Chapter 2
Experimental

2.1 Experimental Overview

Figure 2.1 shows a schematic of the experimental setup. The experiments were performed at the Research Center for Development of Far-Infrared Region at the University of Fukui, Japan.

The basic operating scheme of this experiment is as follows:

1. A gyrotron oscillator radiates a millimeter wave Gaussian beam with a power ranging from 100 to 550 W.
2. A Fabry–Pérot resonance cavity accumulates the Gaussian beam to obtain an equivalent power of over 10 kW.
3. Filled with a specific gas as the electron source the Fabry–Pérot resonance cavity forms Ps within the cavity from positrons emitted by a $^{22}$Na source.
4. The millimeter wave radiation stimulates the transition from $o$-Ps to $p$-Ps.
5. The increase in the number of $\gamma$-ray pairs emitted from the $p$-Ps decays is measured using a $\gamma$-ray detector.
6. To measure the cross-section of the Ps-HFS transition, the frequency was tuned from 201 to 206 GHz by changing the RF cavity in the gyrotron oscillator.

2.2 Millimeter Wave Optics System

2.2.1 Gyrotron Oscillator

This experiment needed high power (>100 W), monochromatic (<0.1 %) and high-duty (>30 %) millimeter waves. The solution was to use a gyrotron oscillator.
2.2.1.1 Principle of the Gyrotron Oscillator

A gyrotron oscillator is a cyclotron-resonance-maser fast-wave device [1–3]. It is not an amplifier, but a self-exciting oscillator of millimeter waves. The maser oscillation of the gyrotron is caused by transitions between the inversely distributed Landau levels of the gyrating electrons instead of the atomic energy levels used in other maser devices. Because a quantum mechanical treatment is not practical, gyrotron theory is usually casted in classical terms, which has been relegated to an appendix, (Appendix A). In this section, an overview of the gyrotron mechanism is described.

Figure 2.2 shows a schematic of the gyrotron oscillator used in this experiment (model serial: FU CW GI). The gyrotron is composed of three parts: a RF cavity (which in fact does not produce radio frequencies but is so called by convention) in a superconducting solenoid, a magnetron injection gun (MIG) and a Gaussian beam converter. These parts were all under high vacuum (about $10^{-5}$ Pa). Electrons were emitted from an electron beam emitter under a high applied voltage (cathode voltage, $V_k = -18$ kV), and their trajectory was controlled with a gun-coil magnet (≈0.1 T) and a 1st anode voltage (≈10 kV). The electrons gyrate in the RF cavity where a magnetic field ($B_0$) is applied.

Figure 2.3 shows a cross-sectional view of the electrons in the RF cavity. The cyclotron angular frequency of the electron is expressed as:

$$\omega_C = \frac{eB_z}{m_e\gamma} = \frac{\Omega_0}{\gamma}, \quad (2.1)$$
Fig. 2.2 Schematic of the gyrotron oscillator, FU CW GI

where \( \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \) (with \( v \) the electron velocity) is the Lorentz factor of an electron. The electron beam with a current of \( I_b \) was cylindrically distributed with a radius \( (R_b) \) corresponding to the shape of the emitter ring of the MIG. The Larmor radius (gyrating radius) is:

\[
r_L = \frac{v_{\perp}}{\omega_c},
\]

which is small compared with \( R_b \) and its thickness. The electrons have a helical motion with a random phase \( (\psi) \) because they were thermally emitted from the emitter.

The gyrating electrons and electric fields in the RF cavity (radius = 5 mm, length = 24 mm) interact with each other in accordance with Maxwell’s equations and the Lorentz force. These interactions excited strong millimeter wave radiation (100–550 W) in the waveguide mode (TE\(_{mn}\) mode). The radius of the RF cavity and the excited mode (TE\(_{52}\) mode in this experiment) determine the output frequency. The waveguide mode was converted to a bi-Gaussian beam by the Gaussian beam converter and was output from a window.
The various simulation plots in Fig. 2.4 show the interactions between the electrons and the RF field at different heights in the cavity. Note that the electrons are in momentum space here (they are in real space in Fig. 2.3). We introduce a relative phase ($\theta$) between the electric field and the electron momentum. At the entrance of the cavity ($z = 0$ mm), the momenta of the electrons were random in phase ($\theta$) through thermal emission from the MIG. They all have the same energy (normalized to unity) as determined by the cathode voltage. When the electrons starts to interact with the RF field inside the RF cavity ($z = 6$ mm), some electrons were accelerated (gaining energy and went outside the circle of radius 1 marked in the left plot of Fig. 2.4) whereas others were decelerated (losing energy and went inside the circle) depending on their initial phases. Because of the Lorentz factor in the cyclotron angular frequency, Eq. (2.1), $\omega_c$ for the accelerated electrons decreases whereas for the decelerated electrons it increases.

If the RF frequency ($\omega_r$) is slightly higher than the initial value of the cyclotron frequency, $\omega_c$ for only the decelerated electrons moves closer to the exact resonance of the RF field, thereby losing an increasing amount of energy in each successive cycle. Automatic bunching of the deceleration phase occurs at $z = 12$ mm (center of the RF cavity) in Fig. 2.4. Finally, at $z = 20$ mm corresponding to the end of the interaction region in the RF cavity, most of the electrons have lost their initial energy.
Fig. 2.4 Simulated interactions between electrons and RF field. Electron momenta were calculated at $z = 0, 6, 12$ and $20$ mm heights in the RF cavity (see Figs. A.1 and A.3) with a cathode voltage $V_k = -18$ kV, an electron-beam current $I_b = 500$ mA, a magnetic field strength $B_0 = 7.4$ T, a cavity radius $R_2 = 2.475$ mm, a beam radius $R_b = 0.7 \cdot R_2$, and a pitch factor $=1.2$. The output power $P_{\text{out}}$ was $2.5$ kW. The left plots show the normalized electron momenta, which initially commenced with a random phase. The middle plots show histograms of the electron phase. The right plots show that the normalized electron energies decrease as they passed through the RF cavity, i.e., the electromagnetic field has gained energy.
as indicted with the plots on the right in Fig. 2.4. The RF field ideally gains 30\% of the initial energy of the electron beam. This is the operating principle of the gyrotron oscillator as given by the classical interpretation of cyclotron resonance in masers.

The excitation conditions of the gyrotron oscillator are as follows:

- A (weakly) relativistic electron beam is used
- The magnetic field strength is set so that the relativistic cyclotron frequency is slightly smaller than the resonance frequency of the RF cavity.

The remarkable feature of the gyrotron oscillator is that it uses a relativistic effect of electron cyclotron motion. Phase bunching occurs even if the electrons are non-relativistic. However, non-relativistic electrons never bunch in the deceleration phase and they do not excite millimeter wave radiation as occurs under gyrotron conditions.

The difference between the cyclotron frequency and the RF frequency is called frequency detuning. Frequency detuning determines the non-linearity of the gyrotron oscillation. The excitation efficiency is a single-valued function of the electron-beam current ($I_b$) for small frequency detuning. The output power increases as $I_b$ increases; this condition is called soft excitation. However, if the frequency detuning is large, the excitation efficiency is a multivalued function of $I_b$; this condition is called hard excitation. Moreover, high efficiencies are obtained under hard excitation (see Appendix A.5). In our case, the operation condition was soft, but we almost had hard excitation and the results for the output power exhibited non-trivial behavior. Under hard excitation, the power begins to be outputted at large electron-beam current and increases as $I_b$ decreases until $I_b$ becomes the minimum possible value. Then, the power increases as $I_b$ increases (see Fig. A.8). The gyrotron determines how it interacts with the plasma; here the parameters need to be controlled manually to accomplish the best interaction demanded by the gyrotron. Thus, it is difficult to set the output power as a target of feedback controls and to stabilize the output power for long durations (normally one week). Details of the power-stabilization procedure are described in Sect. 2.2.4.

### 2.2.1.2 Properties of the Gyrotron, FU CW GI

Figure 2.5 shows a photograph of the gyrotron (FU CW GI) developed specifically for our measurements [4]. The operation parameters for FU CW GI are summarized in Table 2.1. Table 2.2 summarizes the strengths of the superconducting magnetic field, frequency and power for each cavity radius. The cathode voltage was fixed at $-18$ kV during the experiments. The strength of the superconducting magnetic field was adjusted to the value determined by the cavity radius. The 1st anode voltage and the gun-coil magnetic field were controlled to stabilize the output power, line-width, and frequency drift.

The output power was measured with a calorimeter, which was a 46 ml water tank in a thin Teflon case. The temperature increase of the water pool was measured with a Pt100 resistance thermometer. The calorimeter was placed at an angle of about 45\(^\circ\) to prevent any reflected beams from going back into the RF cavity, because reflected
beams can set up standing waves, reducing the accuracy of the power estimation. The reflected beams can also interfere with the electron interactions in the RF cavity and change the condition of the gyrotron during the measurement (see Sect. 2.2.3). The expected output power ranged from 1 to 2 kW, whereas the measured power was less than 600 W and varied by a factor of four depending on the cavity. The conditions of the gyrotron oscillation were very sensitive to the fabrication accuracy of the RF cavity, the quality of the emitter surface of the MIG, and the alignment of the cavity in the magnetic field. In general, it was difficult to reproduce the same conditions for all gyrotron operations [5] (see Appendix A.6 for details).

