Chapter 2
Efficiency Criteria

In economics several different, though related, notions of efficiency are employed. The most important notion of efficiency is that of Pareto-optimality, which is based on the Pareto-criterion. A social alternative $x$ is defined to be Pareto-superior to another social alternative $y$ iff every individual in the society considers $x$ to be at least as good as $y$ and at least one individual considers $x$ to be better than $y$. According to the Pareto-criterion, alternative $x$ is regarded socially better than alternative $y$ if $x$ is Pareto-superior to $y$. A social alternative is defined to be Pareto-optimal or Pareto-efficient iff there is no feasible alternative which is Pareto-superior to it. That is to say, an alternative is Pareto-efficient iff it is not possible to make some individual better off without making anyone worse off.

From the definition it is clear that the notion of Pareto-efficiency is defined with respect to a particular set of social alternatives and a particular set of individuals. If the set of social alternatives or the set of individuals changes, then the set of Pareto-efficient alternatives would also in general change. In particular, if the set of social alternatives contracts, then an alternative which earlier was Pareto-inefficient might become Pareto-efficient; and if the set of social alternatives expands, then an alternative which earlier was Pareto-efficient might become Pareto-inefficient.\textsuperscript{1} If the set of individuals expands or contracts, then an alternative which earlier was Pareto-inefficient might become Pareto-efficient; and an alternative which earlier was Pareto-efficient might become Pareto-inefficient.

Most people find the value judgment of the Pareto-criterion to be rather compelling. It should, however, be noted that the Pareto-criterion can easily conflict with some important value judgments. Suppose under state $x$ a small number of individuals are better off compared to state $y$ and the remaining individuals are equally well off under the two states. While according to the Pareto-criterion state

\textsuperscript{1}The alternatives which are Pareto-inefficient before the expansion of the set of alternatives must of course remain so after the expansion as well; and the alternatives which are Pareto-efficient before the contraction must continue to remain so after the contraction.
If one accepts the value judgment of the Pareto-criterion, then it follows that the choice of the society or the collective should be from the set of Pareto-efficient alternatives. Choice of a Pareto-inefficient alternative would imply selection of an inferior alternative and rejection of a superior alternative as Pareto-inefficiency of an alternative implies the existence of a Pareto-superior alternative and therefore a socially better alternative in the set of feasible alternatives. Thus, for the purpose of making social choices, only the sets of Pareto-efficient alternatives need be considered if the collective subscribes to the Pareto-criterion. It is, however, rather important to note that the acceptance of the Pareto-criterion does not imply that any arbitrary Pareto-efficient alternative is better than any arbitrary Pareto-inefficient alternative. Pareto-inefficiency of an alternative merely entails that there exists another alternative which is Pareto-superior to it and therefore socially better in view of the Pareto-criterion. Under the Pareto-criterion only some pairs of alternatives can be compared. It is perfectly possible that a particular Pareto-inefficient alternative may be noncomparable with a particular Pareto-efficient alternative in terms of the Pareto-criterion but better in terms of another evaluative criterion. An alternative can be Pareto-efficient and at the same time socially highly undesirable. Pareto-efficiency is discussed formally and more fully in the first section of this chapter.

