Preface

The evolution of the state of many systems modeled by linear partial differential equations (PDEs) or linear delay differential equations can be described by operator semigroups. The state of such a system is an element in an infinite-dimensional normed space, whence the name “infinite-dimensional linear system”.

The study of operator semigroups is a mature area of functional analysis, which is still very active. The study of observation and control operators for such semigroups is relatively more recent. These operators are needed to model the interaction of a system with the surrounding world via outputs or inputs. The main topics of interest about observation and control operators are admissibility, observability, controllability, stabilizability and detectability. Observation and control operators are an essential ingredient of well-posed linear systems (or more generally system nodes). In this book we deal only with admissibility, observability and controllability. We deal only with operator semigroups acting on Hilbert spaces.

This book is meant to be an elementary introduction into the topics mentioned above. By “elementary” we mean that we assume no prior knowledge of finite-dimensional control theory, and no prior knowledge of operator semigroups or of unbounded operators. We introduce everything needed from these areas. We do assume that the reader has a basic understanding of bounded operators on Hilbert spaces, differential equations, Fourier and Laplace transforms, distributions and Sobolev spaces on n-dimensional domains. Much of the background needed in these areas is summarized in the appendices, often with proofs.

Another meaning of “elementary” is that we only cover results, for which we can provide complete proofs. The abstract results are supported by a large number of examples coming from PDEs, worked out in detail. We mention some of the more advanced results, which require advanced tools from functional analysis or PDEs, in our bibliographic comments. One of the glaring omissions of the book is that we do not cover anything based on microlocal analysis.

The concepts of controllability and observability have been set at the center of control theory by the work of R. Kalman in the 1960s and soon they have been generalized to the infinite-dimensional context. Among the early contributors we mention D.L. Russell, H. Fattorini, T. Seidman, A.V. Balakrishnan, R. Triggiani, W. Littman and J.-L. Lions. The latter gave the field an enormous impact with his book [156], which is still a main source of inspiration for many researchers.

Unlike in finite-dimensional control theory, for infinite-dimensional systems there are many different (and not equivalent) concepts of controllability and observability. The strongest concepts are called exact controllability and exact observability, respectively. Exact controllability in time $\tau > 0$ means that any final state can be reached, starting from the initial state zero, by a suitable input signal on the time interval $[0, \tau]$. The dual concept of exact observability in time τ means that if the input is zero, the initial state can be recovered in a continuous way from the output signal on the time interval $[0, \tau]$. We shall establish the exact observability or exact controllability of various (classes of) systems using a variety of techniques. We shall also discuss other concepts of controllability and observability.

Exact controllability is important because it guarantees stabilizability and the existence of a linear quadratic optimal control. Dually, exact observability guarantees the existence of an exponentially converging state estimator and the existence of a linear quadratic optimal filter. Moreover, exact (or final state) observability is useful in identification problems. To include these topics into this book we would have needed at least double the space and ten times the time, and we gave up on them. There are excellent books dealing with these subjects, such as (in alphabetical order) Banks and Kunisch [13], Bensoussan et al. [17], Curtain and Zwart [39], Luo, Guo and Morgul [163] and Staffans [209].

Researchers in the area of observability and controllability tend to belong to either the abstract functional analysis school or to the PDE school. This is true also for the authors, as MT feels more at home with PDEs and GW with functional analysis. By our collaboration we have attempted to combine these two approaches. We believe that such a collaboration is essential for an efficient approach to the subject. More precisely, the functional analytic methods are important to formulate in a precise way the main concepts and to investigate their interconnections. When we try to apply these concepts and results to systems governed by PDEs, we generally have to face new difficulties. To solve these difficulties, quite refined techniques of mathematical analysis might be necessary. In this book the main tools to tackle concrete PDE systems are multipliers, Carleman estimates and non-harmonic Fourier analysis, but results from even more sophisticated fields of mathematics (microlocal analysis, differential geometry, analytic number theory) have been used in the literature.

While we were working on this book, Birgit Jacob from the University of Delft (The Netherlands) with Hans Zwart from the University of Twente (The Netherlands) have achieved an important breakthrough on exact observability for normal semigroups. Birgit has communicated to us their results, so that we could include them (without proof) in the bibliographic notes on Chapter 6.

We are grateful to Emmanuel Humbert from the University of Nancy (France) for accepting to contribute to an appendix on differential calculus. The material in Chapter 14 is to a great extent his work.

Bernhard Haak from the University of Bordeaux has contributed significantly to Section 5.6. Moreover, Proposition 5.4.7 is due to him.

Large parts of the manuscript have been read by our colleagues Karim Ramdani, Takéo Takahashi (both from Nancy) and Xiaowei Zhao (from London) who made many suggestions for improvements. The two figures in Chapter 7, the figure in Chapter 11 and the first figure in Chapter 15 were drawn by Karim Ramdani. Jorge San Martin (from Santiago de Chile) contributed in an important manner to the calculations in Section 15.1. Luc Miller (from Paris) made useful comments on Chapter 6. Sorin Micu (from Craiova) and Jean-Pierre Raymond (from Toulouse) made very useful remarks on Sections 9.2 and 15.2, respectively. Gérald Tenenbaum and François Chargois (both from Nancy) suggested us corrections and simplifications in Sections 8.4 and 14.2. Birgit Jacob, in addition to her help described earlier, has made useful bibliographic comments on Chapters 5 and 6. Other valuable bibliographic comments have been sent to us by Jonathan Partington (from Leeds). Qingchang Zhong (from Liverpool) pointed out some small mistakes and typos. We thank them all for their patience and help.

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