The output frequency of the gyrotron was measured using a hetero-dyne technique with a synthesizer (∼12 GHz) as the local oscillator. An even harmonic mixer (WR5.1EHM, Virginia Diodes, Inc., Virginia, US) made of a Schottky barrier diode was used. Denoting the gyrotron frequency by $f_{RF}$ and the synthesizer frequency by $f_{LO}$, the intermediate frequency (IF) can be expressed as:

Table 2.1 Operation parameters of the gyrotron FU CW GI

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cathode voltage</td>
<td>−18 kV</td>
</tr>
<tr>
<td>1st Anode voltage</td>
<td>−8−11 kV</td>
</tr>
<tr>
<td>2nd Anode voltage</td>
<td>GND</td>
</tr>
<tr>
<td>Gun coil magnetic field</td>
<td>0.1–0.14 T</td>
</tr>
<tr>
<td>Beam current</td>
<td>300–400 mA</td>
</tr>
<tr>
<td>Repetition rate</td>
<td>5 Hz</td>
</tr>
<tr>
<td>Duty ratio</td>
<td>30%</td>
</tr>
</tbody>
</table>
Table 2.2  Operating points used for the measurements

<table>
<thead>
<tr>
<th>mode</th>
<th>$R_2$ (mm)</th>
<th>$B_0$ (T)</th>
<th>$f_{\text{calc}}$</th>
<th>$Q_{\text{calc}}$</th>
<th>$f_{\text{meas}}$</th>
<th>$\Delta f$ (MHz)</th>
<th>$P$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE$_{42}$</td>
<td>2.453</td>
<td>6.57</td>
<td>180.76</td>
<td>2080</td>
<td>180.56</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.481</td>
<td>7.35</td>
<td>202.51</td>
<td>2400</td>
<td>201.83</td>
<td>20</td>
<td>190</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.475</td>
<td>7.37</td>
<td>203.00</td>
<td>2450</td>
<td>202.64</td>
<td>20</td>
<td>240</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.467</td>
<td>7.40</td>
<td>203.66</td>
<td>2500</td>
<td>203.00</td>
<td>40</td>
<td>550</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.467</td>
<td>7.44</td>
<td>203.66</td>
<td>2500</td>
<td>203.25</td>
<td>50</td>
<td>250</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.463</td>
<td>7.42</td>
<td>203.99</td>
<td>2530</td>
<td>203.51</td>
<td>20</td>
<td>350</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.453</td>
<td>7.43</td>
<td>204.82</td>
<td>2600</td>
<td>204.56</td>
<td>50</td>
<td>410</td>
</tr>
<tr>
<td>TE$_{52}$</td>
<td>2.443</td>
<td>7.48</td>
<td>205.65</td>
<td>2650</td>
<td>205.31</td>
<td>40</td>
<td>125</td>
</tr>
</tbody>
</table>

\[ f_{\text{IF}} = |f_{\text{RF}} - nf_{\text{LO}}|, \tag{2.3} \]

where $n$ is the harmonic number. $f_{\text{IF}}$ was measured with an oscilloscope and analyzed on-line using the fast Fourier transform (FFT) method. The lower side band (LSB) corresponds to the IF when $nf_{\text{LO}} < f_{\text{RF}}$, and the upper side band (USB) corresponds to the IF when $f_{\text{RF}} < nf_{\text{LO}}$. Note that, a small LSB at the same LO means that the RF is also small. A harmonic number of $n = 16$ was selected to monitor the RF. The output power of the synthesizer for the optimum measurements ranged from 9 to 11 dBm. In accordance with Table 2.2, the measured frequency ($f_{\text{meas}}$) was always smaller than the expected one ($f_{\text{calc}}$) by a few 100 MHz. The thermal expansion of the cavity radius may have caused the frequency to shift in the steady state operation of the gyrotron.

Figure 2.6 shows examples of the measured FFT spectra, the $f_{\text{IF}}$ of the LSB. Each curve shows the FFT spectrum at certain times during a single gyrotron pulse duration. The measured frequency line-width was greater than 2 MHz and drifted...
randomly at about 20 MHz. Because these effects were very small (<100 ppm) and the gyrotron operation is in principle a complicated plasma interaction, there are many reasons that could explain these phenomena. The line-width may be caused by either the large velocity spread ($\Delta v_{\parallel}$) of the electron beam or interactions with reflected electromagnetic waves [6, 7]. The frequency drift may have been caused by a decrease in the voltage by a space charge or thermal expansion of the cavity radius during one pulse [8]. With current gyrotron technology, these are inevitable and result in systematic errors. Fortunately, these errors are small compared with the level of accuracy needed in this thesis (0.1 % or 200 MHz). A better line-width and drift are needed to address the observed discrepancy in Ps-HFS (15 ppm or 3 MHz) in the future. This technical improvement is feasible as it has been reported that the line-width of a low frequency (<170 GHz) gyrotron is better than that at 100 kHz [9].

To measure the cross-section curve of the Ps-HFS transition, the input radiation should be scanned from 201 to 206 GHz. Table 2.2 shows that this was achieved by changing the gyrotron cavity so that it had a different radius, $R_2$. A gate valve was inserted between the cavity section and the MIG (Fig. 2.2), so that the emitter surface was not exposed to air while the cavity was being replaced. It took more than 1 week after the cavity was replaced to flush the emitter, bake the cavity, carefully align all of the elements and find the optimal operation parameters. One exception was at $f_{\text{meas}} = 180.56$ GHz, where the operation mode was the TE$_{42}$ mode with the same cavity radius as the $f_{\text{meas}} = 204.56$ GHz point to obtain a point far off resonance.

The output radiation from the window was designed to be a bi-Gaussian beam (see Appendix A.7). The power distribution of the bi-Gaussian beam was estimated using the Kirchhoff integral method (Fig. 2.7). The horizontally long distribution changed to vertically long distribution during propagation. The power distribution was measured by the temperature increase in a thin (1 mm) polyvinyl chloride (PVC) plate. Figure 2.8 show photographs of the PVC taken with an infrared (IR) camera. The measured power distribution is in good agreement with the estimates. The beam had a waist size $(w_x, w_y) = (15.1, 8.4$ mm) and a waist position from the output window $(z_{0x}, z_{0y}) = (-102.7, 56.8$ mm), where the z-axis is defined along the light path with the origin at the window. The negative sign of $z_{0x}$ means inside the gyrotron.

![Fig. 2.7](image_url)  
**Fig. 2.7** Calculated output patterns of FU CW GI. The left, center, and right images give the power distributions 200, 400, and 700 mm, respectively, from the output window.
Fig. 2.8 Measured output patterns of FU CW GI. The left, center, and right images are the power distributions 200, 400 and 700 mm, respectively, from the output window.

The bi-Gaussian beam was converted to a Gaussian beam with toroidal mirrors, according to the Gaussian lens formula [10]. The focal points were different, depending on the two radii of curvature $R_1$ and $R_2$, as shown in Fig. 2.9. When the beam is reflected by the toroidal mirror at a reflection angle $\theta$, the focal points $F_1$ and $F_2$ are

$$F_1 = R_1 \frac{\cos \theta}{2} \quad (2.4)$$
$$F_2 = R_2 \frac{1}{2 \cos \theta} \quad (2.5)$$

Two mirrors, M1 ($R_1 = 2902$ mm, $R_2 = 1776$ mm) and M2 ($R_1 = 679$ mm, $R_2 = 1482$ mm), made of aluminum, are shown in Fig. 2.1. The reflection angle was fixed at $45^\circ$. M1 was placed 550 mm from the output window and M2 was placed 600 mm from M1. A new beam waist ($w_0 = 8.4$ mm) was established at 600 mm from M2, where the input side mirror of the Fabry–Pérot resonance cavity was placed. Figure 2.10 show the measured power distribution of the converted beam. The beam was almost a perfect Gaussian and efficiently coupled to the cavity.

Fig. 2.9 Schematic of the toroidal mirror. $R_1$ and $R_2$ are the two radii of curvature.
2.2 Millimeter Wave Optics System

Fig. 2.10  Measured power distributions of the Gaussian beam. The left, center, and right figures are the power distributions 600, 750 and 900 mm, respectively, from M2

2.2.2 Fabry–Pérot Resonant Cavity

The output of the gyrotron was at most 550 W. The Ps-HFS direct transition requires over 10 kW. A Fabry–Pérot resonance cavity was developed to obtain such a high equivalent power. In this section, the general theory of this resonance cavity is derived and then specific problems associated with millimeter wave radiation are discussed.

2.2.2.1 Theory of the Fabry–Pérot Resonance Cavity

A Fabry–Pérot resonance cavity is composed of two parallel mirrors. Figure 2.11 shows a schematic of a Fabry–Pérot resonance cavity. An electromagnetic wave with electric field $E_{\text{in}}$ is introduced from the left-hand side. The front mirror (left-hand side) is a half mirror with a reflection coefficient amplitude of $r_f$ and a transmission coefficient of $t_f$. For simplicity, $r_f$ and $t_f$ were assumed to be independent of the
propagation direction. The right-bound and left-bound propagating waves in the cavity are denoted $E_r$ and $E_l$, respectively. A wave $E_{re}$ is reflected from the cavity. The end mirror (right-hand side) has a reflection coefficient of $r_e$ and a very small transmission $t_e$, to sample $E_r$. A transmitted wave is denoted as $E_{tr}$. The boundary conditions for these electric fields are:

$$E_{tr}(d) = t_e E_r(d),$$  
(2.6)

$$E_{re}(0) = -r_f E_{in}(0) + t_f E_l(0).$$  
(2.7)

Note that the negative signs arise from a phase shift of $\pi$, caused by the wave reflecting from each mirror.

