A Two-Warehouse Inventory Model with Exponential Demand Under Permissible Delay in Payment

Trailokyanath Singh and Hadibandhu Pattanayak

Abstract The objective of the proposed paper is to develop an optimal policy of an inventory model that minimizes the total relevant cost per unit time. In this model, a two-warehouse system considers an owned warehouse (OW) with limited storage capacity and a rented warehouse (RW) with unlimited storage capacity. The demand rate is an exponential function of time, the rate of deterioration of OW is more than that of RW and the supplier provides the purchaser a permissible delay of payment. The results have been validated with the help of numerical examples.

Keywords Deterioration · Exponentially increasing demand · Permissible delay in payment · Two-warehouse model

2000 Mathematics Subject Classification 90B05

1 Introduction

In recent years, most of the inventory researchers have been trying to develop the more realistic and practicable inventory models for deteriorating items. In the past few decades, several researchers have studied the inventory model for deteriorating item. Deterioration refers the change, decay, damage, spoilage, vaporization, etc. of the products. Ghare and Schrader [1], the earliest researchers who developed an exponentially decaying inventory model. Their model led the foundation for

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modeling the inventory items by the differential equation considering demand rate as a function of time. An extensive survey of literature concerning the advances of inventory models for deteriorating items was conducted by Raafat [2], Goyal and Giri [3] and Li et al. [4]. In the traditional EOQ model, it was assumed that the purchaser must pay for items as soon as it is received by the system. But actually now-a-days a supplier grants a certain fixed period to the retailer to increase the demand. During this fixed period, no interest is charged by the supplier. Therefore, this delay period is known as trade credit period. During the trade credit period, customers can sell items, accumulate revenues and finally earn interest. The main purpose of the permissible delay period is to encourage the customers to buy more, to increase market share or to deplete inventories of certain items. For the business scenario, different delay period with different price discounts are offered by the suppliers to encourage the customers to order more quantities. In this respect, Goyal [5], the first researcher who developed an EOQ model under the condition of permissible delay in payment. Aggarwal and Jaggi [6], Khanra et al. [7] and Singh and Pattanayak [8] established their models for deteriorating items under permissible delay in payments.

Traditionally, in the EOQ model, a single warehouse is used to store with inventories. But the single warehouse with unlimited capacity is not always true or the assumption of unlimited capacity of it is unrealistic in real life situations. On the other hand, it is practical to consider another warehouse to store excess items for seasonal production or price discount for bulk purchase etc. called the rented warehouse (RW) while the first is called the own warehouse (OW). Because of better preserving facility and a lower deterioration rate, RW charges a higher holding cost (including the material handling cost and deterioration cost) than OW. Therefore, for the economical point of view, RW is stored after OW and RW is cleared before OW. Generally, the two-warehouse inventory models are developed for the storage of deteriorating inventory. In the real life situations, when some new products are launched to the markets or the seasonal product such as the output of the harvest or alternative price discounts for bulk purchase is available or the demand of the items is very high or the cost of procuring items is higher than the other inventory related cost etc., the management may purchase more items at a time. These items cannot be accommodated in the existing storehouse located at busy market place known as OW and for storing the excess items; an additional warehouse called RW is hired on the rental basis which may be located little away from it. In few decades, several researchers have studied in the field of two-warehouse model. Initially, Hartley [9], considered the effect of a two warehouse model with RW storage policy in his research. Later, Sharma [10] developed a model by assuming a single deteriorating item, constant demand and deterioration rate. Pakkala and Achary [11] studied the two-warehouse inventory model for deteriorating items with finite replenishment rate. In these models they considered the demand as constant. Benkherouf [12] extend Sharma's model and relaxed he assumptions of a fixed length of cycle and a fixed stored item in OW. The demand rate is taken as the function of time in their model. A two-warehouse model with constant demand rate, different deterioration rates and shortages under inflation was studied by Yang [13]. In most of literature of two-warehouse inventory models, it is a customary for the enterprisers to store the goods in OW first and then RW and clears the goods of RW first and then OW. Yang and Chang [14] proposed a two-warehouse inventory model for deteriorating items with partial backlogging and permissible delay in payment under inflation. Singh and Pattnayak [15] studied a two-warehouse inventory model for deteriorating items with linear increasing demand under conditionally permissible delay in payment.

