical compact connected manifolds by Goswami and Joardar, along with the new techniques and auxiliary results used in the proof, has brought the theory of quantum isometry to a level of maturity which qualifies for the second criterion.

A unique feature of this book is the emphasis on the interaction of C^* algebraic compact quantum groups with the operator-algebraic (i.e., a la Connes) noncommutative geometry. There are several excellent books on noncommutative geometry as well as quantum groups, but to the best of our knowledge, none of them deals with both these areas together in a significant way. The only notable exception is perhaps the classic treatise [1] by Manin which does present a pioneering formulation of quantum symmetry groups as suitable universal bialgebras or Hopf algebras in the context of noncommutative algebraic geometry, and it is indeed the precursor of the (C^* algebraic) quantum automorphism groups of finite dimensional spaces and algebras defined and studied by Wang, Banica, Bichon et al., serving also as a prime motivation for our theory of quantum isometry groups of more general noncommutative Riemannian manifolds. However, Manin's approach was mostly algebraic and did not deal with the analytic or spectral setup of noncommutative geometry or the setup of compact quantum groups a la Woronowicz. In fact, there does not seem to be any book combining the algebraic noncommutative geometry and compact quantum groups, barring stray discussions or passing references to examples of compact quantum group equivariant spectral triples in some of the recent books on noncommutative geometry. Our book proposes to fill this gap in some sense.

However, there are a few very important topics in the interface of noncommutative geometry and quantum groups which we have not at all touched, as we have mainly dealt with the metric aspects of spectral triples which are relevant for defining and studying isometries given by quantum group actions. These include the theory of Hopf algebra symmetry and the corresponding Hopf-cyclic homology and cohomology [2–6], fusion categories [7], Connes–Kreimer Hopf algebras in the context of renormalization [8], quantum group equivariant KK-theory [9], Baum–Connes conjecture [10, 11], covariant and bi-covariant differential calculus on quantum groups in the sense of Woronowicz, Tsygan [12–15] and others.

We have tried our best to make the exposition as self-contained as possible. However, a background in the basics of operator algebras, differential geometry, and the Peter–Weyl theory of compact groups will be desirable. For the convenience of readers, we have collected the basic concepts and results (mostly without proofs) in the first two chapters. The list of references at the end of the book is by no means exhaustive and we apologize for any inadvertent omission of some relevant reference.

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