We now consider the simplest case, a monochromatic plane wave (with angular frequency $\omega$ and wave number $k$). A right-bound propagating wave in the cavity can be expressed as:

$$E_r(z) = t_f E_{in}(0)e^{ikz} \sum_{j=0}^{\infty} \left[ r_e r_f e^{2ikd} \right]^j = \frac{t_f E_{in}(0)e^{ikz}}{1 - r_f r_e e^{2ikd}}. \quad (2.8)$$

Similarly, a left-bound propagating wave in the cavity is:

$$E_l(z) = t_f r_e E_{in}(0)e^{-ik(z-2d)} \sum_{j=0}^{\infty} \left[ r_e r_f e^{2ikd} \right]^j = \frac{t_f r_e E_{in}(0)e^{-ik(z-2d)}}{1 - r_f r_e e^{2ikd}}. \quad (2.9)$$

By imposing the boundary conditions, Eqs. (2.6) and (2.7), the transmitted and reflected waves can be calculated:

$$E_{tr}(z) = t_e E_r(d)e^{ik(z-d)} = \frac{t_f t_e}{1 - r_f r_e e^{2ikd}} E_{in}(0)e^{ikz} \quad (2.10)$$

and

$$E_{re}(z) = [-r_f E_{in}(0) + t_f E_l(0)]e^{-ikz} = \left[ -r_f + \frac{t_f^2 r_e e^{2ikd}}{1 - r_f r_e e^{2ikd}} \right] E_{in}(0)e^{-ikz}, \quad (2.11)$$

respectively.

Using the power reflection coefficients ($R_f = r_f^2$ and $R_e = r_e^2$) and the transmission coefficients ($T_f = t_f^2$ and $T_e = t_e^2$), the transmitted power ($P_{tr} \propto |E_{tr}|^2$) and reflected power ($P_{re} \propto |E_{re}|^2$) can be expressed as:

$$P_{tr} = \frac{P_{in}}{1 - \sqrt{R_f R_e}} \frac{1}{1 + F \sin^2 kd}. \quad (2.12)$$
\[
\frac{P_{\text{re}}}{P_{\text{in}}} = \left[ \frac{\sqrt{R_f} - (T_f + R_f) \sqrt{R_e}}{1 - \sqrt{R_f} R_e} \right]^2 + \frac{(T_f + R_f) F \sin^2 k d}{1 + F \sin^2 k d}, \tag{2.13}
\]

where \( P_{\text{in}} \propto |E_{\text{in}}|^2 \) is the input power from the gyrotron oscillator, and \( F \) is defined as
\[
F = \frac{4\sqrt{R_f R_e}}{(1 - \sqrt{R_f} R_e)^2}. \tag{2.14}
\]

This cavity resonates when the cavity length (\( d \)) is a half-integer multiple of the wavelength (\( \lambda (k = 2\pi/\lambda) \)), as shown in Fig. 2.12.

The reflection and transmission coefficients of each mirror, which determine the properties of the cavity, are not known. Two observable parameters are introduced instead of these coefficients. One is finesse, \( \mathcal{F} \), defined as:
\[
\mathcal{F} = \frac{\pi}{2} \frac{\sqrt{F}}{\sin \frac{2\pi}{1 - R_f R_e}}. \tag{2.15}
\]

Using the full width at half maximum (FWHM) of \( P_{\text{tr}} \) at one resonance, \( \mathcal{F} \) can be obtained as
\[
\mathcal{F} = \frac{\pi}{\text{FWHM [radian]}} = \frac{\text{FSR [\mu m]}}{\text{FWHM [\mu m]}}, \tag{2.16}
\]

where FSR is the free spectral range, which indicates the separating distance of resonances, as shown in Fig. 2.12. Note that \( \mathcal{F} \) is related to the cavity-loaded quality.
factor ($Q$) and the number of axial nodes ($n$)

$$F = \frac{Q}{n}. \quad (2.17)$$

The other parameter is the input coupling coefficient ($\beta$). To define $\beta$, an input reflection coefficient ($|\rho_{in}|$) was introduced, defined as:

$$|\rho_{in}| = \frac{\sqrt{P_{re}(\text{on resonance})}}{\sqrt{P_{re}(\text{off resonance})}} = \frac{\sqrt{R_f - (T_f + R_f)\sqrt{R_e}}}{1 - \sqrt{R_f R_e}} \frac{1}{\sqrt{R_f + (T_f + R_f)\sqrt{R_e}}}. \quad (2.18)$$

Then, $\beta$ can be expressed as:

$$\beta = \begin{cases} 1 - |\rho_{in}| & < 1 \quad \text{(under-coupled)} \quad (2.19) \\ 1 + |\rho_{in}| & > 1 \quad \text{(over-coupled).} \quad (2.20) \end{cases}$$

$\beta$ corresponds to the effective power input into the resonator and is obtained by measuring the reflected power. Under- or over-coupling should be determined before calculating $\beta$.

The cavity gain is expressed as a function of $F$ and coupling $\beta$. At first, the round-trip reflectance ($\rho$) will be introduced as a function of $F$

$$\rho(F) = R_f R_e = \left(1 + \frac{\pi^2}{2F^2}\right) - \sqrt{\frac{\pi^2}{2F^2} \left(1 + \frac{\pi^2}{2F^2}\right)} \sim 1 - \frac{2\pi}{F}. \quad (2.21)$$

The last approximation is valid if $F \gg 1$, i.e. $\rho \sim 1$. The ratio ($T_f/R_f$) can be expressed as a function of $\beta$ and $\rho$

$$\frac{T_f}{R_f} = \frac{1}{\sqrt{\rho(F)}} \frac{\beta [1 - \rho(F)]}{1 + \beta \sqrt{\rho(F)}}. \quad (2.22)$$

where under-coupling is assumed. The accumulated equivalent power ($P_{acc}$) inside the cavity can be expressed as:

$$\frac{P_{acc}(z)}{P_{in}} = \frac{|E_r + E_l|^2}{|E_{in}|^2} = T_f \left(\frac{1 + \sqrt{R_e}}{1 - \sqrt{R_f R_e}}\right)^2 - \frac{1}{\sqrt{R_f}} F \sin^2 k(z - d). \quad (2.23)$$
The cavity gain \( G \) at resonance is defined by averaging the accumulated power, \( \langle P_{\text{acc}} \rangle \)

\[
G = \left. \frac{\langle P_{\text{acc}} \rangle}{P_{\text{in}}} \right|_{\text{reso}} = \frac{1}{d} \int_0^d T_f \left( \frac{1 + \sqrt{R_e}}{1 - \sqrt{R_f} R_e} \right)^2 - \frac{1}{\sqrt{R_f}} F \sin^2 k(z - d) \frac{1}{1 + F \sin^2 k d} \right|_{\text{reso}} dz
\]

\[
= \frac{T_f}{1 + F \sin^2 k d} \left[ \left( \frac{1 + \sqrt{R_e}}{1 - \sqrt{R_f} R_e} \right)^2 - \frac{2 \sqrt{R_e}}{(1 - \sqrt{R_f} R_e)^2} \left( 1 + \sin 2 k d \right) \right] \Bigg|_{\text{reso}}
\]

\[
\sim \frac{T_f (1 + R_e)}{(1 - \sqrt{R_f} R_e)^2}.
\]

The last approximation is valid if the cavity length \( d \) is much longer than the wavelength. From Eqs. (2.21) and (2.22), we obtain:

\[
G(\mathcal{F}, \beta) = \sqrt{\rho(\mathcal{F})} \frac{\beta [1 - \rho(\mathcal{F})]}{(1 - \sqrt{\rho(\mathcal{F})})^2} \left( 1 + \frac{\beta}{R_e} \right).
\]

If the resonance is in the over-coupled regime, \( \beta \) can be replaced with \( 1/\beta \). As \( R_e \) is usually close to unity, the cavity gain can be determined by the observable \( \mathcal{F} \) and \( \beta \).

Thus far, we have considered the response of a Fabry–Pérot cavity with a plane wave. To stabilize the resonance, the cavity should resonate a Gaussian beam [10].

The cavity contains a plane mirror (front mirror) and a concave mirror (end mirror). A Gaussian beam is uniquely determined if the beam waist size \( w_0 \) and its position are given. The waist position is at the plane mirror and \( w_0 \) is expressed as:

\[
w_0 = \sqrt{\frac{2}{\pi}} \sqrt{d(R - d)},
\]

where \( R \) denotes the radius of curvature of the end mirror and \( d \) is the cavity length. The resonance has low diffraction loss if

\[
0 < \left( 1 - \frac{d}{R} \right) < 1
\]

is fulfilled. A Fabry–Pérot cavity also couples to higher-order Gaussian beams (Hermite-Gaussian beam). The input beam should be shaped (for instance with toroidal mirrors) and carefully aligned (with an accuracy of 1 mm) so as not to excite these higher modes (mode matching).
2.2.2.2 Mirror Design of the Fabry–Pérot Resonance Cavity

In accordance with Eq. (2.26), a cavity should be optimized to have a high $F$ and a reasonable $\beta$. The coupling of $\beta$ should be near $\beta = 1$ (critical coupling), at which all of the power is dumped into the cavity. To achieve these conditions, the reflectivity and loss of the two mirrors should be considered as well as the stabilization condition in Eq. (2.28).

The end mirror should have a reflectance ($R_e$) as high as possible and for this reason a copper concave mirror was selected (expected $R_e = 0.9985$ with $R = 300$ mm). The frequency dependence of the reflectance was negligible from 201 to 206 GHz. The mirror has a small hole (diameter = 0.6 mm) to monitor the transmitted power ($P_t$). The designed $d$ was approximately 156 mm, which satisfies Eq. (2.28) with $R = 300$ mm.