This study proposes a two-warehouse inventory model of deteriorating items with exponentially increasing demand rate is considered under conditionally permissible delay in payment. The demand rate is likely to increase in the case of some new electronic products lunched to the markets like computer chips, modern TV sets etc., harvest items like paddy, wheat etc. and seasonal fruits like mango, oranges etc. For such items the demand is likely to increase very fast, almost exponentially with time. As the demand for such products increases with time, the present model is applicable. Finally an optimal policy is developed for the determination of optimal ordering time and the total relevant cost with exponentially increasing demand rate. Recently, Liang and Zhou [16] established a two-warehouse model for deteriorating items under conditionally permissible delay in payment. As demand pattern is always dynamic, therefore the demand rate is considered an exponentially increasing demand pattern and conditionally delay in payment is permitted in order to make the model relevant and more realistic.

2 Assumptions and Notations

The following assumptions are used to develop the mathematical model:

(i) The demand rate for the item is deterministic and increasing exponentially with respect to time.
(ii) The deterioration rate for the item in OW and RW are different rates and RW offers better preserving facility than OW.
(iii) The inventory model deals with a single item.
(iv) The OW provides a fixed capacity while RW provides unlimited capacity. In order to reduce the inventory costs, it is better to consume the goods of RW than that of OW.
(v) Shortages are not permitted.
(vi) The initial inventory level is zero and lead time is taken as zero.
(vii) The replenishment rate is infinite and replenishment is instantaneous.
(viii) There is no replacement or repair of deteriorated units during the cycle.
(ix) The inventory holding cost of RW is higher than that of OW.
(x) The unit selling price is greater than unit purchase cost.
(xi) For the optimal solution, the maximum deteriorating quantity for the items in the OW is less than exponentially increasing demand rate.

(xii) The supplier offers the retailer a delay period in paying for purchasing cost and the retailer can accumulate revenues by selling items and by earning interests. Total relevant costs include cost of placing orders, cost of carrying inventory, costs of deterioration, interest payable opportunity cost and opportunity interest earned.

The model is developed with the following notations:

(i) D(t): the time-dependent demand rate, $D(t) = A_1 e^{\lambda t}$, where $A_1 > 1$ is the initial demand and $0 < \lambda < 1, (A_1 > \lambda)$ are constants.
(ii) A_o: the ordering cost of inventory per order where $A_o > 0$.
(iii) P: the unit selling price of the item where $P > 0$.
(iv) C: the unit purchase cost of the item where $C > 0$ and $C < P$.
(v) w: the fixed capacity of OW.
(vi) $I_o(t)$: the level of inventory at any instant of time t, in the interval $[0, T]$ in the OW.
(vii) $I_r(t)$: the level of inventory at any instant of time t, in the interval $[0, t_w]$ in the RW.
(viii) h_o: the inventory carrying cost per unit per unit time in OW (excluding interest charges).
(ix) h_r: the inventory carrying cost per unit per unit time in RW (excluding interest charges) where $h_r > h_o$.
(x) γ: the constant rate of deterioration in OW where $0 < \gamma \ll 1$.
(xi) β: the constant rate of deterioration in RW where $0 < \beta \ll 1, \gamma > \beta$ and $(h_r - h_o) > C(\gamma - \beta)$.
(xii) M: the permissible delay period (fraction of the year) in settling the accounts with the suppliers.
(xiii) I_c: interest charged per rupee in stock per cycle by the supplier.
(xiv) I_e: interest that can be earned per rupee per cycle.
(xv) t_w: the time at which the inventory level reaches to w.
(xvi) T: the length of the replenishment cycle.
(xvii) $Z_i, i = 1, 2, 3$: the total relevant costs.

