Preface

This book is designed to present some recent results on some nonlinear parabolic-hyperbolic coupled systems arising from physics, mechanics and material science such as the compressible Navier-Stokes equations, thermo(visco)elastic systems and elastic systems. Some of the content of this book is based on research carried out by the author and his collaborators in recent years. Most of it has been previously published only in original papers, and some of the material has never been published until now. Therefore, the author hopes that the book will benefit both the interested beginner in the field and the expert.

All the models under consideration in Chapters 2–10 are built on nonlinear evolution equations that are parabolic-hyperbolic coupled systems of partial differential equations with time \( t \) as one of the independent variables. This type of partial differential equations arises not only in many fields of mathematics, but also in other branches of science such as physics, mechanics and materials science, etc. For example, some models studied in this book, such as the compressible Navier-Stokes equations (a 1D heat conductive viscous real gas and a polytropic ideal gas) from fluid mechanics, and thermo(visco)elastic systems from materials science, are typical examples of nonlinear evolutionary equations.

It is well known that the properties of solutions to nonlinear parabolic-hyperbolic coupled systems are very different from those of parabolic or hyperbolic equations. Since the 1970s, more and more mathematicians have begun to focus their interests on the study of local well-posedness, global well-posedness and blow-up of solutions in a finite time. Local well-posedness means that, for any given initial datum, a solution exists locally in time, and if it exists locally in time, it is unique and stable in some sense in the considered class. Generally speaking, we have two powerful tools to derive the local existence of solutions to a wide class of nonlinear evolutionary equations, i.e., the contraction mapping theorem and the Leray-Schauder fixed point theorem. Once a local solution in some sense has been established, we may talk about the global well-posedness of solutions, i.e., the global-in-time existence, uniqueness and stability of global solutions. Since the 1960s, many methods of studying global well-posedness have been developed, among which are two powerful tools to derive the global existence of solutions; one is continuation of local solutions, the other is the global iteration method.

In the 1980s, more interest was focused on the global existence of “small solutions”. However, knowledge about the global existence of a “small solution” is usually far from being enough for physical and mechanical problems. Thus we have to look for global
solutions with arbitrary (not necessarily small) initial data. It turns out that an important step is to derive uniform a priori estimates on the solutions by using the special constitutive relations of the equations under consideration. Once global existence and uniqueness have been established, then the main interest should be focused on topics related to the asymptotic behavior of solutions, multiplicity of equilibria, convergence to an equilibrium, dynamical systems such as absorbing sets, the maximal compact attractor, etc. The study of asymptotic behavior of solutions can be divided into two categories. The first category comprises investigations of asymptotic behavior of the global solution for any given initial datum. The second category comprises investigations of asymptotic behavior of all solutions when the initial data vary in any bounded set. There are essential differences between these two categories. The first category deals with only one orbit starting from the datum in the phase space, while the second category deals with a family of orbits starting from any bounded set in the phase space.

For the basic theories of infinite-dimensional dynamical systems, we refer readers to the works by Babin [16], Babin and Vishik [17, 18], Ball [22, 23], Bernard and Wang [38], Chepyzhov, Gatti, Grasselli, Miranville and Pata [56], Chepyzhov and Vishik [57], Constantin and Foias [63], Constantin, Foias and Temam [64], Dlotko [84], Eden and Kalantarov [90], Edendiev, Zelik and Miranville [92], Feireisl [97, 98, 100], Feireisl and Petzeltova [101, 102], Ghidaglia [117, 118], Hale and Perissinotto [136], Haraux [138], Hoff and Ziane [151], Ladyzhenskaya [207], Liu and Zheng [240], Lu, Wu and Zhong [242], Ma, Wang and Zhong [246], Miranville [265, 266], Miranville and Wang [267], Moise and Rosa [269], Moise, Rosa and Wang [270], Pata and Zelik [307], Robinson [362], Sell [369], Sell and You [370, 371], Temam [407], Vishik and Chepyzhov [413, 414], Wang [421], Wang, Zhong and Zhou [422], Wu and Zhong [429], Zhao and Zhou [445], Zheng [450], Zheng and Qin [451, 452], Zhong, Yang and Sun [457], and references therein.

There are 10 chapters in this book. Chapter 1 is a preliminary chapter in which we collect some basic results from nonlinear functional analysis, basic properties of Sobolev spaces, some differential and integral inequalities in analysis, the basic theory of semigroups of linear operators and the basic theory for global attractors. Some results in this chapter will be used in the subsequent chapters, other results, though not used in the subsequent chapters, will be very beneficial to the readers for further study.

The first topic studied in this book is compressible Navier-Stokes equations which describe the fluid motion of conservation of mass, momentum and energy. Chapters 2–5 are devoted to the study of this challenging topic. Chapter 2 will concern the global existence, asymptotic behavior of solutions and the existence of universal attractors for the compressible Navier-Stokes equations of a nonlinear 1D viscous and heat-conductive real gas. In Chapter 3, we shall establish the global existence, asymptotic behavior of solutions to initial boundary value problems and the Cauchy problem of the compressible Navier-Stokes equations of a 1D polytropic viscous and heat-conductive gas. In Chapter 4, we shall investigate the global existence, asymptotic behavior of solutions and the existence of maximal attractors for the compressible Navier-Stokes equations of a polytropic vis-
cous and heat-conductive gas in bounded annular domains in $\mathbb{R}^n$ ($n = 2, 3$). Chapter 5 will be concerned with the global existence and asymptotic behavior of solutions to a polytropic viscous and heat-conductive gas with cylinder symmetry in $\mathbb{R}^3$.