The front mirror was a half mirror with high $R_f$ and low loss, $L_f = 1 - R_f - T_f$. Although in the optical range a high reflectance can be achieved by coating a surface with multiple thin layers of dielectric materials, they are too lossy and useless in the millimeter wave region. Instead, a thin mesh mirror made of gold was developed. Figure 2.13 shows a schematic of a thin gold film with many holes forming the mesh pattern. The line width is $a$ and the gap interval is $g$. The mirror has a high reflectivity with $g \ll \lambda = 1.47$ mm and the periodic array of holes prevent diffraction losses of the Gaussian beam. For the mirror, the thickness is $t = 1 \mu$m, which is thicker than the skin depth of the gold of around 200 GHz (0.18 $\mu$m) and is thin enough to reduce the Ohmic losses when a millimeter wave passes through it.

The mesh itself was too thin to be free-standing and therefore was put on a substrate of thickness, $t_{Sub}$, using photo-lithography and a liftoff technique. High-resistivity silicon was selected as base material because its thermal conductivity (150 W K$^{-1}$ m$^{-1}$) enables adequate cooling power with water cooling. One problem is that the high refractive index (3.54) of silicon severely interferes with the reflectivity of the gold mesh. The reflectance inevitably depends on the frequency. Optimization was needed to obtain a reasonable cavity gain with minimum frequency dependence.

The mesh parameters ($a$, $g$, and $t_{Sub}$) were carefully optimized using CST MW Studio [11]. A plane wave was calculated with a frequency domain solver using periodic boundary conditions. The left graph of Fig. 2.14 shows the result of the

![Cross-sectional view of the mesh-mirror structure on a substrate](image-url)
Fig. 2.14  Simulated properties of the mesh mirror, calculated using CST MW Studio. (left) Comparison between the reflectance (red line) and transmittance (blue line). (right) Comparison between the finesse (red line) and coupling (blue line)

simulation. The optimized parameters are $a = 200 \mu m$, $g = 140 \mu m$, and $t_{Sub} = 1960 \mu m (t = 1 \mu m)$. In accordance with Eqs. (2.15) and (2.19), the expected finesse and coupling were obtained ($R_e = 0.9985$) and are plotted in the right graph of Fig. 2.14. The finesse was always over 450 and the coupling was near the critical angle ($\beta \sim 1$). The expected gain is shown in Fig. 2.15 and is about 400 at around 203 GHz. Note that the frequency dependence of the gain was only 10% around 203.4 GHz regardless of the strong frequency dependence of the finesse. If the input power is 300 W, the accumulated power is expected to be over 100 kW. Figure 2.16 shows a photograph of the fabricated gold mesh on a silicon base. The blank space around the mesh is for water cooling. Validation of the simulation requires resonance tests using the Fabry–Pérot cavity with a controlled cavity length. This is described in the next section.

Fig. 2.15  Expected gain of the Fabry–Pérot cavity with a plane wave input
Fig. 2.16  Photograph of the gold mesh on a silicon substrate. The inset shows the mesh pattern and size

2.2.2.3 Performing Tests with the Fabry–Pérot Resonance Cavity

Controlling the cavity length requires both a long travelling range (\(\sim 1 \text{ mm}\)) and high resolution (\(< 1 \mu \text{m}\)). The Fabry–Pérot cavity resonates when \(d\) is a half-integer multiple of the wavelength \(\lambda = 1.47 \text{ mm}\), as shown on the left side of Fig. 2.12. The exact resonance point can be found by changing the cavity length by more than the FSR. The cavity length should also stay at the resonance point with a precision of better than the FWHM, as shown in the right graph of Fig. 2.12. The simulated finesse \(\mathcal{F} > 450\), corresponding to a FWHM of shorter than 1.5 \(\mu\text{m}\).

The end mirror was mounted on a special X-axis stage (TS102-G, NANO CONTROL Co., Ltd., Tokyo, Japan), which satisfies the above requirements. The actuator is a piezoelectric element exploiting the smooth-impact drive mechanism (SIDM) to provide the driving force. Figure 2.17 shows the principle of operation of the SIDM. In the coarse movement regime, the X-stage moves similar to an inchworm, using the friction of the piezoelectric element and the inertia of the X-stage to achieve a 15 mm travel distance. In the slight movement regime, the position can be controlled at the nanometer level by applying a voltage to the piezoelectric element.

Both the transmitted \(P_{tr}\) and reflected \(P_{re}\) powers should be monitored to control the cavity length. There are many power sensors in the millimeter wave region, such as a Golay cell, a Schottky barrier diode, a bolometer, and a thermopile detector. A detector should withstand 100 mW-class millimeter wave radiation and also have a fast time response to catch up with the gyrotron pulse operation (60 ms, 5 Hz). Most of the sensors saturate or respond slowly compared with our requirement.

The pyroelectric detector selected was a thermal detector made of lithium tantalate (\(\text{LiTaO}_3\)) and satisfied the above requirements. \(\text{LiTaO}_3\) is a pyroelectric crystal whose ends become oppositely charged when heated. The output current of the pyroelectric detector, which is almost proportional to the change in temperature, is converted to a voltage via an operational amplifier and a feedback resistor. The dynamic range can be controlled by changing the resistance (\(100 \text{M}\Omega\), 470 \(\text{M}\Omega\) and 1 \(\text{G}\Omega\) resistances
2.2 Millimeter Wave Optics System

Fig. 2.17 Principle of operation of a SIDM™ (smooth impact drive mechanism) actuator. Reprinted with permission from NANO CONTROL Co., Ltd. Copyright 2008 by NANO CONTROL Co., Ltd

Fig. 2.18 Two pyroelectric detectors (Spectrum Detector Inc. SPH-49)

are usually used). Figure 2.18 shows a photograph of the pyroelectric detector (SPH-49, Spectrum Detector Inc., Oregon, US). Although it is expected to be small, the frequency dependence of the pyroelectric detector is unknown and is discussed in Sect. 2.2.3.
Figure 2.19 shows the setup of the Fabry–Pérot resonance cavity including the piezoelectric stage and the pyroelectric detectors. The front mirror, the optimized gold mesh, was fixed at the beam waist position of the Gaussian beam. The end mirror, the copper concave mirror, was mounted on the piezoelectric stage placed 156 mm away from the front mirror. The Gaussian beam from the gyrotron was split in two by a beam splitter (this is not shown in Fig. 2.1). 99% of the beam went to the Fabry–Pérot cavity and 1% was sampled by one of the pyroelectric detectors as the incident beam (output voltage = $V_{in}$). The beam reflected from the cavity was again split and sampled by another pyroelectric detector (output voltage = $V_{re}$). These two detectors were arranged so that they did not reflect the beam back to the gyrotron, because the reflected beam would cause undesired interference. Part of the accumulated power passed through a small hole (diameter = 0.6 mm,
2.2 Millimeter Wave Optics System

Fig. 2.20 Measured resonance of the Fabry–Pérot cavity arrangement. The red dots represent voltage data of the transmitted power ($V_{tr}$) and the blue dots represent data of the reflected power ($V_{re}$). The solid lines are the best fits for each data set. The peak position was adjusted to the origin of the horizontal axis. Input frequency $= 203.7$ GHz

length $= 0.5$ mm) at the center of the end mirror. The other pyroelectric detector measured the transmitted power (output voltage $= V_{tr}$).

Figure 2.20 shows the result of the resonance test at 203.7 GHz. The measured FWHM was about 1.7 $\mu$m and according to Eq. (2.16), the corresponding finesse is about 430 (the expected value is 600). The coupling coefficient was 0.23 (the expected value is 1.2). This inconsistency cannot be explained with either an underestimation of the reflectance or additional losses. It turns out that the monochromatic assumption in the theory is too ideal for the gyrotron used to test the Fabry–Pérot cavity. The line-width and drift shown in Fig. 2.6 reduce the effective finesse and coupling. As a result, the simulation of the mesh mirror cannot be validated by performing measurements with the current gyrotron. Moreover, instead of using the gain formula, Eq. (2.26), a new method is needed to estimate the power inside the Fabry–Pérot resonance cavity. This new method is described in the next section.

2.2.3 Estimation of the Accumulated Power

The transmitted power samples part of the accumulated power according to the boundary conditions in Eq. (2.6),

$$P_{tr} = T_e \frac{1}{1 + R_e} P_{acc}. \quad (2.29)$$

Equation 2.29 is valid, even when the input electromagnetic waves are not perfectly monochromatic. The transmittance of the end mirror through the small hole ($T_e$) is too complicated to calculate. It is also non-trivial to estimate the response of
the pyroelectric detector and the interference of the transmitted beam in the base material of the detector and the end mirror. An absolute power-calibration factor \( (C) \) that includes all of the difficulties is defined as:

\[
P_{\text{acc}} = \frac{1 + R_e}{T_e} P_{\text{tr}} = CV_{\text{tr}}.
\]

(2.30)

\( C \) depends on the frequency and can be determined experimentally as follows:

- A known Gaussian beam is prepared with a shape correction
- \( V_{\text{tr}} \) and the power of the Gaussian beam \( (P_{\text{in}}) \) are measured simultaneously using a chopper
- The reflection effects from the end mirror are corrected.

Details on the measurements are described in the next section.