3 Mathematical Model

During the interval $[0, t_w]$, the instantaneous inventory level in RW and OW at any t are governed by the following differential equations:

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -D(t), \qquad 0 \le t \le t_w \tag{1}$$

where $D(t) = A_1 e^{\lambda t}, A_1 > 1$ is the initial demand and $0 < \lambda < 1 (A_1 > \lambda)$ with the boundary condition $I_r(t_w) = 0$ and

$$\frac{dI_o(t)}{dt} + \gamma I_o(t) = 0, \qquad 0 \le t \le t_w \tag{2}$$

with initial condition $I_o(0) = w$, respectively.

Using the conditions above, the solutions of Eqs. (1) and (2) are given by

$$I_r(t) = \frac{A_1}{\beta + \lambda}\left[e^{(\beta+\lambda)t_w - \beta t} - e^{\lambda t}\right], \qquad 0 \le t \le t_w \tag{3}$$

$$\text{and } I_o(t) = we^{-\gamma t}, \qquad 0 \le t \le t_w\,. \tag{4}$$

Further, when $t \in [t_w, T]$, the inventory level in OW is governed by the following differential equation:

$$\frac{dI_o(t)}{dt} + \gamma I_o(t) = -D(t), \qquad t_w \le t \le T \tag{5}$$

with the boundary condition $I_o(T) = 0$.

The solution of Eq. (5) is given by

$$I_0(t) = \frac{A_1}{\gamma + \lambda}\left[e^{(\gamma+\lambda)T - \gamma t} - e^{\lambda t}\right], \qquad t_w \le t \le T. \tag{6}$$

Now, the total annual relevant cost Z consists of the following elements:

(i) Cost of placing orders: $(CO) = \frac{A_o}{T}$.
(ii) Annual cost of carrying inventory: (CC)

The annual cost of carrying inventory in RW during the interval $[0, t_w]$ and the annual cost of carrying inventory in OW during $[0, T]$ are

$$\begin{aligned}\frac{h_r}{T}\int_0^{t_w} I_r(t)dt &= \frac{h_r}{T}\int_0^{t_w}\left[\frac{A_1}{\beta+\lambda}\left\{e^{(\beta+\lambda)t_w - \beta t} - e^{\lambda t}\right\}\right]dt \\ &= \frac{A_1 h_r}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left\{e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right\} + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right]\end{aligned} \tag{7}$$

and $\frac{h_o}{T}\int_0^T I_o(t)dt = \frac{h_o}{T}\left[\int_0^{t_w} I_o(t)dt + \int_{t_w}^T I_o(t)dt\right]$

$$= \frac{A_1 h_o}{(\gamma+\lambda)T}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T - \gamma t_w} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda t_w} - e^{\lambda T}\right)\right] + \frac{wh_o}{\gamma T}\left(1 - e^{-\gamma t_w}\right) \tag{8}$$

respectively.

(i) Annual costs of deteriorating units: CD

The total annual cost of deteriorated units CD is C times the sum of the amounts of the deteriorated items in both RW and OW during the interval $[0, T]$, i.e.,

$$\begin{aligned} CD &= \frac{C}{T}\left[\beta\int_0^{t_w} I_r(t)dt + \gamma\int_0^{T} I_o(t)dt\right] \\ &= \frac{\beta A_1 C}{(\beta+\lambda)T}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right] \\ &\quad + \frac{\gamma A_1 C}{(\gamma+\lambda)T}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T} + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right)\right] + \frac{wC}{T}\left(1 - e^{-\gamma t_w}\right). \end{aligned} \tag{9}$$

(ii) The annual interest chargeable cost:

In this respect, there arise three possibilities for the annual interest payable opportunity costs.

Case (I): $M \le t_w < T$.

The annual interest chargeable cost (IC_1)

$$\begin{aligned} &= \frac{CI_c}{T}\left[\int_M^{t_w} I_r(t)dt + \int_M^{T} I_o(t)dt + \int_{t_w}^{T} I_o(t)dt\right] \\ &= \frac{A_1 CI_c}{(\beta+\lambda)}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(e^{\lambda M} - e^{\lambda t_w}\right)\right] \\ &\quad + \frac{A_1 CI_c}{(\gamma+\lambda)T}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T} + \frac{1}{\lambda}\left(e^{\lambda T} - e^{\lambda t_w}\right)\right)\right] + \frac{wCI_c}{\gamma T}\left(e^{-\gamma M} - e^{-\gamma t_w}\right). \end{aligned} \tag{10}$$

Case (II): $t_w < M \le T$.