For the compressible Navier-Stokes equations, we consult the works by Duan, Yang and Zhu [87], Ducomet and Zlotnik [88], Feireisl and Petzeltova [103], Feireisl, Novotny and Petzeltova [104], Frid and Shelukhin [106], Fujita-Yashima and Benabdallah [110, 111], Fujita-Yashima, Padula and Novotny [112], Galdi [115], Hoff [142–146], Hoff and Serre [147], Hoff and Smoller [148], Hoff and Zarnowski [149], Hsiao and Luo [158], Huang, Matsumura and Xin [160], Itaya [161], Jiang [164–167, 169–171], Jiang and Zhang [174–177], Jiang and Zlotnik [178], Kanel [182], Kawashima [188, 189], Kawashima, Nishibata and Zhu [190], Kawashima and Nishida [191], Kawohl [192], Kazhikhov [193–195], LeFloch and Shelukhin [219], Lions [235], Matsumura [252], Matsumura and Nishida [253–257], Nagasawa [283–287], Novotny and Straškraba [301, 302], Okada and Kawashima [303], Padula [305], Qin [323, 325, 326], Qin and Hu [329], Qin, Huang and Ma [330], Qin and Kong [331], Qin, Ma, Cavalcanti and Andrade [335], Qin, Ma and Huang [336], Qin and Zhao [346], Valli and Zajaczkowski [412], and the references therein.

The second topic studied in this book is a 1D thermostatic system which describes the motion of conservation of mass, momentum and energy in the thermostatic media. Chapter 6 will be devoted to the study of global existence, asymptotic behavior and the existence of universal attractors for a 1D thermostatic model in materials science.

The third topic considered in this book is that of some viscoelastic models. In Chapter 10, we shall obtain the large-time behavior of energy of multi-dimensional nonhomogeneous anisotropic elastic system.

For the related (thermo)(visco)elastic models, we refer to Andrews [12], Andrews and Ball [13], Chen and Hoffmann [54], Coleman and Gurtin [62], Dafermos [69, 75, 76], Dafermos and Nohel [79, 80], Fabrizio and Lazzari [95], Giorgi and Naso [121], Greenberg and MacCamy [129], Guo and Zhu [132], Kim [197], Lagnese [209], Liu and Zheng [239, 240], Niezgódka and Sprekel [293], Niezgódka, Zheng and Sprekel [294], Qin, Ma and Huang [336], Racke and Zheng [355], Renardy, Hrusa and Nohel [361], Shen and Zheng [373], Shen, Zheng and Zhu [376], Shibata [377], Sprekel and Zheng [390, 391], Sprekel, Zheng and Zhu [392], Watson [424], Zheng [447, 448, 450], Zheng and Shan [452, 453, 454], Zhu [460], and the references therein.

The fourth topic under consideration is an investigation of a classical 1D thermoelastic model. Such a model describes the elastic and the thermal behavior of elastic, heat conductive media, in particular the reciprocal actions between elastic stresses and temperature differences. The classical thermoelastic system is such a thermoelastic model that the elastic part is the usual second-order one in the space variable and the heat flux obeys Fourier’s law, which means that the heat flux is proportional to the temperature gradient. In Chapter 7, we shall establish the global existence and exponential stability of solutions to a 1D classical thermoelastic system of equations with a thermal memory. In
Chapter 9, we shall study the blowup phenomena of solutions to the Cauchy problem of a 1D non-autonomous classical thermoelastic system.

There is much literature on classical thermoelastic model; we refer the readers to Burns, Liu and Zheng [46], Dafermos [67], Dafermos and Hsiao [78], Hale and Perissinetto [136], Hansen [137], Hoffmann and Zochowski [153], Hrusa and Messaoudi [155], Hrusa and Tarabek [156], Jiang, Muñoz Rivera and Racke [172], Jiang and Racke [173], Kim [198], Kirane and Kouachi and Tatar [199], Kirane and Tatar [200], Lebeau and Zuazua [216], Liu and Zheng [238, 240], Messaoudi [260], Muñoz Rivera [274, 275], Muñoz Rivera and Barreto [277], Muñoz Rivera and Oliveira [278], Muñoz Rivera and Qin [279], Qin [315], Qin and Muñoz Rivera [341], Racke [348], Racke and Zheng [355], Slemord [378], Zheng [450], and the references therein.

Recently, Green and Naghdi [127, 128] re-examined the classical thermoelastic models and introduced the so-called models of thermoelasticity of types II and III for which the heat fluxes are different from Fourier’s law. Chapter 8 will concern the global existence and exponential stability of solutions to the 1D thermoelastic equations of hyperbolic type, which is in fact a 1D thermoelastic system of type II with a thermal memory.

