## Contents

1 Introduction of Rotordynamics .................................................. 1  
1.1 Vibration Problems in Rotating Machinery .......................... 1  
  1.1.1 Varieties of Rotating Machinery .............................. 1  
  1.1.2 Bearings .................................................. 4  
  1.1.3 Defects in Various Elements and Induced Vibration .............. 6  
  1.1.4 Rotordynamics ........................................... 6  
1.2 Types of Vibration in Rotating Machinery .......................... 8  
1.3 Classification of Vibration by Mechanism of Occurrence ........... 9  
1.4 Simplifying Complicated Phenomena ................................. 11  

2 Basics for a Single-Degree-of-Freedom Rotor ......................... 13  
2.1 Free Vibrations .................................................. 13  
  2.1.1 Natural Frequency ....................................... 13  
  2.1.2 Calculation of Spring Constant .............................. 14  
  2.1.3 Conservation of Energy .................................. 15  
  2.1.4 Mass Effects of Spring on Natural Frequency ............... 15  
2.2 Damped Free Vibration ............................................. 18  
  2.2.1 Mass-Spring-Viscous Damped System .......................... 18  
  2.2.2 Measurement of Damping Ratio ............................... 20  
  2.2.3 Phase Lead/Lag Corresponding to Damping Ratio ............... 24  
2.3 Unbalance Vibration of a Rotating Shaft .......................... 25  
  2.3.1 Complex Displacement and Equation of Motion ............... 25  
  2.3.2 Complex Amplitude of Unbalance Vibration ................... 26  
  2.3.3 Resonance Curves ........................................ 27  
  2.3.4 Nyquist Plot ............................................. 28  
  2.3.5 Bearing Reaction Force at Resonance ......................... 30  
  2.3.6 Transmissibility of Unbalance Vibration to Foundation ...... 32
## Contents

### 2.4 Evaluation of \( Q \)-Value

- 2.4.1 \( Q \)-Value Criterion ..................................... 34
- 2.4.2 Measurement of \( Q \)-Value by the Half Power Point Method ..................................... 35
- 2.4.3 Measurement of \( Q \)-Value Using a Nyquist Plot .............. 34
- 2.4.4 Re-evaluation of \( Q \)-Value for Rapid Acceleration ............ 36
- 2.4.5 Vibration in Passing Through a Critical Speed .............. 39

### 3 Modal Analysis of Multi-Degree-of-Freedom Systems .............. 41

- 3.1 Equation of Motion for a Multi-dof System .......................... 41
  - 3.1.1 Multiple Mass Systems .................................. 41
  - 3.1.2 Equation of Motion for a Two-dof System .................... 42
  - 3.1.3 Equation of Motion for a Multi-dof System .................. 43
- 3.2 Modal Analysis (Normal Mode Method) ............................ 46
  - 3.2.1 Eigenvalue Analysis ..................................... 46
  - 3.2.2 Orthogonality ............................................. 46
  - 3.2.3 Reduced Order Modal Model ................................ 47
  - 3.2.4 Vibration Response ...................................... 48
- 3.3 Modal Analysis of Beams ........................................... 53
  - 3.3.1 Natural Frequencies and Eigenmodes ......................... 53
  - 3.3.2 Correspondence of the Modal Analyses for Multi-dof Systems and Continua .................. 53
  - 3.3.3 Reduced Modal Models .................................... 55
  - 3.3.4 Modal Eccentricity ....................................... 57
- 3.4 Physical Models from Reduced Modal Models ......................... 60
  - 3.4.1 Modal Mass ................................................. 60
  - 3.4.2 Equivalent Mass Method ................................... 62
- 3.5 Approximation of Natural Frequencies .............................. 63
  - 3.5.1 Rayleigh’s Method ........................................ 63
  - 3.5.2 Method Using Influence Coefficients ......................... 65
  - 3.5.3 Dunkerley’s Formula ....................................... 67
  - 3.5.4 Iterative Method (Power Method) [B4] ....................... 68
  - 3.5.5 Stiffness Matrix Method ..................................... 70
  - 3.5.6 Transfer Matrix Method ..................................... 73

### 4 Mode Synthesis and Quasi-modal Method .............................. 79

- 4.1 Mode Synthesis Models ............................................ 79
  - 4.1.1 Why Mode Synthesis? ...................................... 79
  - 4.1.2 Guyan Reduction Method .................................... 80
  - 4.1.3 Mode Synthesis Models ................................... 84
- 4.2 Quasi-modal Models ................................................ 90
  - 4.2.1 Principle of the Quasi-modal Model ......................... 90
  - 4.2.2 Examples of Quasi-modal Models ......................... 97
- 4.3 Plant Transfer Function ............................................ 99
Chapter 5: Unbalance and Balancing

5.1 Unbalance in a Rigid Rotor

5.1.1 Static Unbalance and Dynamic Unbalance

5.1.2 Static Unbalance and Couple Unbalance

5.1.3 Adverse Effects of Unbalance Vibration

5.1.4 Residual Permissible Unbalance in a Rigid Rotor

5.2 Field Single-Plane Balancing (Modal Balancing)

5.2.1 Relationships among Rotational Pulse, Unbalance and Vibration Vector

5.2.2 Linear Relationship

5.2.3 Identifying the Influence Coefficient

5.2.4 Correction Mass

5.3 Balancing by the Influence Coefficient Method

5.4 Modal Balancing

5.5 n-Plane Balancing or (n + 2)-Plane Balancing?