The annual interest chargeable cost (IC_2)

$$= \frac{CI_c}{T}\int_M^{T} I_o(t)dt = \frac{CI_c A_1}{T(\gamma+\lambda)}\left[\frac{1}{\gamma}\left\{e^{(\gamma+\lambda)T-\gamma M} - e^{\lambda T}\right\} + \frac{1}{\lambda}\left(e^{\lambda M} - e^{\lambda T}\right)\right]. \tag{11}$$

Case (III): $M > T$.

No interests are charged for the items.

(v) The annual opportunity interest

Case (I): $M \le T$.

The annual interest earned is (IE_1)

$$= \frac{PI_e}{T}\int_0^M tD(t)dt = \frac{PI_eA_1}{T\lambda}\left[Me^{\lambda M} + \frac{1}{\lambda}\left(1 - e^{\lambda M}\right)\right]. \tag{12}$$

Case (II): $M > T$.

The annual interest earned is (IE_2)

$$\begin{aligned} &= \frac{PI_e}{T}\left[\int_0^T tD(t)dt + (M - T)\int_0^M D(t)dt\right] \\ &= \frac{PI_eA_1}{T\lambda}\left[Te^{\lambda T} + \left(M - T - \frac{1}{\lambda}\right)\left(e^{\lambda T} - 1\right)\right]. \end{aligned} \tag{13}$$

According to the assumptions, the annual relevant cost $Z(t_w, T)$ for the retailers = cost of placing orders + inventory carrying cost in RW + inventory holding cost in OW + cost of deteriorating items + interest payable opportunity cost − opportunity interest earned.

$$\text{i.e., } Z(t_w, T) = \begin{cases} Z_1, & M \le t_w \le T, \\ Z_2, & t_w < M \le T, \\ Z_3, & M > T, \end{cases} \tag{14}$$

where

$$\begin{aligned} Z_1 = {} & \frac{A_o}{T} + \frac{(h_r + \beta C)A_1}{T(\beta + \lambda)}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right] \\ & + \frac{(h_o + \gamma C)A_1}{T(\gamma + \lambda)}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda t_w} - e^{\lambda T}\right)\right] \\ & + \frac{CI_cA_1}{(\beta + \lambda)T}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w-\beta M} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(e^{\lambda M} - e^{\lambda t_w}\right)\right] \\ & + \frac{CI_cA_1}{(\gamma + \lambda)T}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda t_w} - e^{\lambda T}\right)\right] \\ & + \frac{W}{\gamma T}\left[(h_o + \gamma C)(1 - e^{-\gamma t_w}) + CI_c\left(e^{-\gamma M} - e^{-\gamma t_w}\right)\right] \\ & - \frac{PI_eA_1}{\lambda T}\left[Me^{\lambda M} + \frac{1}{\lambda}\left(1 - e^{\lambda M}\right)\right], \end{aligned} \tag{15}$$

$$\begin{aligned}Z_2 = {} & \frac{A_o}{T} + \frac{(h_r + \beta C)A_1}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right] \\ & + \frac{(h_o + \gamma C)A_1}{T(\gamma+\lambda)}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda t_w} - e^{\lambda T}\right)\right] \\ & + \frac{CI_cA_1}{T(\gamma+\lambda)}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma M} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda M} - e^{\lambda T}\right)\right] \\ & + \frac{w}{\gamma T}[(h_o + \gamma C)(1 - e^{-\gamma t_w})] - \frac{PI_eA_1}{\lambda T}\left[Me^{\lambda M} + \frac{1}{\lambda}\left(1 - e^{\lambda M}\right)\right],\end{aligned} \tag{16}$$