One difficulty with the Pareto-criterion is that its scope is rather limited. If there is a person in the collective who prefers alternative \( x \) to alternative \( y \) and another person who prefers \( y \) to \( x \), then under the Pareto-criterion \( x \) and \( y \) cannot be compared regardless of the preferences of the remaining individuals. Thus, in general, only a small proportion of all pairs of alternatives would be comparable under the Pareto-criterion. Because of this one would also generally expect the sets of Pareto-efficient alternatives to be large. In many contexts, therefore, the notion of efficiency that is used is that which is based on the Kaldor compensation principle rather than that based on the Pareto-criterion. An alternative \( x \) is defined to be Kaldor-superior to another alternative \( y \) if \( y \) is Kaldor-superior to \( x \) then the gainers can compensate the losers and still be better off. Actual compensation, however, need not be paid. Under the Kaldor criterion alternative \( x \) is regarded as socially better than alternative \( y \) if \( x \) is Kaldor-superior to \( y \). Intuitively, it seems that if the gainers can compensate the losers and still be better off, then the total wealth after the change must be larger than before the change. It should be noted that if actual compensation is not paid then there is no consensual basis to the change. The underlying, though often implicit, premise in using this notion of efficiency is that the questions of total wealth and its distribution can somehow be separated. The main drawback of the Kaldor compensation principle lies in the fact that the Kaldor-superior relation fails to be asymmetric. It is possible that of the two social alternatives \( x \) and \( y \), each is Kaldor-superior to the other. That is to say, examples can be constructed such that if there is a move from alternative \( y \) to alternative \( x \), the gainers can compensate the losers and still be better off; and also that if there is a
move from alternative $x$ to alternative $y$, the gainers can compensate the losers and still be better off. One way to resolve this difficulty is to modify the compensation principle along the lines suggested by Scitovsky. Under the Kaldor compensation principle, one infers ‘social alternative $x$ is better than social alternative $y$’ from the statement ‘if there is a move from social alternative $y$ to social alternative $x$, then the gainers can compensate the losers and still be better off’. Under the Scitovsky criterion, one infers ‘social alternative $x$ is better than social alternative $y$’ from ‘if there is a move from social alternative $y$ to social alternative $x$, then the gainers can compensate the losers and still be better off; and if there is a move from social alternative $x$ to social alternative $y$, then it is not the case that the gainers can compensate the losers and still be better off’. Unlike the Kaldor-superior relation, the Scitovsky-superior relation is asymmetric; and therefore it can never happen that of the two alternatives $x$ and $y$, each is Scitovsky-superior to the other. The social ‘at least as good as’ relation generated by the Scitovsky criterion, however, fails to satisfy transitivity.\(^2\) The logical difficulties associated with Kaldor and Scitovsky compensation principles are particular instances of the Arrow paradox relating to aggregation of individual preferences into social preferences. Another way to look at these difficulties is to see them as manifestations of the fact that the notion of wealth in general cannot be separated from its distribution. The second section of the chapter discusses the compensation principles in greater detail.\(^3\)

### 2.1 Pareto-Efficiency

Let $\mathcal{N} = \{1, \ldots, n\}, n \geq 2$, denote the set of individuals comprising the society; and let $\mathcal{S}$ be the nonempty set of social alternatives. Let $R_i, i \in \mathcal{N}$, denote the ‘at least as good as’ binary relation of individual $i$ on $\mathcal{S}$. Thus, $x R_i y, x, y \in \mathcal{S}$, will stand for ‘individual $i$ considers $x$ to be at least as good as $y$’. ‘At least as good as’ binary relation of the society will be denoted by $R$. The asymmetric parts of binary relations $R_i, R$, etc., will be denoted by $P_i, P$, etc., respectively; and the symmetric parts by $I_i, I$, etc., respectively.\(^4\) Each $R_i, i \in \mathcal{N}$, will be assumed to be an ordering, i.e. a reflexive, connected, and transitive binary relation.\(^5\)

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\(^2\)See Arrow (1963).


\(^4\)Let $R$ be a binary relation on a set $\mathcal{S}$. The asymmetric part $P$ and the symmetric part $I$ of $R$ are defined by: $(\forall x, y \in \mathcal{S})(x P y \iff x R y \land \sim y R x) \land (x I y \iff x R y \land y R x)$. Thus, if $R$ stands for ‘at least as good as’, then $P$ and $I$ stand for ‘preferred to’ and ‘indifferent to’.