5.5.1 Comparison

5.5.2 Number of Correction Planes Needed for Universal Balancing

5.5.3 What Is the “2” in the (n + 2)-Plane Method?

5.6 Balancing of a Rotor Supported by Magnetic Bearings

5.6.1 Balancing by Feed-Forward (FF) Excitation

5.6.2 Case Study: Centrifugal Compressor Supported by AMBs [VB245]

5.7 Balancing without Rotational Pulses

5.7.1 Four Run Method

5.7.2 Balancing by Placing a Trial Mass at a Regular Phase Pitch

5.8 Solution of Two-Plane Balancing

5.8.1 Principle of Calculation

5.8.2 In-Phase and Out-of-Phase Balancing

Chapter 6: Gyroscopic Effect on Rotor Vibrations

6.1 Rotodynamics

6.2 Gyroscopic Moment and the Motion of a Top

6.2.1 Gyroscopic Moment

6.2.2 Equation of Motion of a Top and Whirling Solution

6.3 Natural Vibration of a Rotor System

6.3.1 Natural Frequency of Whirling

6.3.2 Influence of the Gyroscopic Factor

6.3.3 Calculation of the Natural Frequency of Whirling in Multi-dof Rotor System
6.4 Unbalance Vibration and Resonance ........................ 163
6.4.1 Condition for Unbalance Resonance and Critical Speed ........ 163
6.4.2 Resonance Curves for Unbalance Vibration .......... 165
6.4.3 Calculation of Critical Speed of a Multi-dof Rotor System .......... 167
6.5 Vibration and Resonance with Base Excitation .......... 168
6.5.1 Resonance Conditions ................................ 168
6.5.2 Forced Vibriational Solution for Base Excitation .... 170
6.5.3 Resonance Curves and Whirling Trajectories ........ 172
6.5.4 Case Study: Aseismic Evaluation of a High-Speed Rotor .......... 173
6.6 Ball Passing Vibration and Resonance Due to Ball Bearing Defects .................. 176
6.6.1 Ball Bearing Specifications .................. 176
6.6.2 Excitation by a Recess on Outer Race ............ 176
6.6.3 Excitation by a Recess on Inner Race ............. 177
6.6.4 Resonance Conditions .................. 178
6.6.5 Case Study: Hard Disk Drive (HDD) [VB218] ........ 178
7 Approximate Evaluation for Eigenvalues of Rotor-Bearing Systems ........ 181
7.1 Equation of Motion for a Single-Degree-of-Freedom Rotor System ........ 181
7.2 Vibration Characteristics of a Symmetrically Supported Rotor System .... 183
7.2.1 Natural Frequencies of a Conservative System .... 184
7.2.2 Effects of Non-conservative System Parameters .... 185
7.2.3 Parameter Survey ................................ 187
7.3 Natural Frequencies of a Rotor Supported by Anisotropic Bearings ........ 189
7.3.1 Natural Frequency of a Conservative System .... 189
7.3.2 Elliptical Whirling of a Conservative System .... 190
7.3.3 Influence of Gyroscopic Effect .................. 192
7.3.4 Shape of Elliptical Whirling Orbit ............. 193
7.3.5 Effects of Non-conservative Parameters .......... 195
7.3.6 Parameter Survey ................................ 197
7.4 Vibration Characteristics of a Jeffcott Rotor .......... 199
7.4.1 Equation of Motion ................................ 199
7.4.2 Vibration Characteristics .................. 200
7.4.3 Real Mode Analysis .................. 202
7.4.4 Complex Mode Analysis ............ 204
7.5 Analysis of Characteristics of Unbalance Vibration ........ 205
7.5.1 Equation of Motion .................. 205

7.6 Analysis of Characteristics of Unbalance Vibration ........ 205
7.6.1 Equation of Motion .................. 205
7.5.2 Unbalance Vibration of an Isotropically Supported Rotor System ................................................. 205
7.5.3 Unbalance Vibration of a Rotor Supported by Anisotropic Bearings ........................................... 206
7.6 Case Study: Vibrations of a Flexible Rotor with Cylindrical Bearings ............................................. 208
7.6.1 Critical Speed Map .............................................. 208
7.6.2 Calculation of Complex Eigenvalues and Q-Values .......................................................... 209
7.6.3 Root Loci .......................................................... 210
7.6.4 Resonance Curves for Unbalance Vibration ................................................................. 211