and

$$\begin{aligned}Z_3 = {} & \frac{A_o}{T} + \frac{(h_r + \beta C)A_1}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left(e^{(\beta+\lambda)t_w} - e^{\lambda t_w}\right) + \frac{1}{\lambda}\left(1 - e^{\lambda t_w}\right)\right] \\ & + \frac{(h_o + \gamma C)A_1}{T(\gamma+\lambda)}\left[\frac{1}{\gamma}\left(e^{(\gamma+\lambda)T-\gamma t_w} - e^{\lambda T}\right) + \frac{1}{\lambda}\left(e^{\lambda t_w} - e^{\lambda T}\right)\right] \\ & + \frac{w}{\gamma T}[(h_o + \gamma C)(1 - e^{-\gamma t_w})] - \frac{PI_eA_1}{\lambda T}\left[Te^{\lambda T} + \left(M - T - \frac{1}{\lambda}\right)\left(e^{\lambda T} - 1\right)\right].\end{aligned} \tag{17}$$

The objective of the model is to find the optimal values of t_w^* and T^* in order to minimize the total relevant cost $Z(t_w, T)$.

For the minimization of Z_1, the necessary conditions are

$$\begin{aligned}\frac{\partial(Z_1)}{\partial t_w} = {} & \frac{(h_r + \beta C)A_1}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left\{(\beta+\lambda)e^{(\beta+\lambda)t_w} - \lambda e^{\lambda t_w}\right\} - e^{\lambda t_w}\right] \\ & + \frac{(h_o + \gamma C)A_1}{T(\gamma+\lambda)}\left[e^{\lambda t_w} - e^{(\gamma+\lambda)T-\gamma t_w}\right] + \frac{w}{T}(h_o + \gamma C + CI_c)e^{-\gamma t_w} \\ & + \frac{CI_cA_1}{(\beta+\lambda)T}\left[\frac{1}{\beta}\left((\beta+\lambda)e^{(\beta+\lambda)t_w-\beta M} - \lambda e^{\lambda t_w}\right) - e^{\lambda t_w}\right] \\ & + \frac{CI_cA_1}{(\gamma+\lambda)T}\left[e^{\lambda t_w} - e^{(\gamma+\lambda)T-\gamma t_w}\right] = 0.\end{aligned} \tag{18}$$

and

$$\frac{\partial(Z_1)}{\partial T} = \frac{(h_o + \gamma C + CI_c)A_1}{(\gamma+\lambda)T}\left[\frac{1}{\gamma}\left((\gamma+\lambda)e^{(\gamma+\lambda)T-\gamma t_w} - \lambda e^{\lambda T}\right) - e^{\lambda T}\right] - \frac{Z_1}{T} = 0. \tag{19}$$

Let t_w^* and T_1^* be the optimal solutions of Eqs. (18) and (19) and the solution set $\left(t_w^*, T_1^*\right)$ will be optimal solution if its Hessian matrix $Z_1\left(t_w^*, T_1^*\right)$ is positive definite provided $\left[\frac{\partial^2(Z_1)}{\partial t_w^2}\right]_{(t_w^*,T_1^*)} > 0$, $\left[\frac{\partial^2(Z_1)}{\partial T^2}\right]_{(t_w^*,T_1^*)} > 0$ and $\left[\frac{\partial^2(Z_1)}{\partial t_w^2}\cdot\frac{\partial^2(Z_1)}{\partial T^2} - \frac{\partial^2(Z_1)}{\partial t_w \partial T}\cdot\frac{\partial^2(Z_1)}{\partial T \partial t_w}\right]_{(t_w^*,T_1^*)} > 0.$

Similarly, for the minimization of Z_2, the necessary conditions are

$$\begin{aligned}\frac{\partial(Z_2)}{\partial t_w} &= \frac{(h_r+\beta C)A_1}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left\{(\beta+\lambda)e^{(\beta+\lambda)t_w} - \lambda e^{\lambda t_w}\right\} - e^{\lambda t_w}\right] \\ &+ \frac{(h_o+\gamma C)A_1}{T(\gamma+\lambda)}\left[e^{\lambda t_w} - e^{(\gamma+\lambda)T-\gamma t_w}\right] + \frac{W}{T}(h_o+\gamma C)e^{-\gamma t_w} = 0.\end{aligned} \tag{20}$$

and

$$\begin{aligned}\frac{\partial(Z_2)}{\partial T} &= \frac{(h_o+\gamma C)A_1}{(\gamma+\lambda)T}\left\{\frac{1}{\gamma}\left((\gamma+\lambda)e^{(\gamma+\lambda)T-\gamma t_w} - \lambda e^{\lambda T}\right) - e^{\lambda T}\right\} \\ &+ \frac{CI_cA_1}{(\gamma+\lambda)T}\left\{\frac{1}{\gamma}\left((\gamma+\lambda)e^{(\gamma+\lambda)T-\gamma M} - \lambda e^{\lambda T}\right) - e^{\lambda T}\right\} - \frac{Z_2}{T} = 0.\end{aligned} \tag{21}$$