\(^5\)A binary relation $R$ over a set $\mathcal{S}$ is (i) reflexive iff $(\forall x \in \mathcal{S})(x R x)$, (ii) connected iff $(\forall x, y \in \mathcal{S})[x \neq y \rightarrow x R y \lor y R x]$, (iii) transitive iff $(\forall x, y, z \in \mathcal{S})[x R y \land y R z \rightarrow x R z]$, and (iv) an ordering iff it is reflexive, connected, and transitive.
Let \( Q \) be the unanimity relation defined by: \((\forall x, y \in S)(xQy \leftrightarrow (\forall i \in N)(xR_i y))\). Thus, \( xQy \) iff everyone in the society considers \( x \) to be at least as good as \( y \). An alternative \( x \in S \) is defined to be Pareto-superior to another alternative \( y \in S \) iff \([xQy \land \sim yQx]\), i.e. iff \([(\forall i \in N)(xR_i y) \land (\exists i \in N)(xP_i y)]\). That is to say, \( x \in S \) is Pareto-superior to \( y \in S \) iff everyone considers \( x \) to be at least as good as \( y \) and at least one person considers \( x \) to be better than \( y \). An alternative \( x \in S \) is defined to be Pareto-indifferent to another alternative \( y \in S \) iff \([xQy \land yQx]\), i.e. iff \([(\forall i \in N)(xI_i y)]\). That is to say, \( x \in S \) is Pareto-indifferent to \( y \in S \) iff everyone considers \( x \) to be as good as \( y \). From the definition of unanimity relation, it follows that it is a reflexive and transitive binary relation on \( S \).

**Proposition 2.1.** Let \( N \) and \( S \) be the set of individuals and the set of alternatives, respectively. If every individual belonging to \( N \) has an ordering on \( S \), then unanimity relation is reflexive and transitive.

**Proof.** As by definition \( xQx \leftrightarrow (\forall i \in N)(xR_i x) \), the reflexivity of \( Q \) follows from the reflexivity of individual \( R_i \)’s.

Let \( xQy \land yQz \).

\[
xQy \rightarrow (\forall i \in N)(xR_i y) \\
yQz \rightarrow (\forall i \in N)(yR_i z) \\
(\forall i \in N)(xR_i y) \land (\forall i \in N)(yR_i z) \rightarrow (\forall i \in N)(xR_i z), \text{ as each } R_i \text{ is transitive.} \\
(\forall i \in N)(xR_i z) \rightarrow xQz.
\]

Thus, \( Q \) is transitive. \( \square \)

From the transitivity of the unanimity relation \( Q \), it follows that ‘Pareto-superior’ relation and ‘Pareto-indifferent’ relation are transitive.\(^6\) As ‘Pareto-superior’ relation is the asymmetric part of the unanimity relation, it follows that it is irreflexive.\(^7\) ‘Pareto-indifference’ relation is reflexive as \( Q \) is reflexive.

An alternative \( x \in S \) is defined to be Pareto-optimal or Pareto-efficient iff

\[
\sim (\exists y \in S)(\forall i \in N)(yR_i x) \land (\exists i \in N)(yP_i x)].
\]

That is to say, an alternative \( x \in S \) is Pareto-efficient iff there is no alternative which is Pareto-superior to it. An alternative is Pareto-inoptimal or Pareto-inefficient iff it is not Pareto-optimal.

The following example illustrates the definitions of Pareto-superiority and Pareto-optimality.

**Example 2.1.** Let \( S = \{x, y, z, w\} \) and \( N = \{1, 2, 3, 4\} \).

Let the preference orderings of individuals in \( N \) be as follows:

- \( R_1 = xyzw \)
- \( R_2 = w(yz)x \)
- \( R_3 = (xy)zw \)
- \( R_4 = (xyz)w \)

\(^6\)If a binary relation is transitive, then its asymmetric and symmetric parts are transitive. See Sen (1970).

\(^7\)A binary relation \( R \) over a set \( S \) is irreflexive iff \((\forall x \in S)(\sim xRx)\).
(Notation: Alternatives within the parentheses are indifferent to each other. If an alternative is left of another, then the former is preferred to the latter. Thus, individual 3 is indifferent between \( x \) and \( y \), prefers both \( x \) and \( y \) to both \( z \) and \( w \), and prefers \( z \) to \( w \).)