8 Rotor System Evaluation Using Open-Loop Characteristics ....................................................... 213
8.1 Open-Loop Analysis of a Single-dof System .............................................................................. 213
8.1.1 Open-Loop Frequency Response of a Single-dof System .................................................. 213
8.1.2 Measurement of Open-Loop Frequency Response .......................................................... 221
8.2 Modal Open-Loop Frequency Response ................................................................................. 222
8.2.1 Modal Model .................................................................................................................. 222
8.2.2 Modal Open-Loop Frequency Response ........................................................................... 224
8.3 Open-Loop Frequency Response of a Jeffcott Rotor ............................................................... 228
8.3.1 Series Coupling and Phase Lead Function ........................................................................ 228
8.3.2 Open-Loop Frequency Response ................................................................................... 229
8.3.3 Gain Cross-Over Frequency and Phase Margin ............................................................. 230
8.3.4 Precision of Approximate Solutions ............................................................................... 232
8.3.5 Optimal Damping ......................................................................................................... 234
8.3.6 Frequency Response ..................................................................................................... 237

9 Bridge Between Inertial and Rotational Coordinate Systems .................................................... 241
9.1 Vibration Waveforms (Displacement and Stress Caused by Strain) ............................................. 241
9.2 Natural Frequencies ............................................................................................................. 243
9.3 Resonance Conditions ....................................................................................................... 245
9.4 Representation of Equation of Motion ................................................................................... 246
9.4.1 Gyroscopic Moment and Coriolis Force ........................................................................ 246
9.4.2 Case Study: Multi-blade Fan (Sirocco Fan) [VB55] ......................................................... 248

10 Vibration Analysis of Blade and Impeller Systems ..................................................................... 253
10.1 Natural Frequencies of Rotating Structure Systems .............................................................. 253
10.1.1 Natural Frequencies of a Thin Disk ........................................................................... 253
10.1.2 Natural Frequencies of Blades ..................................................................................... 257
10.1.3 Vibration Analysis of Cyclic Symmetry Structural Systems ............................................. 259
10.1.4 General Vibration Analysis of Blades and Impellers in a Rotational Coordinate System ...... 266
10.2 Vibration and Resonance of Blades and Impellers ........... 268
  10.2.1 Conditions for Blade-Shaft Coupled Vibration ....... 268
  10.2.2 Natural Vibration Modes of Blades and Blade Wheels ............................................. 269
  10.2.3 External Forces Acting on Blades and Impellers ........ 269
  10.2.4 Resonance Conditions of Blades .................. 270
  10.2.5 Criterion of Blade Resonance: Campbell Diagram ... 273
  10.2.6 Case Study: Resonance in Impeller Blades of Centrifugal Compressor [VB958] .................. 278
10.3 Blade/Impeller Vibrations Excited at Stationary Side ......... 281
  10.3.1 Difference in Excitation Methods and Resonance Conditions .............................................. 281
  10.3.2 Representation of Vibration of Blades and Impellers in an Inertial Coordinate System ....... 281
  10.3.3 Resonance Condition 1 ............................. 283
  10.3.4 Resonance Condition 2 ............................. 285

11 Stability Problems in Rotor Systems .......................... 287
  11.1 Unstable Vibration Due to Internal Damping of a Rotor .............................................. 287
    11.1.1 Equation of Motion ............................ 287
    11.1.2 Stability Condition ............................. 289
    11.1.3 Stability Analysis ............................. 290
  11.2 Unstable Vibration of an Asymmetric Rotor System ........... 293
    11.2.1 Equation of Motion ............................ 293
    11.2.2 Overview of Vibration in an Asymmetric Rotating Shaft .............................................. 295
    11.2.3 Simulation of Vibration of Asymmetric Rotor .......... 303
  11.3 Vibration Due to Thermal-Bow by Contact Friction ........... 306
    11.3.1 Thermal-Bow ................................. 306
    11.3.2 Thermal-Bow Model ............................ 307
    11.3.3 Stability Analysis ............................. 309
    11.3.4 Physical Interpretation of Stability .............. 310
    11.3.5 Simulation of Thermal-Bow Induced Vibration ....... 312
  11.4 Thermal-Bow Induced Vibration of an Active Magnetic Bearing Equipped Rotor .................. 314
    11.4.1 Thermal-Bow Model ............................ 314
    11.4.2 Stability Analysis ............................. 315
    11.4.3 Physical Interpretation of Stability .............. 317
    11.4.4 Simulation of Thermal Bow Induced Vibrations ... 318

12 Rotor Vibration Analysis Program: MyROT .................. 321
  12.1 Data on Rotor Systems ............................. 321
    12.1.1 Rotor Drawing and Discretization .................. 321
    12.1.2 Data Organization of a Rotor System .............. 323
Vibrations of Rotating Machinery
Volume 1. Basic Rotordynamics: Introduction to Practical Vibration Analysis
Matsushita, O.; Tanaka, M.; Kanki, H.; Kobayashi, M.; Keogh, P.
2017, XIII, 360 p. 318 illus., 203 illus. in color., Hardcover
ISBN: 978-4-431-55455-4