Let t_w^* and T_2^* be the optimal solutions of Eqs. (20) and (21) and the solution set $\left(t_w^*, T_2^*\right)$ will be optimal solution if its Hessian matrix $Z_2\left(t_w^*, T_2^*\right)$ is positive definite provided $\left[\frac{\partial^2(Z_2)}{\partial t_w^2}\right]_{(t_w^*,T_2^*)} > 0$, $\left[\frac{\partial^2(Z_2)}{\partial T^2}\right]_{(t_w^*,T_2^*)} > 0$ and $\left[\frac{\partial^2(Z_2)}{\partial t_w^2}\cdot\frac{\partial^2(Z_2)}{\partial T^2} - \frac{\partial^2(Z_2)}{\partial t_w \partial T}\cdot\frac{\partial^2(Z_2)}{\partial T \partial t_w}\right]_{(t_w^*,T_2^*)} > 0.$

The minimization of Z_3, the necessary conditions are

$$\begin{aligned}\frac{\partial(Z_3)}{\partial t_w} &= \frac{(h_r+\beta C)A_1}{T(\beta+\lambda)}\left[\frac{1}{\beta}\left\{(\beta+\lambda)e^{(\beta+\lambda)t_w} - \lambda e^{\lambda t_w}\right\} - e^{\lambda t_w}\right] \\ &+ \frac{(h_o+\gamma C)A_1}{T(\gamma+\lambda)}\left[e^{\lambda t_w} - e^{(\gamma+\lambda)T-\gamma t_w}\right] + \frac{W}{T}(h_o+\gamma C)e^{-\gamma t_w} = 0.\end{aligned} \tag{22}$$

and

$$\begin{aligned}\frac{\partial(Z_3)}{\partial T} &= \frac{(h_o+\gamma C)A_1}{(\gamma+\lambda)T}\left[\frac{1}{\gamma}\left((\gamma+\lambda)e^{(\gamma+\lambda)T-\gamma t_w} - \lambda e^{\lambda T}\right) - e^{\lambda T}\right] \\ &- \frac{PI_eA_1}{\lambda T}\left\{\lambda e^{\lambda T} - e^{\lambda T} + 1\right\} - \frac{Z_3}{T} = 0.\end{aligned} \tag{23}$$

Let t_w^* and T_3^* be the optimal solutions of Eqs. (22) and (23) and the solution set $\left(t_w^*, T_2^*\right)$ will be optimal solution if its Hessian matrix $Z_3\left(t_w^*, T_3^*\right)$ is positive definite provided $\left[\frac{\partial^2(Z_3)}{\partial t_w^2}\right]_{(t_w^*,T_3^*)} > 0$, $\left[\frac{\partial^2(Z_3)}{\partial T^2}\right]_{(t_w^*,T_3^*)} > 0$ and $\left[\frac{\partial^2(Z_3)}{\partial t_w^2}\cdot\frac{\partial^2(Z_3)}{\partial T^2} - \frac{\partial^2(Z_3)}{\partial t_w \partial T}\cdot\frac{\partial^2(Z_3)}{\partial T \partial t_w}\right]_{(t_w^*,T_3^*)} > 0.$

4 Algorithms for Finding the Optimal Solution

The Newton-Rapson's method is applied to find the optimal solution of the model. The following steps are as follows:

Step 1: Determine t_w^{1*} and T_1^* from Eqs. (18) and (19) where $t_w^* = t_w^{1*}$ and $T^* = T_1^*$. If the condition $M \le t_w^{1*} < T_1^*$ is satisfied, then determine $Z_1^* = Z_1^*\left(t_w^{1*}, T_1^*\right)$ from Eq. (15); otherwise go to Step 2.