### 2.2.3.1 Shape Correction with a PVC Sheet and an IR Camera

The only choice for the high-power Gaussian beam was one from the gyrotron. The shape of the radiation was converted efficiently and was almost a perfect Gaussian beam. However, it was necessary to correct for the shape difference, mainly because of the limited accuracy of the millimeter wave beam alignment \( (\pm 3 \text{ mm}) \). The left plot in Fig. 2.21 shows a typical measured mode pattern and the right plot shows a corresponding theoretical calculation. The data were fitted with a bi-Gaussian and appeared to have a larger beam waist size than predicted by a factor of 1.4 and a center displacement of 1 mm. These were corrected as follows: The temperature of the fitted function at the origin \( (T_0) \) was obtained and the temperature \( (T) \) inside three standard deviations was integrated. Then, a hole-to-beam power ratio \( (R_{\text{beam}}) \) was calculated using:

![Fig. 2.21](image-url)  

Fig. 2.21 Mode pattern at the end mirror in the Fabry–Pérot resonance cavity as determined by: (Left) measuring the temperature distribution of a PVC plate with an IR camera (the white line is a fitted bi-Gaussian function), (Right) by a theoretical calculation
\[ R_{\text{beam}} = \frac{\pi r_{\text{hole}}^2 T_0}{\int_0^{2\pi} d\theta \int_0^{3\sigma} T dr}, \]  

(2.31)

where \( r_{\text{hole}} \) denotes the radius of the small hole and is 0.3 mm. Systematic errors were obtained by displacing the origin within the accuracy of the alignment, 3 mm, and calculating the minimum and maximum relative changes in \( R_{\text{beam}} \).

The hole-to-cavity-mode power ratio in the Fabry–Pérot cavity \( R_{\text{cavity}} \) was estimated by integrating the theoretical function

\[
R_{\text{cavity}} = \frac{\int_0^{r_{\text{hole}}} \exp\left(-2(r/w_z)^2\right) r dr}{\int_0^{\infty} \exp\left(-2(r/w_z)^2\right) r dr} \cdot \frac{1}{1 + R_e}
\]

(2.32)

\[
= \left\{1 - \exp\left[-2(r_{\text{hole}}/w_z)^2\right]\right\} \frac{1}{1 + R_e},
\]

(2.33)

where \( w_z \) denotes the beam size at the end mirror, which is expressed as:

\[
w_z = \sqrt{\frac{\lambda}{\pi} R \sqrt{\frac{d}{R - d}}},
\]

(2.34)

Table 2.3 summarizes some examples of the measured shape ratios, taken at the same position as the end mirror in the Fabry–Pérot cavity. It is expected that \( R_{\text{beam}} \approx R_{\text{cavity}} \). All values of \( R_{\text{beam}} \) look consistent and are in good agreement with \( R_{\text{cavity}} \).

### 2.2.3.2 Simultaneously Measuring \( V_{\text{Tr}} \) and \( P_{\text{in}} \)

The gyrotron has a pulsed operation (with a duty ratio of 30% and a repetition rate of 5 Hz). To take simultaneous measurements, the output power should be divided into two optical paths. A beam splitter, the same as that used to sample \( P_{\text{in}} \) and \( P_{\text{re}} \) (see Fig. 2.19), is useless because its frequency dependence is unknown. A chopper

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( R_{\text{cavity}} )</th>
<th>( R_{\text{beam}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>201.83</td>
<td>( 6.10 \times 10^{-4} )</td>
<td>( 6.91^{+0.03}_{-0.35} \times 10^{-4} )</td>
</tr>
<tr>
<td>202.64</td>
<td>( 6.12 \times 10^{-4} )</td>
<td>( 6.16^{+0.08}_{-0.39} \times 10^{-4} )</td>
</tr>
<tr>
<td>203.51</td>
<td>( 6.15 \times 10^{-4} )</td>
<td>( 6.05^{+0.06}_{-0.35} \times 10^{-4} )</td>
</tr>
<tr>
<td>204.55</td>
<td>( 6.18 \times 10^{-4} )</td>
<td>( 6.49^{+0.02}_{-0.28} \times 10^{-4} )</td>
</tr>
<tr>
<td>205.31</td>
<td>( 6.21 \times 10^{-4} )</td>
<td>( 6.29^{+0.13}_{-0.44} \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Measurements were performed at the end mirror position in the Fabry–Pérot cavity.
A schematic and photograph of the simultaneous measurements of $V_{tr}$ and $P_{in}$ (top view) was developed to synchronously divide the gyrotron pulse in two (Fig. 2.22). At one phase, the Gaussian beam hits a Teflon box and is absorbed by water, raising the water temperature ($\Delta T$). At the other phase, the Gaussian beam hits the end mirror and is detected by a pyroelectric detector ($V_{tr}$). No power leaks were observed at the chopper. The effective duty ratio was 15% and the repetition rate was 2.5 Hz. The pulse width was still maintained at 60 ms so that the response-time effects of the pyroelectric detector were the same as the original case.

The reflection at the Teflon box was 5% (measured and assigned as a systematic uncertainty). All of the other reflections were dumped inside a box filled with 46 ml of water. The temperature increase of the water, carefully stirred with a stick, was
Fig. 2.23 Data fitting for the power calibration. (left) $P_{\text{in}}$, measured by the increase in temperature of the water. The black points relate to measured data and the red line is the best fit. (right) Measurement of $V_{\text{tr}}$, performed by moving the piezoelectric stage near the mesh mirror position. The black points are data obtained when the chopper was closed and the red points are the measured data. The red line is the best fit measured with a resistance thermometer Pt100 during the experiments (60 s). The left plot in figure in Fig. 2.23 shows a typical result of the increasing temperature of the water. The data were fitted with a function that included the thermal diffusion from water to the Teflon box, which is expressed as:

\begin{align}
  f(t < t_0) &= T_0 \\
  f(t > t_0) &= T_0 + (T_{\text{inf}} - T_0) \left[ 1 - e^{-\alpha(t-t_0)} \right],
\end{align}

where $t_0$ is the start time, $T_0$ the initial temperature, $T_{\text{inf}}$ the asymptotic temperature after a long time, and $\alpha$ a constant that determines the velocity of the thermal diffusion. The input power is expressed as:

\begin{equation}
  P_{\text{in}} = \frac{(T_{\text{inf}} - T_0) \times \alpha \times 4.2 \times 46}{0.95 \times \text{duty}/2},
\end{equation}

where 0.95 is the transmittance of the Teflon box and the duty ratio is divided by 2, corresponding to chopping. In Fig. 2.23, $P_{\text{in}} = 280$ W.

The systematic uncertainties in this measurement were estimated as follows:

- The reflectivity of the Teflon box is 5/95 %.
- The accuracy of the amount of water is 2/46 ml.
- The accuracy of the time measurement is 2/60 s.
- The accuracy of temperature measurement is 0.2/10 °C.

These values can be combined by taking the square root of the sum of their squares. The transmitted power ($V_{\text{tr}}$) was measured for the other phase of the chopper. The reflected beam at the end mirror was again reflected either at the RF cavity in the gyrotron or at the output window to form a standing-wave. The piezoelectric...
Experimental stage was advanced within a few wavelengths to measure the standing wave. The standing-wave formula Eq. (2.12) was used to fit $V_{tr}$ and can be re-written as:

$$f(x) = \frac{V_{tr}}{[1 - \sqrt{\rho}]^2 + 4\sqrt{\rho} \sin^2 [k(x - \theta)],} \quad (2.38)$$

where $\rho$ is the round-trip reflectance and $k$ is the wave number, which is fixed. The right plot in Fig. 2.23 shows the result of this fitting.

### 2.2.3.3 Correction of the Reflection Effect

There is one problem caused by the reflection at the end mirror, other than producing a standing wave. The beam reflected from the end mirror reduces the oscillation efficiency, but only when measuring $V_{tr}$. It was assumed in describing in the gyrotron oscillation in Sect. 2.2.1 that there were no reflections. The reflected radiation changes the boundary conditions (see Eq. (A.7) for details) and disturbs the electric field in the RF cavity. This phenomenon is affected by both the electron-beam alignment and the reflection conditions, and the same phenomenon has been reported in a MW-class gyrotron [5].

This effect was corrected by sampling the incident power ($V_{in}$) with the beam splitter shown in Fig. 2.19 before the chopper. Although the splitting efficiency itself is unknown, it can be used to determine $V_{tr}$, a relative correction at the same frequency. The $V_{in}$ during the $V_{tr}$ measurements (defined as $V_{in}'$) was fitted by Eq. (2.38), while $V_{in}$ during the water measurement phase was fitted with a constant. The $P_{in}$ value was corrected with $V_{in}' / V_{in}$ to simulate the real value for $P_{in}'$ by measuring $V_{tr}$ with the reflection. Figure 2.24 show two typical results for the $V_{in}$ measurements with different conditions. Systematic uncertainties were estimated by the fitting errors of $V_{tr}$, $V_{in}'$ and $V_{in}$.

![Fig. 2.24](image.png)

**Fig. 2.24** Data fitting for the reflection correction. The black points are the data for the phase of the $P_{in}$ measurement (no reflection) and the red points are the data for that of the $V_{tr}$ measurement. The solid lines are the best fits for each data set. The end point mirror was moved a few wavelengths near the mesh mirror position of the Fabry–Pérot cavity. (left) Small correction case. (right) Large correction case.
In summarizing the above discussion, the calibration constant \( C \) is expressed as:

\[
C = \frac{P_{\text{acc}}}{V_{\text{tr}}} = \frac{R_{\text{beam}}}{R_{\text{cavity}}} \frac{P_{\text{in}}'}{V_{\text{tr}}} = \frac{R_{\text{beam}}}{R_{\text{cavity}}} \frac{V_{\text{in}}'}{V_{\text{in}}} \frac{P_{\text{in}}}{V_{\text{tr}}}. \tag{2.39}
\]

We measured the calibration constant \( C \) at three different distances (600, 750, and 900 mm) from the toroidal mirror (M2) to check the correlation of the reflection with \( V_{\text{in}} \). Examples of the \( C \) measurements are listed in Table 2.4. At 201.83 GHz, the reflection correction, \( V_{\text{in}}'/V_{\text{in}} \) was almost independent of the distance and \( C \) was within one standard deviation. The reflection corrections at 202.64 GHz differed by a factor of five for four independent measurements. However, \( C \) was still within two standard deviations. This means that the reflection correction is necessary to obtain proper \( C \) values and works well in all cases.

