The purpose of the present monograph is to discuss briefly what summability theory is like when the underlying field is not \( \mathbb{R} \) (the field of real numbers) or \( \mathbb{C} \) (the field of complex numbers) but a field \( K \) with a non-Archimedean or ultra-metric valuation, i.e., a mapping \( \nu: K \to \mathbb{R} \) satisfying the ultrametric inequality
\[
\nu(x + y) \leq \max\{\nu(x), \nu(y)\}
\]
instead of the usual triangle inequality \( |x + y| \leq |x| + |y|, x, y \in K \).

To make the monograph really useful to those who wish to take up the study of ultrametric summability theory and do some original work therein, some knowledge of real and complex analysis, functional analysis and summability theory over \( \mathbb{R} \) or \( \mathbb{C} \) is assumed.

Some of the basic properties of ultrametric fields—their topological structure and geometry—are discussed in Chap. 1. In this chapter, we introduce the \( p \)-adic valuation, \( p \) being prime and prove that any valuation of \( \mathbb{Q} \) (the field of rational numbers) is either the trivial valuation, a \( p \)-adic valuation or a power of the usual absolute value \( \nu(x) \) on \( \mathbb{R} \), i.e., \( \nu(x) = \nu_0(x) \), where \( 0 < \nu_0 \leq 1 \). We discuss equivalent valuations too. In Chap. 2, we discuss some arithmetic and analysis in \( \mathbb{Q}_p \), the \( p \)-adic field for a prime \( p \). In Chap. 2, we also introduce the concepts of differentiability and derivatives in ultrametric analysis and very briefly indicate how ultrametric calculus is different from our usual calculus.

In Chap. 3, we speak of ultrametric Banach space, and also mention the many results of the classical Banach space theory, viz., the closed graph, the open mapping and the Banach-Steinhaus theorems carry over in the ultrametric set-up. However, the Hahn-Banach theorem fails to hold. To salvage the Hahn-Banach theorem, the concept of a “spherically complete field” is introduced and Ingleton’s version of the Hahn-Banach theorem is proved. The lack of ordering in an ultrametric field \( K \) makes it quite difficult to find a substitute for classical “convexity”. However, classical convexity is replaced, in the ultrametric setting, by a notion called “\( K \)-convexity”, which is briefly discussed towards the end of the chapter.

In the main Chap. 4, our survey of the literature on “Ultrametric Summability Theory”, starts with the paper of Andree and Petersen of 1956 (it was the earliest known paper on the topic) to the present. As far as the author of the present monograph knows, most of the material discussed in the survey has not appeared in book form earlier. Almost all of Chap. 4 consists entirely of the work of the
author of the present monograph. Suitable references have been provided at appropriate places indicating where further developments may be found.

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