Step 2: Determine t_w^{2*} and T_2^* from Eqs. (20) and (21) where $t_w^* = t_w^{2*}$ and $T^* = T_2^*$. If the condition $t_w^{2*} < M \le T_2^*$ is satisfied, then determine $Z_2^* = Z_2^*\left(t_w^{2*}, T_2^*\right)$ from Eq. (16); otherwise go to Step 3.

Step 3: Determine t_w^{3*} and T_3^* from Eqs. (22) and (23) where $t_w^* = t_w^{3*}$ and $T^* = T_3^*$. If the condition $t_w^{3*} < M \le T_3^*$ is satisfied, then determine $Z_3^* = Z_3^*\left(t_w^{3*}, T_3^*\right)$ from Eq. (17); otherwise go to Step 4.

Step 4: If $\left(t_w^*, T^*\right) = \arg\min\left\{Z_1^* = Z_1^*\left(t_w^{1*}, T_1^*\right), Z_2^* = Z_2^*\left(t_w^{2*}, T_2^*\right), Z_3^* = Z_3^*\left(t_w^{3*}, T_3^*\right)\right\}$, then determine t_w^*, T^* and Z^*.

5 Numerical Examples

Example 1: Let us consider the parameters of the two-warehouse inventory model as $A_1 = 2000$ units per year, $\lambda = 0.4$ units per year, $A_o = \text{Rs.}\,1600$ per order, $h_r = \text{Rs.}\,4$ per unit per order, $h_o = \text{Rs.}\,1$ per unit per order, $w = 120$ units, $C = \text{Rs.}\,10$ per unit, $P = \text{Rs.}\,16$ per unit per year, $I_c = \text{Rs.}\,0.16$ per rupee per year, $I_e = \text{Rs.}\,0.12$ per rupee per year, $M = 0.25$ year, $\gamma = 0.2$ and $\beta = 0.08$:

Using the step-by-step procedure, the optimal solutions are $t_w^* = 0.300324$ year and $T^* = 0.646559$ year and the corresponding $Z^* = \text{Rs.}\,4271.69$.

Therefore, the both warehouses will be empty after 0.346235 year as RW vanishes at 0.300324 year.

Example 2: Let us consider the parameters of the two-warehouse inventory model as $A_1 = 2000$ units per year, $\lambda = 0.3$ units per year, $A_o = \text{Rs.}\,1550$ per order, $h_r = \text{Rs.}\,3$ per unit per order, $h_o = \text{Rs.}\,1$ per unit per order, $w = 120$ units, $C = \text{Rs.}\,10$ per unit, $P = \text{Rs.}\,15$ per unit per year, $I_c = \text{Rs.}\,0.15$ per rupee per year, $I_e = \text{Rs.}\,0.12$ per rupee per year, $M = 0.25$ year, $\gamma = 0.1$ and $\beta = 0.05$:

Using the step-by-step procedure, the optimal solutions are $t_w^* = 0.422787$ year and $T^* = 0.933838$ year and the corresponding $Z^* = \text{Rs.}\,3764.23$.

Therefore, the both warehouses will be empty after 0.511051 year as RW vanishes at 0422787 year.

6 Conclusions

To be precise, an approach is made to associate costs and to determine the inventory control policy which minimizes the total relevant costs. In this paper, a two-warehouse inventory model of deteriorating items with exponentially increasing demand rate is considered under conditionally permissible delay in payment. The demand rate is likely to increase in the case of some new electronic products lunched to the markets like computer chips, modern TV sets etc., harvest items like paddy, wheat etc. and seasonal fruits like mango, oranges etc. For such items the demand is likely to increase very fast, almost exponentially with time. For selling more items, suppliers offer delay periods. Finally an optimal policy is developed for the determination of optimal ordering time and the total relevant cost with exponentially increasing demand rate.

Further extensions of this two-warehouse inventory model can be done for generalized demand pattern, stock-dependent demand, quantity discount, a bulk release pattern etc. Another possible future direction of research is to consider the inflation and time value of money.

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