We consult the works by Messaoudi [261], Racke [350, 351], Racke and Wang [354] for thermoelastic models with second sound, which means that the heat flux is given by Cattaneo’s law (i.e., the heat flux \( q \) satisfies \( \tau q_t + q + \kappa \nabla \theta = 0 \) with \( \tau > 0, \kappa > 0 \) constants), instead of Fourier’s law of the classical thermoelastic models in which \( \tau = 0 \). For the thermoelastic models of type II, we refer to the works by Green and Naghdi [127, 128], Gurtin and Pipkin [133], and Qin and Muñoz Rivera [340], and the references therein. For the thermoelastic models of type III, we refer to the works by Green and Naghdi [127, 128], Quintanilla and Racke [347], Reissig and Wang [360], and Zhang and Zuazua [444], and the references therein.

I sincerely hope that readers will learn the main ideas and essence of the basic theories and methods in deriving global well-posedness, asymptotic behavior and existence of global (universal) attractors for the models under consideration in this book. Also I hope that readers will be stimulated by some ideas from this book and undertake further study and research after having read the related references.

I appreciate my former Ph.D. advisor, Professor Songmu Zheng from Fudan University for his constant encouragement, useful advice and great support and help. Special thanks go to Professor Bert-Wolfgang Schulze for his interest in my research and for acting as the initiator for publication of this book. I would like also to acknowledge the NNSF of China for its support. Currently, this book project is being supported by the National Jie Chu Qing Nian Grant (No. 10225102), Grant (No. 10571024) of the NNSF of China, by a grant from the Institute of Mathematical Sciences, The Chinese University of Hong Kong, and by Grant (No. 0412000100) of Prominent Youth from Henan Province of China. Also I hope to take this opportunity to thank my teachers Professors Daqian Li (Ta-tsien Li) (one of my former advisors for the Master Degree), Jiaxing Hong, Weixi Shen, Tiehu Qin, Shuxing Chen, Yongji Tan, Jin Cheng from Fudan University. I appreciate the help from Professors Boling Guo, Ling Hsiao, Zhouping Xin, Tong Yang,
Preface xv

Yi Zhou, Hua Chen, Jingxue Yin, Song Jiang, Ping Zhang, Changxing Miao, Zheng-an Yao, Junning Zhao, Weike Wang, Huijiang Zhao, Changjiang Zhu, Zhong Tan, Jinghua Wang, Guowang Chen, Mingxin Wang, Sining Zheng, Chengkui Zhong, Xiaoping Yang, Huicheng Yin, Daoyuan Fang, Dexing Kong, Ting Wei, Yachun Li, Shu Wang, Xiangao Liu, Yaguang Wang, Yongqian Zhang, Wenyi Chen, Yaping Wu, Quansen Jiu, Hailiang Li, Xi-nan Ma, Feimin Huang, Xiaozhou Yang, Lixin Tian, Yong Zhou, Hao Wu, Zhenhua Guo, Yeping Li, Feng Xie, Jing Wang, Chunqing Xie and Ting Zhang for their constant help. Also I would like to thank Professors Herbert Amann, Michel Chipot from Switzerland, Professors Guiqiang Chen, Irena Lasiecka, Chun Liu, Hailiang Liu, Tao Luo and Dening Li from the USA, Professor Hugo Beirao da Veiga, Maurizio Grasselli, Cecilia Cavaterra from Italy, Professors Jaime E. Muñoz Rivera, Abimael F. Dourado Loula, Alexandre L. Madureira, Frédéric G. Christian Valentin, Tufa Ma, M.M. Cavalcanti, D. Andrade from Brazil, Professors Tzon Tzer Lü, Jyh-Hao Lee, Chun-Kong Law, Ngai-Ching Wong, John Men-Kai Hong and Kin-Ming Hui from Chinese Taiwan, Professors Reinhard Racke, Michael Reissig, Jürgen Sprekels, Pavel Krejci and Peicheng Zhu from Germany, and Professors Alain Miranville, Yuejun Peng, Bopeng Rao from France for their constant and great help.

Last but not least, I want to take this opportunity to express my deepest thanks to my parents, Zhenrong Qin and Xilan Xia, and to my elder brother Yuxing Qin and sisters Yuyuan Qin and Yuzhou Qin for their constant concern, encouragement and great help in all aspects of my life. My deepest gratitude goes to my wife, Yu Yin and my son, Jia Qin, for their constant advice and support in my career.

Professor Yuming Qin
Department of Applied Mathematics
College of Science
Donghua University
Shanghai 201620, China
E-mails: yuming@dhu.edu.cn
yuming.qin@hotmail.com

and

Visiting Professor Yuming Qin
The Institute of Mathematical Sciences
The Chinese University of Hong Kong
Shatin, N.T., Hong Kong, China
E-mail: ymqin@ims.cuhk.edu.hk
Nonlinear Parabolic-Hyperbolic Coupled Systems and Their Attractors
Qin, Y.
2008, XV, 468 p., Hardcover
ISBN: 978-3-7643-8813-3
A product of Birkhäuser Basel