Checking pairwise we find: (i) As individual 1 prefers \( x \) to \( y \) and individual 2 prefers \( y \) to \( x \), it follows that neither \( x \) is Pareto-superior to \( y \) nor \( y \) is Pareto-superior to \( x \). (ii) As individual 1 prefers \( x \) to \( z \) and individual 2 prefers \( z \) to \( x \), it follows that neither \( x \) is Pareto-superior to \( z \) nor \( z \) is Pareto-superior to \( x \). (iii) As individual 1 prefers \( x \) to \( w \) and individual 2 prefers \( w \) to \( x \), it follows that neither \( x \) is Pareto-superior to \( w \) nor \( w \) is Pareto-superior to \( x \). (iv) Individuals 1 and 3 prefer \( y \) to \( z \) and individuals 2 and 4 are indifferent between \( y \) to \( z \); therefore it follows that \( y \) is Pareto-superior to \( z \). (v) As individual 1 prefers \( y \) to \( w \) and individual 2 prefers \( w \) to \( y \), it follows that neither \( y \) is Pareto-superior to \( w \) nor \( w \) is Pareto-superior to \( y \). (vi) As individual 1 prefers \( z \) to \( w \) and individual 2 prefers \( w \) to \( z \), it follows that neither \( z \) is Pareto-superior to \( w \) nor \( w \) is Pareto-superior to \( z \).

Thus, from (i) to (vi) we have: (a) There is no alternative which is Pareto-superior to \( x \). (b) There is no alternative which is Pareto-superior to \( y \). (c) There is an alternative which is Pareto-superior to \( z \). (d) There is no alternative which is Pareto-superior to \( w \). Consequently, alternatives \( x, y, w \) are Pareto-optimal and alternative \( z \) is not Pareto-optimal.

Remark 2.1. The notion of Pareto-optimality or Pareto-efficiency is defined relative to particular sets of individuals and alternatives. If these sets change, then in general the set of Pareto-optimal alternatives would also change even when there is no change in the preference orderings of individuals. Consider a given \( N \) and preferences of individuals in \( N \). If the set of alternatives \( S \) expands and becomes \( S' \supset S \), then it is possible for an alternative that was Pareto-optimal before the expansion to become Pareto-inoptimal after the expansion. Such would be the case if one of the alternatives in \( S' - S \) happens to be Pareto-superior to the alternative in question. An alternative that was Pareto-inoptimal before the expansion cannot of course become Pareto-optimal. If an alternative is inoptimal with respect to \( S \), then there must exist an alternative belonging to \( S \) which is Pareto-superior to it. As \( S'' \supset S \), there would exist a Pareto-superior alternative after the expansion as well. If the set of alternatives \( S \) contracts and becomes \( S'' \subset S \), then it is possible for an alternative that was Pareto-inoptimal before the contraction to become Pareto-optimal after the contraction. Such would be the case if alternatives Pareto-superior to the alternative in question do not belong to \( S'' \). An alternative that was Pareto-optimal before the contraction would of course remain Pareto-optimal even after the contraction. Non-existence of a Pareto-superior alternative in \( S \) implies non-existence of a Pareto-superior alternative in \( S'' \). When \( N \) expands or contracts, both kinds of changes are possible. A previously Pareto-optimal alternative might become Pareto-inoptimal, or a previously Pareto-inoptimal alternative might become Pareto-optimal. The following examples illustrate these points.
Example 2.2. Let $S = \{x, y, z, w, t, v\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in $N$ be as follows:

$R_1 = vxzytw$
$R_2 = vxzwyt$
$R_3 = xwyztv$

As every individual prefers $x$ to all of $y, z, w, t$, it follows that none of $y, z, w, t$ is Pareto-optimal. As individual 3 prefers $x$ to $v$, it follows that no alternative in $S$ is Pareto-superior to $x$, and consequently $x$ is Pareto-optimal. Individual 1 prefers $v$ to any other alternative in $S$; therefore it follows that no alternative in $S$ is Pareto-superior to $v$ implying that it is Pareto-optimal.

