Preface

The present book deals with canonical factorization problems for different classes of matrix and operator functions. Such problems appear in various areas of mathematics and its applications. The functions we consider have in common that they appear in the state space form or can be represented in such a form. The main results are all expressed in terms of the matrices or operators appearing in the state space representation. This includes necessary and sufficient conditions for canonical factorizations to exist and explicit formulas for the corresponding factors. Also, in the applications the entries in the state space representation play a crucial role.

The theory developed in the book is based on a geometric approach which has its origins in different fields. One of the initial steps can be found in mathematical systems theory and electrical network theory, where a cascade decomposition of an input-output system or a network is related to a factorization of the associated transfer function.

Canonical factorization has a long and interesting history which starts in the theory of convolution equations. Solving Wiener-Hopf integral equations is closely related to canonical factorization. The problem of canonical factorization also appears in other branches of applied analysis and in mathematical systems theory, in $H_\infty$-control theory in particular.

The first book devoted to the state space factorization theory was published in 1979 as the monograph “Minimal factorization of matrix and operator functions,” Operator Theory: Advances and Applications 1, Birkhäuser Verlag, written by the first three authors. Some of the factorization results published in the 1979 book appeared there in print for the first time.

The present book is the second book written by the four of us in which the state space factorization method is systematically used and developed further. In the earlier book [20], published in 2008, the emphasis is on non-canonical factorizations and degree 1 factorizations, in particular. In the present book we concentrate on canonical factorizations. Together both books present a rich and far reaching update of the 1979 monograph [11].

In the present book the emphasis is on canonical factorization and symmetric factorization with applications to different classes of convolution equations. For
the latter we have in mind the transport equation, singular integral equations, equations with symbols analytic in a strip, and equations involving factorization of non-proper rational matrix functions. A large part of the book will deal with factorization of matrix functions satisfying various symmetries. A main theme will be the effect of these symmetries on factorization and how the symmetries can be used in effective ways to get state space formulas for the factors. Applications to $H_\infty$-control theory, which have been developed in the 1980s and 1990s, will also be included. The text is largely self-contained, and will be of interest to experts and students in mathematics, sciences and engineering.

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The authors

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Postscript

On Monday October 12, 2009, Israel Gohberg, the second author of this book, passed away at the age of 81. At that time the preparation of the book was in a final phase and only some minor work had to be done. Israel Gohberg was one of the initiators using state space methods in solving problems appearing in various branches of mathematical analysis and its applications. His fundamental insights and inspiring leadership have been driving forces in our joint work.
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