For all frequencies, the results were averaged for the three different distance settings and are summarized in Table 2.5 and Fig. 2.25. \( C \) started to increase roughly with decreasing frequency because the transmittance of the evanescent mode was reduced (the cutoff frequency was about 290 GHz). The discrete structure near 203 GHz may

### Table 2.4 Examples of the \( C \) measurements and the \( V_{\text{in}} \) correction

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Distance (mm)</th>
<th>( \frac{R_{\text{beam}}}{R_{\text{cavity}}} )</th>
<th>( P_{\text{in}} ) (W)</th>
<th>( \frac{V_{\text{in}}'}{V_{\text{in}}} )</th>
<th>( V_{\text{tr}} ) (mV)</th>
<th>( C ) (kW/V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>201.83</td>
<td>600</td>
<td>1.880±0.195−0.367</td>
<td>280(22)</td>
<td>0.793</td>
<td>17.53(28)</td>
<td>23.7±3.1−5.0</td>
</tr>
<tr>
<td>201.83</td>
<td>900</td>
<td>0.516±0.002−0.015</td>
<td>239(19)</td>
<td>0.864</td>
<td>5.41(09)</td>
<td>19.7±1.6−1.7</td>
</tr>
<tr>
<td>202.64</td>
<td>600</td>
<td>1.842±0.177−0.254</td>
<td>271(21)</td>
<td>0.634</td>
<td>18.04(15)</td>
<td>17.5±1.6−2.7</td>
</tr>
<tr>
<td>202.64</td>
<td>600</td>
<td>1.638±0.0070−0.196</td>
<td>582(46)</td>
<td>0.280</td>
<td>10.92(07)</td>
<td>24.4±2.7−3.7</td>
</tr>
<tr>
<td>202.64</td>
<td>900</td>
<td>0.505±0.002−0.015</td>
<td>240(18)</td>
<td>1.010</td>
<td>5.42(05)</td>
<td>22.5±1.8−1.8</td>
</tr>
<tr>
<td>202.64</td>
<td>900</td>
<td>0.583±0.039−0.039</td>
<td>484(38)</td>
<td>0.388</td>
<td>4.17(03)</td>
<td>26.3±2.7−2.7</td>
</tr>
</tbody>
</table>

### Table 2.5 Measured absolute calibration constant, \( C \)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>( C ) (kW/V)</th>
<th>Relative accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>180.56</td>
<td>32.4±7.8−9.4</td>
<td>27</td>
</tr>
<tr>
<td>201.83</td>
<td>23.2±3.4</td>
<td>16</td>
</tr>
<tr>
<td>202.64</td>
<td>23.4±4.5</td>
<td>20</td>
</tr>
<tr>
<td>203.00</td>
<td>14.6±2.2−2.3</td>
<td>15</td>
</tr>
<tr>
<td>203.25</td>
<td>13.0±1.3−1.5</td>
<td>12</td>
</tr>
<tr>
<td>203.51</td>
<td>10.2±1.3−1.5</td>
<td>14</td>
</tr>
<tr>
<td>204.56</td>
<td>11.3±2.3</td>
<td>21</td>
</tr>
<tr>
<td>205.31</td>
<td>11.4±1.4−1.6</td>
<td>14</td>
</tr>
</tbody>
</table>
Fig. 2.25 Results of the $C$ measurements

be from interference behind the end mirror and is expected to give a proper result if the mirror structure is fixed. The value of $C$ was measured using an independent method given in Sect. 4.1 and consistent results were obtained.

A systematic error was obtained from the square root of the sum of the squares of $R_{\text{beam}}$, $P_{\text{in}}$, $V_{\text{fr}}$, $V_{\text{in}}$, and $V_{\text{in}}'$. In addition, the standard deviation for each $C$ value for different conditions was combined in a similar way to introduce the remaining fluctuations in each measurement. The systematic errors in the $C$ measurements directly affected the Ps-HFS value. The objective of this experiment, the first direct measurement of the Ps-HFS, could be achieved with this uncertainty. In the future, the precision should be improved by 100 times to address the observed discrepancy in the Ps-HFS. A possible improvement is discussed in Sect. 4.3.

2.2.4 Power Stabilization

The accumulated power in the Fabry–Pérot resonance cavity should be stabilized to properly determine the cross-section of the Ps-HFS transition. The resonance power is a result of the output power of the gyrotron, the stability of the output frequency, and the length of the Fabry–Pérot cavity. The first two are automatically determined by non-linear plasma-wave interactions in the gyrotron. The controllable parameters are the magnetic field strength, electron-beam current, electron-beam trajectories, and the length of the Fabry–Pérot cavity.

2.2.4.1 Stabilization of the Electron Beam Current

The electron beam current ($I_{b}$) in the gyrotron was controlled with the heater temperature at the beam-emitter ring (thermionic electron emission). The heater temperature reached around 1000°C when a voltage $V_{\text{H}} \sim 10$ V was applied to the heater. $I_{b}$ was stabilized with a proportional integral (PI) control method. The voltage of the heater was controlled with an AC high-power supply (SRJ500, MATSUSADA Precision,
Inc., Shiga, Japan) connected to a PC via a RS232C cable. The output voltage of the power supply ($V_{AC}$) was determined using the following equation:

$$V_{AC}(n) = V_{AC}(n-1) + K_P (I_b(n-1) - I_b(n)) + K_I (I_b^{\text{target}} - I_b(n)),$$  \hspace{1cm} (2.40)

where $V_{AC}(n)$ is the supply voltage set at the $n$th operation, $I_b(n)$ the beam current during the $n$th measurement, $K_P$ the proportional gain, $K_I$ the integral gain, and $I_b^{\text{target}}$ the target voltage. $K_P$ was set to 40 and $K_I$ was set to 0.08 (a typical voltage is $V_{AC} \sim 135 \text{ V}$, corresponding to $V_H \sim 10 \text{ V}$ and $I_b \sim 400 \text{ mA}$). Figure 2.26 is a plot of $I_b$ and $V_H$ for this stabilization system. The gyrotron beam current was stable within $\pm 10\%$ for both a short time (1 min) and a long time (1 day). The gyrotron oscillation gradually became stable and outputted a steady power with a constant beam current. Note that the output power itself is not a proper parameter that can be stabilized because of strong non-linearities in the high-power gyrotron operation. As was mentioned in Sect. 2.2.1, the output power can be a multivalued function of $I_b$ for a nearly hard excitation \cite{2}.

### 2.2.4.2 Stabilization of the Resonance of the Fabry–Pérot Resonance Cavity

The cavity length varies without moving the piezoelectric X-stage because of fluctuations in the temperature of the gas chamber. The gyrotron frequency drifted and moved off-resonance. The cavity length needs to be controlled to accomplish on-resonance for long periods of time. Stabilizing the accumulated power integrated with one pulse (60 ms) was performed as follows. Two thresholds, $V_{\text{high}}$ and $V_{\text{low}}$, were defined and the cavity length was controlled such that $V_{\text{tr}}$ was between $V_{\text{high}}$ and $V_{\text{low}}$. The cavity length was rescanned to search for the resonance position, only when the $V_{\text{tr}}$ fell below $V_{\text{low}}/2$. Figure 2.27 shows a plot of $V_{\text{tr}}$ and the mirror position.
Even if the cavity length is well controlled, the accumulated power in one pulse is not necessarily stabilized because the line-width (a few MHz) and the drift in frequency (about 20 MHz) in one pulse are broader than the bandwidth of the Fabry–Pérot cavity. Control of the cavity length is unable to keep up with such fast fluctuations. This resulted in power fluctuations in the pulses that depended on the conditions of the gyrotron operation. This effect was inevitable with the current gyrotron and was corrected in the data analysis described in Sect. 3.4.

2.3 Positronium Assembly and $\gamma$-Ray Detectors

Photographs of the Ps assembly and $\gamma$-ray detectors are shown in Fig. 2.28. Figures 2.29 and 2.30 show the top and side views of a the gas chamber, respectively. Ps is formed with a positron from a $^{22}$Na source and an electron from a gas molecule. If formed in the beam region in the Fabry–Pérot cavity, Ps undergoes the transition from $o$-Ps to $p$-Ps occurred. The $\gamma$-rays from the decay of Ps were detected by $\gamma$-ray detectors.

2.3.1 Positronium Formation Assembly

A $^{22}$Na positron source was mounted on a UV-transparent acrylic light-guide (Fig. 2.28). The source emitted a positron (end point energy $= 546$ keV, probability $= 90\%$), then subsequently emitted one 1275 keV $\gamma$-ray. A thin plastic scintillator (NE102, thickness $= 0.1$ mm) was placed next to the source to tag the emission time of the positron. Optical photons went divided right and left by light-guides to reach one of two photo-multipliers (PMT), labeled Pla-0 and Pla-1 in Fig. 2.30. Coincidence of the two PMTs reduces accidental events from dark currents or electric noise. A
2.3 Positronium Assembly and γ-Ray Detectors

Fig. 2.28 Photograph of the chamber

Fig. 2.29 Schematic of the gas chamber (top view)

fine-mesh type PMT (R5924-70, 2 inch, HAMAMATSU Photonics K. K., Shizuoka, Japan) was used to reduce the effects of a residual magnetic field (1 mT) from the superconducting magnet in the gyrotron. A lead plate (thickness = 20 mm) shielded the 1275 keV and the 511 keV γ-rays from the positrons that were annihilated near the source.
The gas chamber was filled with 1 atm gas. The gold mesh mirror on the silicon substrate was used as the window of the gas chamber. The silicon substrate was not transparent to optical photons, so that it shielded ambient light outside the chamber to the plastic scintillator and the light-guide. The gas temperature was cooled with a cooling pipe (20°C water flow) and a fan was placed in the chamber (Fig. 2.29). The positron was decelerated by ionizing the gas molecules and finally stopped in the chamber. The number of stopped positrons at the beam region was about 2% of all emitted positrons and 7% of the positrons that were tagged by the plastic scintillator. About half of the stopped positrons formed Ps, 25% of which were $p$-Ps and immediately decayed (lifetime = 125 ps) into two $\gamma$-rays (511 keV), and 75% were $o$-Ps with long lifetimes. Because 7.9% of the $o$-Ps were annihilated by colliding with an electron in a gas molecule (pick-off annihilation), the lifetime of $o$-Ps was reduced from 142 to 131 ns. Neopentane gas was selected to eliminate a background, which had been overlooked in previous measurements. This is discussed in more detail in the next section.