Now, consider a contraction of the set of alternatives to $S' = \{y, z, w, t, v\}$.
As no alternative belonging to $S'$ is Pareto-superior to any alternative belonging to $S' - \{t\}$, it follows that all of $y, z, w, v$ are Pareto-optimal. As every individual prefers all of $y, z, w$ to $t$, $t$ is Pareto-inoptimal.

In this example when the set of alternatives expands from $S'$ to $S$, (i) alternatives $y, z, w$ Pareto-optimal with respect to $S'$ become Pareto-inoptimal with respect to $S$, and (ii) $t$, Pareto-inoptimal alternative with respect to $S'$, is Pareto-inoptimal with respect to $S$ as well. When the set of alternatives contracts from $S$ to $S'$, (i) alternatives $y, z, w$ Pareto-inoptimal with respect to $S$ become Pareto-optimal with respect to $S'$, and (ii) $v$, Pareto-optimal alternative with respect to $S$, is Pareto-optimal with respect to $S'$ as well. ♦

Example 2.3. Let $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in $N$ be as follows:

$R_1 = xyz$
$R_2 = yzx$
$R_3 = (xy)z$

As every individual prefers $y$ to $z$, it follows that $z$ is not Pareto-optimal. As individual 1 prefers $x$ to both $y$ and $z$, it follows that no alternative in $S$ is Pareto-superior to $x$, and consequently $x$ is Pareto-optimal. Individual 2 prefers $y$ to any other alternative in $S$; therefore it follows that no alternative in $S$ is Pareto-superior to $y$ implying that it is Pareto-optimal.

Now, consider an expansion of the set of individuals to $N' = \{1, 2, 3, 4\}$ with $R_4 = zyx$. Individual 4 prefers $z$ to any other alternative in $S$; therefore it follows that no alternative in $S$ is Pareto-superior to $z$ implying that it is Pareto-optimal. Thus, all alternatives are Pareto-optimal with respect to $S$ and $N'$.

In this example when the set of individuals expands from $N$ to $N'$, alternative $z$ Pareto-inoptimal with respect to $N$ becomes Pareto-optimal with respect to $N'$. When the set of individuals contracts from $N'$ to $N$, alternative $z$ Pareto-optimal with respect to $N'$ becomes Pareto-inoptimal with respect to $N$. ♦
Example 2.4. Let $S = \{x, y, z\}$ and $N = \{1, 2, 3\}$.
Let the preference orderings of individuals in $N$ be as follows:

$R_1 = (xy)z$
$R_2 = z(yx)$
$R_3 = (xy)z$

As individual 2 prefers $z$ to both $x$ and $y$, it follows that no alternative in $S$ is Pareto-superior to $z$, and consequently $z$ is Pareto-optimal. Every individual is indifferent between $x$ and $y$, and individual 1 prefers both $x$ and $y$ to $z$; therefore there is no alternative in $S$ which is Pareto-superior to $x$ or $y$. Thus, all alternatives in $S$ are Pareto-optimal.

Now, consider an expansion of the set of individuals to $N' = \{1, 2, 3, 4\}$ with $R_4 = zyx$. Individual 4 prefers $y$ to $x$ and individuals 1, 2, 3 are indifferent between $x$ and $y$, implying $y$ is Pareto-superior to $x$. Consequently $x$ is not Pareto-optimal with respect to $S$ and $N'$.

In this example when the set of individuals expands from $N$ to $N'$, alternative $x$ Pareto-optimal with respect to $N$ becomes Pareto-inoptimal with respect to $N'$. When the set of individuals contracts from $N'$ to $N$, alternative $x$ Pareto-inoptimal with respect to $N'$ becomes Pareto-optimal with respect to $N$. ◊

These examples make it clear that while discussing Pareto-efficiency or otherwise of alternatives, it is crucial that the set of individuals and the set of alternatives remain fixed throughout the discourse to avoid the possibility of erroneous inferences.

2.1.1 Paretian Value Judgment

The Pareto-criterion is a value judgment according to which, if alternative $x$ is Pareto-superior to $y$ then $x$ is socially better than $y$, and if alternative $x$ is Pareto-indifferent to $y$ then $x$ is socially as good as $y$. Being based on unanimity, Pareto-criterion is generally regarded as noncontroversial. Because the unanimity relation $Q$ is reflexive and transitive, it follows that the social ‘at least as good as’ relation induced by the Pareto-criterion is reflexive and transitive, but in general not complete. Two alternatives can be compared in terms of the Pareto-criterion if one of them is Pareto-superior to the other or if the two are Pareto-indifferent to each other; but not otherwise. In particular, if some individual prefers alternative $x$ to alternative $y$ and some other individual prefers $y$ to $x$, then in terms of the Pareto-criterion $x$ and $y$ cannot be compared. Pareto-criterion implies that if an alternative

\[\text{\footnotesize\ref{footnote}}\]

\footnote{It is rather important to note that merely from the fact of $x$ being Pareto-superior to $y$, one cannot infer that $x$ is socially better than $y$. The persuasive nature of a normative criterion cannot make a normative inference from a purely factual premise possible.}
is Pareto-inefficient, then there exists an alternative which is socially better than the alternative in question. From this it follows that if the social choice has to be a best alternative, i.e. an alternative at least as good as every feasible alternative, then it must be from among the subset of Pareto-efficient alternatives.

While from the Pareto-criterion it can be inferred that for every Pareto-inefficient alternative there exists a socially better alternative, one cannot infer that every Pareto-efficient alternative is better than every Pareto-inefficient alternative. In Example 2.1 we found that alternatives $x$, $y$, $w$ are Pareto-efficient and alternative $z$ is Pareto-inefficient. $z$, however, is Pareto-incomparable with $x$ and $w$. So, although $z$ is Pareto-inefficient and $x$ and $w$ Pareto-efficient, one cannot infer that $x$ and $w$ are better than $z$. From this it follows that a movement from a Pareto-inefficient state to a Pareto-efficient state does not imply that the society has moved to a socially better state than before.

Because the Pareto-criterion is based on unanimity, in general there will be few pairs of alternatives which would be comparable. There are two ways by which greater comparability could be attained. If one is essentially unwilling to make use of any value judgment other than the Pareto-criterion, then one way to make all pairs of alternatives comparable is to declare all Pareto-incomparable alternatives to be socially indifferent to each other. If the value judgment of the Pareto-criterion is supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, then it follows that all Pareto-efficient alternatives are equally good; and every Pareto-efficient alternative is at least as good as any Pareto-inefficient alternative. Thus, all Pareto-efficient alternatives are best and none of Pareto-inefficient alternatives is best. For instance, in the case of Example 2.1, by the use of the value judgment of the Pareto-criterion supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, we obtain that alternatives $x$, $y$, $w$ are equally good; $y$ is socially better than $z$; and $z$ is socially indifferent to $x$ and $w$. When Pareto-criterion is supplemented by the value judgment that all Pareto-incomparable alternatives are equally good, the social ‘at least as good as’ relation is no longer transitive as is clear from Example 2.1.10

If for comparing alternatives, which are incomparable in terms of the Pareto-criterion, one makes use of other value judgments, then there is no longer any guarantee that every Pareto-efficient alternative will be socially at least as good as every Pareto-inefficient alternative. The example given below illustrates the point.

**Example 2.5.** Let $N = \{1, 2, 3, 4\}$ and $S = \{(x_1, x_2, x_3, x_4) \mid \sum_{i \in N} x_i \leq 4\}$, where $x_i$ denotes the amount of money held by individual $i$.