### 2.3.2 Selection of the Gas Used

Selecting an appropriate gas was one of the most important points in this experiment. A mixed gas of nitrogen and isobutane was used in previous measurements [12]. It appeared that a non-negligible systematic error remained when using this gas mixture. We broke down the gas effects into three separate issues:
2.3 Positronium Assembly and $\gamma$-Ray Detectors

- Reduction of the slow positrons
- Power absorption by the gas
- Positron acceleration by millimeter waves and collisions with gas molecules

The related errors are described in the following sections.

2.3.2.1 Slow Positron Annihilation

According to the Ps formation theory by Ore, a positron can produce Ps (ionization potential $= 6.8$ eV) if its kinetic energy ($E$) satisfies

$$I_{1st} - 6.8 \text{ eV} < E < E_{ex},$$

where $I_{1st}$ and $E_{ex}$ are the first ionization potential and the excitation energy of the gas molecule, respectively. This condition is called the Ore gap. If the kinetic energy of the positron is too small, the positron cannot capture an electron to form the Ps atom. Less than half of the positrons that are stopped in the gas can form Ps. The others become slow positrons and remain in the gas. The annihilation rate of the slow positrons is expressed as:

$$\Gamma_{\text{slow}} = \pi r_0 cn Z_{\text{eff}},$$

where $r_0$ is the classical radius of an electron, $n$ is the number density of the gas, and $Z_{\text{eff}}$ is the effective electron number, which contributes to the annihilation. $Z_{\text{eff}}$ depends on the gas molecules; see Table 2.6 [13, 14].

Figure 2.31 shows the time spectra of a slow positron and $o$-Ps in 1 atm nitrogen separated with energy information. When some of the $o$-Ps transitions into $p$-Ps, its decay looks the same as a slow positron (back-to-back $\gamma$-ray pairs of 511 keV). The

![Fig. 2.31](image-url)  

Fig. 2.31 Time spectra of a slow positron and $o$-Ps in 1 atm nitrogen. The red line shows a slow positron (lifetime $= 167$ ns), measured by selecting a back-to-back 511 keV $\gamma$-ray. The black line shows a $o$-Ps (lifetime $= 133$ ns in nitrogen), measured by selecting a Compton-free (from 360 to 450 keV) $\gamma$-ray.
signal to noise ratio (S/N) was too low to observe this transition in pure nitrogen. The lifetime of a slow positron should be smaller than that of o-Ps.

In accordance with Table 2.6, alkane molecules are favored because of their short lifetimes for slow positrons. Isobutane gas has been used to quench the slow positrons in some experiments [12, 15]. However, we could not use pure isobutane in our experiments because of its power absorption. This is described in the next section.

### 2.3.2.2 Absorption of 203 GHz Radiation

Polyatomic gases have rotational levels and vibrational levels resulting from their internal degrees of freedom. If a molecule also has an electric dipole moment ($d$), it absorbs electromagnetic radiation via transitions between these levels. For 203 GHz (wavelength = 1.47 mm) radiation, the rotational transitions should be considered. The vibrational transitions higher than 1 THz can be ignored.

The energy difference ($\Delta E$) between the rotational levels is expressed as [16]

$$\Delta E = 2 \cdot \frac{\hbar^2}{2I} (J + 1) = 2B(J + 1),$$

where $I$ is the moment of inertia around one axis and $J = 1, 2, 3, \ldots$ the rotational quantum number. $B = \hbar^2 / 2I$ is called the rotational constant and has been measured and/or calculated for many molecules. Some of the $B$ and $d$ values for various gas molecules that are often used in Ps experiments are listed in Table 2.6 [17, 18]. Isobutane has three absorption lines near 203 GHz, at 186.9, 202.5, and 218.1 GHz.

If the absorption line is sharp enough, the gyrotron frequency can be tuned to keep it away from the absorption peaks. The line-width ($\Delta f$) of the power absorption is broadened by collisions with gas molecules and is expressed as [16]

$$\Delta f = 2 \times \frac{1}{2\pi \tau},$$

$$\tau = \frac{1}{\sigma v_{rel} \hbar}$$
where $\tau$ is the lifetime of one rotational level determined by the collision, $\sigma$ the cross-section of the collision, $v_{\text{rel}}$ the relative velocity between the gas molecules, and $n$ the number density of gas molecules. Assuming the scattering is elastic and the isobutane radius is approximately $3.5\,\text{Å}$, then $\sigma \approx 1.5 \times 10^{-18}\,\text{m}^2$. At 1 atm and 300 K, $v_{\text{rel}} = 465\,\text{m/s}$ and $n = 2.45 \times 10^{25}\,\text{m}^{-3}$. The estimated $\Delta f$ is about $5.4\,\text{GHz}$, which covers the frequency range from 197 to 209 GHz (Fig. 2.32). Power absorption by pure isobutane causes the finesse of the Fabry–Pérot cavity to decrease. Although pure isobutane has been studied in detail in other experiments [15], it is not ideal to use it in this experiment.

### 2.3.2.3 Positron Acceleration by Millimeter Waves

Thus far, we have decided not to use pure nitrogen and pure isobutane because of the slow positron and power absorption, respectively. However, a mixed gas containing nitrogen and isobutane seems to work well. The lifetime of a slow positron is $3.4\,\text{ns}$ (in 1.9 atm nitrogen and 0.1 atm isobutane), and the power absorption is strongly suppressed by pressure broadening in the mixed nitrogen gas. This gas mixture was previously used in the first observation of the Ps-HFS transition at 202.9 GHz [12]. A new systematic problem became apparent when the Ps-HFS resonance curve was measured. The high-power millimeter wave field used ($E \sim 200\,\text{kV/m}$ at 20 kW accumulation) increased the amount of Ps formed.

It was reported in the first indirect measurement of Ps-HFS that the Ps formation probability in nitrogen gas increases in the presence of high-power microwaves [19]. It was interpreted that the slow positrons were accelerated to an energy above the lower edge of the Ore gap defined by Eq. (2.41). This effect was studied in detail using a static electric field [20, 21], but has not been performed with high frequency fields so far. The same phenomenon with a millimeter wave was first observed in this experiment. A positron is either accelerated or decelerated under the Lorentz force of the millimeter wave. Because the frequency of this interaction is about $200\,\text{GHz}$, the mean energy gain of a positron in a vacuum is very small (about 1 meV). However,
in about 1 atm gas, the energy gain can exceed the Ore gap, caused by collisions (a few ps cycle) between the positrons and gas molecules. These collisions randomize the phase of a positron during its acceleration and some of the positrons statistically obtain a kinetic energy of a few eV within several ns. A simple theory with a random-walk model is derived in Appendix B. Figure 2.33 shows the simulated positronium formation probability ($G$) in 1 atm nitrogen gas. It can be seen that an increase in the Ps formation probability can occur in high-power millimeter wave radiation.

Using the high-power millimeter waves (24 kW), an increase in the Ps formation was observed (Fig. 2.34), in which the number of o-Ps events increased while the number of slow positron events decreased. Slow positron events were selected with an energy cut-off of $511 \text{ keV} \pm 3\sigma$, and the o-Ps events were selected with an energy range from 360 to 450 keV (Compton free from the 511 keV signal). The increase in the Ps formation was probed by subtracting the beam-OFF events from the beam-ON events (see Sect. 3.3.4). For simple gases such as argon and nitrogen, an increase in Ps formation was observed. The same phenomenon occurred in mixed gases containing nitrogen and isobutane. Therefore, we could not use a gas containing nitrogen in this experiment. In contrast, the Ps formation did not increase in pure alkane gases, as suggested in [20, 21]. An accelerated positron excites many internal modes (rotation
and vibration) of complex gas molecules by inelastic scattering, markedly reducing the energy gain. As isobutane absorbs 203 GHz radiation, the only remaining possibility is pure neopentane.

### 2.3.3 \(\gamma\)-Ray Detectors

Four LaBr\(_3\) (5% Ce) crystal scintillators (BriLanCe\(^\text{TM}\) 380, Saint-Gobain Crystals S. A., Courbevoie, France) were placed around the beam region, denoted as La-0, La-1, La-2, and La-3 (Fig. 2.29). Figure 2.35 shows a photograph of the scintillators. The size of the LaBr\(_3\) (Ce) crystals were \(\phi \times 1.5\) inches \(\times\) 2.0 inches. Each crystal was covered with an aluminum housing (0.5 mm thick) and then connected to a PMT (HAMAMATSU R5924-70). The crystals and PMTs were assembled outside the gas chamber to protect them from becoming contaminated by the neopentane.

The \(\gamma\)-rays emitted from annihilated Ps were detected using a coincidence pair of scintillators. The four back-to-back pairs are (La-0, La-2), (La-0, La-3), (La-1, La-2) and (La-1, La-3). The number of events for each pair was almost the same because the Ps formation region in the gas was wide (about 100 mm) and the shape of the 203 GHz beam accumulated in the Fabry–Pérot cavity was long (156 mm). The energy resolution of the LaBr\(_3\) (Ce) crystal was higher than that of other inorganic scintillators (about 4% at the FWHM) and together with the back-to-back coincidence, two 511 keV \(\gamma\)-ray annihilation events could be enhanced by tight energy selection. Its fast time response (\(\tau = 16\) ns) is also appropriate for experiments with high statistics compared with semiconductor detectors such as a germanium crystal. The high Z of La has a higher stopping power than a NaI scintillator. The characteristics of the scintillator are summarized in Table 2.7.

![Photograph of some LaBr\(_3\)(Ce) crystals](image)
Table 2.7 Properties of the LaBr$_3$(Ce) scintillator

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light yield (photons/keV$_\gamma$)</td>
<td>63</td>
</tr>
<tr>
<td>Primary decay time (ns)</td>
<td>16</td>
</tr>
<tr>
<td>Density (g/cm$^3$)</td>
<td>5.08</td>
</tr>
<tr>
<td>Wavelength of emission max (nm)</td>
<td>380</td>
</tr>
<tr>
<td>Refractive index @ emission max</td>
<td>1.9</td>
</tr>
<tr>
<td>Thickness for 50% attenuation (662 keV) (cm)</td>
<td>1.8</td>
</tr>
</tbody>
</table>

2.4 Electronic and Data Acquisition

2.4.1 Overview

The data acquisition (DAQ) and control system consisted of two parts, a detector and an optical part. The detector part comprised a nuclear instrument module (NIM) and a computer-automated measurement and control (CAMAC) system to acquire the electric signals from the PMTs in the plastic scintillator and the LaBr$_3$(Ce) detectors. The optical part controls the optical system and monitors the experimental conditions (such as the power, temperature, pressure, gyrotron parameters, and the position of the piezoelectric stage). It consists of a data logger (8420-50, HIOKI E. E., Corp., Nagano, Japan), a piezoelectric X-stage (NANO CONTROL TS102-G), pyroelectric detectors, an analog-to-digital converter (ADC, NI PCI-6225, 16 bits, 250 kHz, National Instruments Corp., Texas, US), and an interlock (OMRON PLC Cj1). All data are sent to a PC running the Linux operating system and stored on a hard disk drive. Figure 2.36 is a schematic of the whole DAQ system.

The resonance peak of the Fabry–Pérot cavity was searched for by moving the piezoelectric stage actuated by a driver connected to a PC via a RS232C cable. The cavity length was controlled so that the cavity remained at resonance. Once the cavity was at resonance, then the electron-beam current ($I_b$) was stabilized by controlling the heater voltage of the MIG (Sect. 2.2.4). The AC voltage of the heater was supplied by the high-power AC supply and its output voltage was controlled remotely by a PC via a RS232C cable. The power accumulated in the cavity gradually became stable as a steady gyrotron oscillation was achieved with the constant beam current.

After stabilization, the PC sent a start command to the CAMAC controller (CC/NET, Toyo Corp., Tokyo, Japan) and the detector began to acquire data. All of the CAMAC modules were cleared and the latch was released by a reset signal from the output register. This causes all of the systems to become active and the controller waited for the interrupt signal, the so-called LAM (look-at-me) signal, provided by a charge-sensitive ADC (C1205, CAEN S. p. A., Viareggio, Italy). The interrupt signal came only when the trigger condition was satisfied. The data acquisition was triggered when back-to-back $\gamma$-ray signals from the LaBr$_3$(Ce) scintillators were coincident within 40 ns and then when this coincidence was within $-100$ to
1100 ns of the timing of the plastic scintillators. The charge-sensitive ADC (CAEN C1205) measured the energy information from the LaBr₃(Ce) detector. Two charge-sensitive ADCs (7167, PHILLIPS, Amsterdam, Netherlands and RPC-022, REPIC, Tokyo, Japan) were used to measure the energy information of the plastic scintillator with different gate lengths. Time information for the plastic and LaBr₃(Ce) scintillators was recorded using a direct clock (2 GHz) with a count type time-to-digital converter (TDC, KEK GNC-060). After the reading procedures, the controller saved the data through a network file system (NFS) and started the next event cycle.

The environmental conditions (temperature and pressure) were recorded with the data logger (HIOKI 8420-50). The incident ($V_{in}$), reflected ($V_{re}$) and transmitted ($V_{tr}$) powers were monitored with the pyroelectric detectors (Fig. 2.19). The signal waveforms of the pyroelectric detectors (duty ratio $= 30\%$, repetition rate $= 5$ Hz, pulse width $= 60$ ms) were read with the ADC (PCI-6225, National Instruments) at a sampling rate set to 0.5 kHz. The trigger pulse for the gyrotron output and the level of the electron-beam current ($I_b$) were also recorded with the same ADC. These data per gyrotron trigger were recorded on a PC every 100 cycles.
Some parameters that are important for safe operations were recorded with an interlock (PLC Cj1, OMRON Corp., Kyoto, Japan). The interlock could shut down the cathode power supply, voltage amplifier of the 1st anode, gun-coil power supply, and heater power supply when it was not operating properly or there was a sudden blackout. The temperatures of the different parts of the gyrotron were monitored to check for any abnormal electron-beam trajectories and to prevent the components from thermal destruction. The gyrotron vacuum was monitored so that the emitter surface of the MIG was not exposed to accidental out-gassing. The water-cooling flow and temperature were also checked. These data were sent to a PC every 5 s, independent of the other DAQ systems.

Details on the detector part are described in the following sections.

### 2.4.2 Electronics of the Plastic Scintillator

A circuit diagram of the plastic scintillator is shown in Fig. 2.37. The two PMTs for the plastic scintillator operated at +2215 V (Pla-0) and +2250 V (Pla-1). High voltages were supplied by a positive high voltage module (NHQ204M, Iseg Spezialelektronic, Rossendorf, Germany). Their gains were $3.4 \times 10^7$ (Pla-0) and $3.8 \times 10^7$ (Pla-1).

The output of the PMT was divided into three lines by a linear fan-out module (RIS-0255, RIS Corp., Tennessee, US). One of the divided signals was fed into a discriminator (KN246, Kaizu Works, Tokyo, Japan). The threshold value of the discriminator was set to 25 mV, corresponding to about one photoelectron. The others were used to measure the signal amplitude with a charge-sensitive ADC (PHILLIPS 7167) and a short gate (60 ns), and a charge-sensitive ADC (REPIC RPC-022) and a long gate (1000 ns). All of the analog transmission lines were covered with electric shields, and their lengths were prepared as short as possible to reduce electric noise caused by the gyrotron’s pulse operation.

One of the outputs of the discriminator was delayed by 200 ns and provided a stop signal for the TDC (KEK GNC-060). The other output was used to make a coincidence signal with the two PMTs (Pla-and). Dark currents and other accidental noise were suppressed using the coincidence signal. The coincidence signal was used to make a common start signal with the TDC (KEK GNC-060), a short gate signal with the charge-sensitive ADC (PHILLIPS 7167), a long gate signal with the charge-sensitive ADC (REPIC RPC-022), a fast clear, and the main trigger. These are described in Sect. 2.4.4.

### 2.4.3 Electronics of the γ-Ray Detectors

A circuit diagram of the γ-ray detectors is shown in Fig. 2.38. High voltages were supplied to the four PMTs (HAMAMATSU R5924-70) by a positive high-voltage module (MATSUSADA HEER-3R10) and were distributed by a voltage divider
Fig. 2.37 Circuit diagram of the plastic scintillator system
Fig. 2.38  Schematic of the electronics for γ-ray detector system
Fig. 2.39  Schematic of the electronics for the trigger system
(201A-SHV, Technoland Corp., Tokyo, Japan). The high-voltage values were +1450 V for La-0, +1330 V for La-1, +1230 V for La-2, and +1290 V for La-3.

The output of the PMT was divided into two lines by a linear fan-out module (RIS-255). One of the divided signals was fed into a discriminator (KN246). The threshold value was set to 90 mV, corresponding to about 50 keV. The other signal was used to measure the signal amplitude with a charge-sensitive ADC (CAEN C1250) with a 150 ns gate width.

One of the outputs of the discriminator was delayed by 200 ns, providing a stop signal for the TDC (KEK GNC-060). The other output was fed by a logic fan-in/fan-out module and then sent to a coincidence logic unit. The output of the logic unit became the back-to-back coincidence signal of the four $\gamma$-ray detectors (La-b2b). The La-b2b signal was used to provide a gate signal for the charge-sensitive ADC (CAEN C1205) and the main trigger.

### 2.4.4 Electronics of the Trigger System

A circuit diagram of the trigger system is shown in Fig. 2.39. The coincidence signal from the two PMTs for the plastic scintillator (Pla-and) was widened to 1200 ns (Pla-gate). The La-b2b signal was delayed by 100 ns and was coincident with the Pla-gate, triggering the data acquisition (main trigger). The main trigger also produced a gate signal for the charge-sensitive ADC (CAEN C1205), used to record energy information from the LaBr$_3$(Ce) detectors. The trigger signal provided a latch start at the same time. The latch signal vetoed the subsequent gate signals and fast clear signals. After all data had been read and saved, latch reset signals were produced by an output register of CAMAC.

The short and long gate signals for the ADCs for the plastic scintillator were produced by a Pla-gate independent of the main trigger. The data were cleared by the clear signal (fast clear) unless the main trigger signal was produced. The Pla-gate also produced the common start signal for the TDC. The TDC data were cleared by the fast clear in the same way.

### References

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