The preference orderings of individuals in $N$ are given by:

$(\forall i \in N) [(x_1, x_2, x_3, x_4) R_i (x'_1, x'_2, x'_3, x'_4) \iff x_i \geq x'_i].$

Let $a = (4, 0, 0, 0)$ and $b = (1, 1, 1, 0.9999)$. $a$ is Pareto-efficient and $b$ is Pareto-inefficient. Under the Pareto-criterion the two alternatives are not comparable. If one

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9Alternative $x \in S$ is best in $S$ according to binary relation $R$ iff $(\forall y \in S)(x R y)$.
10The ‘socially better than’ relation, however, is transitive. See Sen (1970).
uses some criterion like justice, fairness, or equality to compare $a$ and $b$, then $b$ would be regarded as socially better than $a$. Thus, the use of a criterion to compare Pareto-incomparable alternatives can lead to a Pareto-inefficient alternative being regarded as socially better than a Pareto-efficient alternative.

Some laws, rules, and institutions have the property that when purposive and self-regarding individuals act within their framework, the outcomes are Pareto-efficient. As a shorthand, we can term such laws, rules, and institutions themselves as Pareto-efficient. It is generally taken for granted that Pareto-efficient laws, rules, and institutions are to be preferred over those which do not have the property of invariably giving rise to Pareto-efficient outcomes. Simply on the basis of the Paretian value judgment, such a conclusion is not warranted. This is irrespective of whether Paretian value judgment is given primacy over all other value judgments or not. As we saw above, what is required to reach the conclusion that Pareto-efficient laws, rules, and institutions are to be preferred over the Pareto-inefficient ones is to have only the Paretian value judgment supplemented by the value judgment that all Pareto-incomparable alternatives are socially indifferent, to the exclusion of other value judgments.

2.2 The Kaldor Criterion and Wealth Maximization

Comparing of alternative policies in terms of the Pareto-criterion for any real society is a non-starter. As almost all policies tend to benefit some and harm some others, comparison in terms of Pareto-criterion is not possible. To surmount this difficulty Kaldor (1939) proposed that in order to determine whether a proposed change is desirable, one should find out whether those benefiting from the change are in a position to compensate those who would be losing out from the change and still be better off compared to the pre-change situation. If the gainers can compensate the losers and still be better off than before then the change is preferable; otherwise not. If gainers can compensate the losers and still be better off, then intuitively it seems that the change is wealth increasing, regardless of whether compensation is paid or not. If compensation is paid the post-compensation state is clearly Pareto-superior to the pre-change state as no one is worse off than before and the gainers are better off. But the crucial point is that even if compensation is not paid, from the fact that the gainers can compensate the losers and still be better off, it appears that there is now greater wealth than before.

Let $S(x)$ denote the set of social states accessible from social state $x$ through compensations. Social state $x$ is defined to be Kaldor-superior to social state $y$ iff there exists some $z \in S(x)$ which is Pareto-superior to $y$. The value judgment of the Kaldor criterion declares $x$ to be socially better than $y$ if $x$ is Kaldor-superior to $y$. On the basis of the existence of $z$, which can be reached through compensations from $x$, and under which everyone is at least as well off as under $y$ and some people are better off, the Kaldor criterion declares $x$ itself to be socially better than $y$. 
The value judgment underlying the Kaldor criterion, however, is self-inconsistent. It is possible for \( x \) to be Kaldor-superior to \( y \) and \( y \) to be Kaldor-superior to \( x \).\(^{11}\) As it is not possible to have both \( x \) to be socially better than \( y \) and \( y \) to be socially better than \( x \), the value judgment which declares an alternative to be socially better than another if the former is Kaldor-superior to the latter is incapable of being held in all cases. The example below illustrates the inconsistency of the value judgment of Kaldor-superiority implying social strict preference.

**Example 2.6.** Let \( N = \{1, 2\} \). Let \(((a_1, b_1); (a_2, b_2))\) denote the allocation in which individual 1 has \( a_1 \) amount of good 1 and \( b_1 \) amount of good 2, and individual 2 has \( a_2 \) amount of good 1 and \( b_2 \) amount of good 2. Let \( S = \{x, y, z, w\} \), where:

\[
\begin{align*}
x &= ((9, 1); (1, 4)) \\
y &= ((4, 1); (1, 9)) \\
z &= ((4, 3); (1, 7)) \\
w &= ((7, 1); (3, 4)).
\end{align*}
\]

The preference orderings of individuals in \( N \) are given by:

\[
\begin{align*}
R_1 &= zywx \\
R_2 &= yzwx
\end{align*}
\]

\( x \) is Kaldor-superior to \( y \) as the gainer if there is a move from \( y \) to \( x \), individual 1, can more than compensate individual 2 by giving up 2 units of good 1 to individual 2 and still be better off. Allocation \( w \) is reached from \( x \) when individual 1 transfers 2 units of good 1 to individual 2. \( w \) is preferred to \( y \) by both the individuals.

\( y \) is Kaldor-superior to \( x \) as the gainer if there is a move from \( x \) to \( y \), individual 2, can more than compensate individual 1 by giving up 2 units of good 2 to individual 1 and still be better off. Allocation \( z \) is reached from \( y \) when individual 2 transfers 2 units of good 2 to individual 1. \( z \) is preferred to \( x \) by both the individuals. \( \diamond \)

One way out of the inconsistency, following the Scitovsky suggestion, is to regard of the two social states \( x \) and \( y \), \( x \) to be socially better than \( y \) if \( x \) is Kaldor-superior to \( y \) and \( y \) is not Kaldor-superior to \( x \); \( y \) to be socially better than \( x \) if \( y \) is Kaldor-superior to \( x \) and \( x \) is not Kaldor-superior to \( y \); and \( x \) to be socially indifferent to \( y \) if it is the case that both \( x \) is Kaldor-superior to \( y \) and \( y \) is Kaldor-superior to \( x \) or if it is the case that neither \( x \) is Kaldor-superior to \( y \) nor \( y \) is Kaldor-superior to \( x \). It can be shown that the social indifference relation generated by this modified compensation principle is not transitive.\(^{12}\)

In connection with the use of compensation principles for evaluating social states, two important points need to be noted. First, when one state is declared socially better than another state, it ought to mean that taking into consideration all relevant things from an overall viewpoint, the former is a better state for the society than the latter. In judging social states both aggregate wealth and its distribution

\(^{11}\)That it is possible to have both \( x \) to be Kaldor-superior to \( y \) and \( y \) to be Kaldor-superior to \( x \) was first pointed out by Scitovsky (1941).

\(^{12}\)Arrow (1951).
matter. Thus, comparing social states solely on the basis of aggregate wealth, to the exclusion of its distribution, constitutes a questionable method. Second, even if one is solely interested in aggregate wealth, and not its distribution, there is no way that the notions of magnitude of wealth and its distribution can be separated except in trivial cases.\(^ {13}\) The logical difficulties associated with compensation criteria are a reflection of the general impossibility of defining aggregate wealth independent of its distribution. The use of Kaldor-Hicks criterion in the law and economics literature is quite common, notwithstanding the difficulties discussed above.

References


\(^ {13}\)The difficulties associated with the compensation principles have been elucidated by Arrow (1963, pp. 39–40) as follows:

A matter which immediately springs to mind is the desirability of the goal which Kaldor and Hicks set for themselves, that of separating the production aspects of a desired change in social state from the distribution aspects. Any given choice is made on the basis of both considerations; even if a clear-cut meaning were given to an ordering of social states in terms of production, it would not be in the least obvious what use that ordering would be in relation to the desired ordering of social states in terms of all relevant factors, including distributional elements as well as production.

But a deeper objection is that, in a world of more than one commodity, there is no unequivocal meaning to comparing total production in any two social states save in terms of some standard of value which makes the different commodities commensurable; and usually such a standard of value must depend on the distribution of income. In other words, there is no meaning to total output independent of distribution.
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